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A survey of models for determining optimal audit strategies

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ABSTRACT

The problem studied in this survey is how to optimize the allocation of audit resources over an auditee population with respect to available population statistics. The auditees are assumed to optimize their expected utility based on information about the audit strategy. This survey is limited to models where the auditee can vary the fraud amount along a continuous scale.

If the auditor is not able or willing to announce the audit strategy, a Nash equilibrium can be derived in which the auditor and auditee correctly anticipate each other's strategies. If the auditor announces the audit strategy in advance, the problem is formulated as a sequential game with perfect information which is solved as an optimization problem.

Early models in the literature resulted in unrealistically high degrees of fraud. Later models have incorporated a split into one group of inherently honest auditees and another group of potentially dishonest auditees. The fraction of inherently honest auditees is exogenous.

In this paper, the four combinations of non-announcing/pre-announcing the strategy and all potentially dishonest/some inherently honest auditees are studied. For the case of pre-announcing the strategy with some inherently honest auditees, two new solution methods are presented.

Two main conclusions are as follows. First, models with some inherently honest auditees have greater external validity. Second, when a pre-announced strategy is feasible, as it often is with tax and benefit audits, the pre-announced strategy is preferred by the auditor over the non-announced strategy.

1. Introduction

This paper deals with the use of statistical and mathematical models for determining audit strategies. Such models have primarily been developed for tax auditing, where large populations of auditees are available making it possible to obtain empirical statistics. Large populations may also be available in auditing of social benefits, whereas they are infrequently found in financial statement auditing.

Audits are performed in order to discover and correct fraud and unintentional errors. They may also have a deterrent effect if an economic penalty is imposed when fraud is detected or if the auditee experiences a social penalty upon detection, "guilt and shame".

The term audit strategy is used with some different meanings. It often implies the guidelines which are used in the design of an audit plan for a specific project (www.accountingtools.com). It can also be used for the plan the auditee has for answering questions from an auditor (www.businessdictionary.com). In this paper, audit strategy refers to the allocation of audit resources in response to statistical data about the auditee population. Thus, some auditees will be subject to a higher audit probability (also denoted *audit rate*) whereas other auditees will run a smaller likelihood of being audited.

The average audit probability over an auditee population, denoted *audit density*, may be set by an external budget constraint. Alternatively, it can be determined by an economic model that minimizes the total cost, i.e., the expected fraud loss plus the expected audit cost.

The base-line audit strategy is random auditing, where all auditees are subject to the same audit probability. A natural improvement in comparison to random auditing is a segmentation of the auditee population based on different anticipated degrees of fraud in different auditee segments. This is denoted a *segmentation strategy*.

A more complex audit strategy is obtained when the audit probability is modelled as a function of a *control variable* that is used by the auditee to determine the amount of fraud and which can be observed by the auditor. This survey is limited to models where the control variable is continuous, for instance the declared income of a taxpayer. Other models exist where the auditee makes a choice between discrete alternatives. Several such examples are found in the accounting literature, for instance Shibano (1990), Matsumura and Tucker (1992), Bloomfield (1995), Caplan (1999) and Patterson and Noel (2003).

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Another example is the insurance fraud model developed by Dionne, Giuliano, and Picard (2009) where the auditee makes a choice between no fraud and a fixed amount of fraud.

If the audit probability function is communicated by the auditor, the auditee will have the opportunity to incorporate this information when choosing the amount of fraud to maximize his/her expected pay-off, i.e., expected fraud revenue less expected penalty cost. This type of audit strategy is denoted *information strategy* since it is based on information communicated from the auditor to the auditee.

The two strategy types can be combined, such that a separate audit probability function is applied for each auditee segment.

The information strategy assumes that auditees adapt their behaviour to audit strategy information. This assumption is supported by several studies with full-scale experiments on taxpayers or users of social benefits. Slemrod, Blumenthal, and Christian (2001), Hasseldine, Hite, James, and Toumi (2007) and Kleven, Knudsen, Kreiner, and Saez (2011) studied the effect of general audit threats on declared income, whereas Engström and Hesselius (2007) studied the effect of audit threats on claims of social benefits. Appelgren (2008) studied specifically the effect of an information strategy on taxpayers, i.e., where the threat was directed towards those who declared the lowest income.

The primary purpose of this paper is to present a survey of different applications of information strategies. A second purpose is to provide a solution to a previously unsolved information strategy problem formulation.

The classical tax audit model was developed by Allingham and Sandmo (1972). Assuming risk-averse taxpayers and random auditing with a given audit probability, the optimum taxpayer fraud amount is determined. With risk-neutral auditees, the optimum fraud amount will either be zero or equal to taxpayer income. The model can be used to determine the audit effort that minimizes net expected total cost, i.e., tax loss plus audit costs less penalty revenues.

The models developed by Allingham and Sandmo (1972) and Reinganum and Wilde (1985) and Reinganum & Wilde, 1986) treat all taxpayers as rational utility-maximizing actors and obtain therefore higher degrees of fraud than what is observed empirically. Erard and Feinstein (1994a) and others explain this difference with varying social factors such as moral principles, stigma, guilt, shame etc., i.e., different forms of social penalty.

The limitation of the early models was overcome by Erard and Feinstein (1994b). Their model splits the taxpayer population into two groups, one inherently honest group that always declares the true income and one potentially dishonest group that might commit fraud as part of their utility-maximizing strategy. The share of inherently honest taxpayers is exogenous.

In this paper, we will distinguish between models where all auditees are potentially dishonest and models with a portion of honest auditees who never behave fraudulently.

Another distinction is whether the audit probability function is communicated to the auditees or not. Reinganum and Wilde (1986) and Erard and Feinstein (1994b) assume that the auditor is not able or willing to inform the auditees. The problem is then formulated as a game with a Nash equilibrium in which the auditor and auditee correctly anticipate each other's strategies.

Alternatively, the auditor provides the auditees with full information regarding the audit strategy. This is often realistic for tax and benefit auditing, but highly unrealistic for financial statement auditing. The problem is then formulated as a sequential game with perfect information, which mathematically has the form of an optimization problem. Such sequential game models have been studied for tax auditing by Reinganum and Wilde (1985) and Sanchez and Sobel (1993), for financial statement auditing by Morton (1993) and for benefit auditing by Appelgren (2017). In this paper, a sequential game with perfect information is referred to simply as a sequential game.

We thus have four different cases which are studied below.

	Pre-announced strategy	Non-announced strategy
All potentially dishonest	AP: Morton, 1993	AN: Reinganum & Wilde, 1986
Some inherently honest	HP: Not treated previously	HN: Erard & Feinstein, 1994a

Depending on the situation, the audit problem can be formulated either as the minimization of the fraud loss subject to an audit budget constraint or as the minimization of the total cost, i.e., fraud loss plus audit cost. The case with a budget constraint can be converted to the total cost case with the replacement of the unit audit cost with a shadow price, i.e., a Lagrange multiplier for the constraint. The shadow price is selected such that the budget constraint is satisfied.

The analysis of the four cases above fundamentally employs the assumptions that auditees are risk-neutral, that penalties are proportional to the fraud amount and, for tax auditing, that tax is proportional to income. It is also assumed that all fraud and all errors (uninten-tional misstatements) are discovered in an audit.

In the accounting literature, it is quite common to study models where the pay-off of the auditor is different from the pay-off of the organisation he/she represents. See for instance Shibano (1990), Matsumura and Tucker (1992), Bloomfield (1995), Caplan (1999) and Patterson and Noel (2003). One such example of tax auditing is the paper by Di Porto, Persico, and Sahuguet (2013). In the following, it is assumed that the pay-off of the auditor is identical with the pay-off of the organisation.

The second purpose of this paper is to demonstrate two solution methods for the previously unsolved HP case, i.e., the case with some honest auditees and pre-announced strategy.

The paper is organised as follows. Section 2 describes empirical research on the effect of audit information on auditee behaviour. Section 3 is devoted to the cases of audit strategy problems where the strategy is not announced in advance, whereas section 4 contains the cases with pre-announced strategy. In section 5, numerical examples concerning tax, benefit and accounting fraud are presented. Section 6 contains discussion and conclusions.

2. Empirical research on audit strategy information

Slemrod et al. (2001) studied tax compliance in Minnesota through a large experiment on 47,000 taxpayers using three different letters with varying degrees of audit threat. The strongest letter, sent to 1724 taxpayers, stated that their tax returns, both state and federal, would be closely examined. The difference between the treatment group and a control group in reported taxable income increase from the previous year was used as a non-compliance measure. The effect of the strong letter was positive although not statistically significant for low- and middle-income taxpayers. Surprisingly, the effect was reversed for high income taxpayers.

Hasseldine et al. (2007) studied the effect of five variations of warning letters to approximately 5000 sole proprietors in the UK. The weakest letter contained “We are here to give you advice and support if you need it” whereas the strongest letter announced “Your tax return has been chosen for enquiry”. The authors studied the effect of the letters on the change in net profit from the previous year. They found that the three strongest letters had a statistically significant impact on net profit, especially for those taxpayers who did not use a tax advisor.

Sole proprietors with a turnover of less than 15,000 pounds sterling were allowed to file a simplified tax return. The authors studied the effect of warning letters on the percentage of auditees who reported a turnover exceeding this limit. For those who prepared their tax returns without external help, this percentage increased from 30% in the control group with no letter to 54% in the group with the strongest letter.

The conclusion was that audit information has a significant effect on auditee behaviour.

Engström and Hesselius (2007) studied overutilization of Temporary Parental Benefit (TPB) in Sweden. TPB is available to caregivers (parents or other persons) who lose income when taking care of sick children and is administered by the Swedish Social Insurance Agency.

The measurement was primarily based on the effect of information to the caregivers. After a reference period during which the claims of TPB were measured, a warning letter was distributed, after which a new measurement of claims was made during an audit period. The message of the warning letter was:" You have been selected for special review…". An information letter was also used in the study, describing the regulations concerning TPB.

The reduction of claims between the reference period and the audit period was used as a measure of fraud. In addition, audits were performed of the claims made during the audit period in order to measure remaining errors.

Letters were sent to three groups:

A: Both warning letter and information letter, approximately 29,000 persons, 1271 audits.

B: Information letter only, approximately 7000 persons, 339 audits.

C: Warning letter only, approximately 7000 persons, 356 audits.

The effect of the letters was a statistically significant reduction of claims in groups A and C amounting to 13–14%. For group B, a non-significant increase in claims was recorded, possibly because persons unfamiliar with TPB were informed of their rights.

Appelgren (2008) made an empirical test of the effect of information to taxpayers concerning different audit strategies. The test was carried out by the Swedish Tax Agency in 2003–2004 on approximately 900 sole proprietors. The primary objective was to investigate whether information to taxpayers about a near-optimal audit strategy reduces tax fraud compared to information about a more conventional audit strategy, i.e. pure random audits. Information concerning the use of tax advisors/paid preparers was not collected.

The test was conducted on sole proprietors without employees and with little or no income from employment. These persons were presumed to support themselves from their businesses. The trades included were craftsmen in the building industry, auto-repair shops and hairdressers. Those trades were selected by the Tax Agency as they were the largest groups of sole proprietorships.

In line with the results reported by Reinganum and Wilde (1986), Erard and Feinstein (1994b) and others, the optimal audit strategy for a homogeneous group of taxpayers is to concentrate audits on those who declare the lowest income. In the experiment, however, the total net cash flow of the household was instead used as the basis for audit selection. Net cash flow was defined as declared income after tax, adjusted for non-cash items like depreciation and allocation to tax allocation reserves, as well as for cash items not included in income such as loan repayment and new borrowing.

Three groups were studied, each consisting of around 300 firms:

A. Rational group: The members were informed that audits would concentrate on taxpayers who declare the lowest net cash flow.
B. Random group: The members were informed that taxpayers to be audited would be selected at random.
C. Control group: The members received no information.

The effect of the letters was measured as the income change between the years 2003 and 2002. The average increase was approximately 20% in group A, 11% in group B and 9% in group C. The effect of a letter warning of a random audit was thus quite small, whereas the effect of a letter warning of audits concentrating on low income auditees was statistically significant compared to the control group.

Kleven et al. (2011) studied tax compliance in Denmark. A main result was that evasion of third party reported income is small compared to substantial evasion on self-reported income. Other conclusions were that the marginal tax rate has little effect on evasion and that those who were audited the previous year reduced tax evasion in the following year.

Kleven et al. also studied the effect of audit threats, regrettably only on a group of employees, and not on self-employed persons. A sample of 25,000 persons received a warning letter with a threat of 50% or 100% probability of audit of their self-reported income. The authors found that a threat of 100% auditing had approximately double the impact compared to a threat of 50% auditing.

3. Non-announced strategy cases

3.1. The non-announced Nash equilibrium game model

In the non-announced Nash equilibrium game, one player (the auditor) has the strategy set $\boldsymbol{A}$ and the objective function C to be minimized. The other player (the auditees) has the strategy set $\boldsymbol{B}$ and the objective function U to be maximized. A Nash equilibrium $[\boldsymbol{a}^*,\boldsymbol{b}^*]$ has the properties.

$\boldsymbol{a}^*$ minimizes $C[\boldsymbol{A},\boldsymbol{b}^*]$ and.
$\boldsymbol{b}^*$ maximizes $U[\boldsymbol{a}^*,\boldsymbol{B}]$.

In words, each player is unable to improve his/her expected outcome compared to his/her equilibrium strategy as long as the other player plays his/her equilibrium strategy.

The strategy set $\boldsymbol{A}$ corresponds to the set of audit frequency functions $p_a(x)$ and the strategy set $\boldsymbol{B}$ to the set of declared control variable functions $x(y)$, i.e.,

$p_a^*(x)$ minimizes $C[p_a(x),x^*(y)]$ and
$x^*(y)$maximizes $U[p_a^*(x),x(y)]$

where

$x =$ control variable
$y =$ true value of the control variable

3.2. The AN case: all auditees potentially dishonest, non-announced strategy

In Reinganum and Wilde (1986), the authors presented a tax fraud model (the *RW model*) with a constant tax rate t and a penalty upon detection proportional to the fraud amount with a proportionality factor π. The tax agency maximizes net expected revenue with an audit cost $c(p)$ which depends on the audit probability p. Below, we assume that the audit cost is proportional to the audit probability. The results below correspond to Corollary 3 in the Reinganum & Wilde paper, although notation is slightly different.

Reinganum and Wilde assumed that the tax agency is not able or permitted to pre-commit to an audit strategy. The model thus belongs to the AN case. The control variable is the declared income x, with a true value y. Assuming a linear auditee utility function, the auditees maximize their expected fraud gain:

$$U = t(y-x)[1-p_a(x)]-\pi t(y-x)p_a(x)$$

which trivially is rewritten as

$$U = t(y-x)[1-(1+\pi)p_a(x)] \tag{1}$$

In this paper, we introduce the relative audit frequency function $p(x)$,

$$p(x) = (1+\pi)\, p_a(x)$$

which modifies the taxpayer utility function into

$$U = t(y - x)[1 - p(x)] \tag{2}$$

As shown by Sanchez and Sobel (1993) and others, the optimal audit frequency $p_a(x)$ will never exceed the *critical audit level* $1/(1+\pi)$. The relative audit frequency $p(x)$ will therefore never exceed unity.

As the taxpayer wishes to maximize its utility U, differentiation of Eq. (2) renders that a taxpayer with true income y will select the declared income x which satisfies

$$-p'(x)/(1 - p(x)) = 1/(y - x) \tag{3}$$

By integration of Eq. (2) over the true income distribution of the studied population, the expected net loss of taxable income per taxpayer is

$$L = \int (y - x)[1 - p(x)]f(y)dy \tag{4}$$

where $f(y)$ is the true income frequency function on the interval $y_{min} \leq y \leq y_{max}$.

The average number of audits per taxpayer, i.e., the audit density, is

$$D_\pi = \int p_a[x(y)]f(y)dy = \int p[x(y)]f(y)dy/(1+\pi) = D/(1+\pi)$$

where

$$D = \int p[x(y)]f(y)dy \tag{5}$$

If the unit audit cost is c_a, the audit cost per taxpayer becomes

$$C_a = c_a D/(1+\pi)$$

The total cost becomes

$$tL + c_a D/(1+\pi) = t\{L + c_a D/[t(1+\pi)]\}$$

In order to obtain results independent of the tax rate and the penalty rate, we normalize the unit audit cost by setting

$$c = c_a/[t\ (1+\pi)] \tag{6}$$

The auditor then wishes to find the functions $x(y)$ and $p(x)$ which minimize

$$C = L + cD$$

i.e.,

$$C = \int [y - x(y)]\{1 - p[x(y)]\}f(y)dy + c\int p[x(y)]f(y)dy \tag{7}$$

For a Nash equilibrium, the derivative with respect to $p(x)$ of the total cost C has to be zero in order to find an equilibrium in the interior, i.e., where $0 < p(x) < 1$. Thus,

$$y - x(y) - c = 0 \tag{8}$$

Combining (3) and (8) renders a differential equation for $p(x)$

$$-p'(x)/(1 - p(x)) = 1/c \tag{9}$$

which has the solution

$$p(x) = 1 - Ae^{x/c} \tag{10}$$

Reinganum & Wilde permitted the declared income x to fall below the minimum true income y_{min} such that Eq. 10 is valid for $y_{min} - c \leq x \leq y_{max} - c$. They proved that the optimum value of A is such that the $p(x)$ function hits zero at $x = y_{max} - c$.

In this paper, we prevent the declared income from falling below the minimum true income by setting

$$p(x) = 1 \text{ for } x < y_{min}$$

In such a revised model, we are unable to prove that the optimum audit function hits $p(x) = 0$ at $x = y_{max} - c$. Let z denote the declared income where the audit function hits zero, i.e., $1 - Ae^{z/c} = 0$. We then have three separate sections of the declared income interval, where

$$p(x) = 1 \text{ for } x < y_{min}$$

$$p(x) = 1 - Ae^{x/c} \text{ for } y_{min} \leq x < z$$

$$p(x) = 0 \text{ for } x \geq z$$

The constant A is determined through a numerical minimization of the total cost C. Finally, the actual audit rate is calculated,

$$p_a(x) = p(x)/(1+\pi)$$

A tax audit example is shown in section 5.1 below.

A variation of the AN case was studied by Newman, Rhoades, and Smith (1996). The paper treated a case of management fraud where an agent may overstate a control variable having a known probability distribution. Nash equilibria were calculated for scenarios with different assumptions regarding the relations among audit effort, detection likelihood and audit cost.

3.3. The HN case: some inherently honest auditees, non-announced strategy

In the model presented by Erard and Feinstein (1994b), denoted *the EF model*, the taxpayer population is split into two groups, one honest group that always declares the true income and one potentially dishonest group that optimizes its utility as in the earlier models.

The share of inherently honest taxpayers, denoted Q, is exogenous in this model. The model allows for risk-neutral or risk-averse taxpayers as well as a penalty proportional to the amount of tax evaded. It is based on the assumption that the tax authority is not able or willing to inform the taxpayers about the audit strategy. The problem is therefore formulated as a game where a Nash equilibrium solution can be obtained in which the auditor and auditee correctly anticipate each other's strategies. The model thus belongs to the HN case.

The EF model is able to determine optimal audit strategies and taxpayer behaviour that fairly resembles what is observed empirically.

It should be noted that Erard & Feinstein formulate the problem with a budget constraint which is included in the objective function via a Lagrange multiplier λ. The current paper uses a total cost formulation, with the same normalization of audit function and audit cost as in Section 3.2. Erard & Feinstein use a different penalty rate definition θ, where $\theta = t\pi$.

Compared to the AN case, the expected net loss of taxable income per taxpayer is reduced by a factor $1 - Q$,

$$L = (1 - Q)\int (y - x)[1 - p(x)]f(y)dy \tag{11}$$

Another difference from the AN case is the calculation of the number of audits, since some taxpayers declare the income $x(y)$ and the others report their true income y. The average number of audits per taxpayer, i.e., the audit density, becomes

$$D_\pi = (1-Q)\int p_a[x(y)]f(y)dy + Q\int p_a(x)f(x)dx$$

i.e.,

$$D_\pi = \left[(1-Q)\int p[x(y)]f(y)dy + Q\int p(x)f(x)dx\right]/(1+\pi) = D/(1+\pi)$$

where

$$D = (1-Q)\int p[x(y)]f(y)dy + Q\int p(x)f(x)dx \tag{12}$$

If the unit audit cost is c_a, the audit cost per auditee becomes $c_aD/(1+\pi)$. The total cost becomes

$$tL + c_aD_\pi = t\{L + c_aD/[t(1+\pi)]\}$$

In order to obtain results independent of the tax rate and the penalty rate, we normalize the unit audit cost by setting

$$c = c_a/[t(1+\pi)] \tag{13}$$

The expected total cost becomes

$$C = (1-Q)\int [y-x(y)]\{1-p[x(y)]\}f(y)dy + c(1-Q)\int p[x(y)]f(y)dy + cQ\int p(x)f(x)dx \tag{14}$$

The derivative with respect to $p(x)$ of the total cost C has to be zero in order to find an equilibrium in the interior, i.e., where $0 < p(x) < 1$. Thus,

$$-(1-Q)[y-x(y)-c]f(y)dy + cQf(x)dx = 0$$

This can be rewritten as a differential equation for the relation between true and declared income,

$$dx/dy = [(y-x)/c-1](1/Q-1)f(y)/f(x) \tag{15}$$

In a normalized total cost formulation of the EF model, Eq. (6) in Erard and Feinstein (1994b) becomes

$$E(y|x) = x + c$$

By combining this equation with Eq. (7) in the Erard & Feinstein paper, our Eq. (14) is obtained.

Eq. (14) is preferably integrated forwards starting at $x(y_a) = y_{min}$, where y_{min} is the lower end of the true income interval and $y_a \geq y_{min}$. Eq. (3) is thereafter integrated backwards starting at $x = x_b$ and $p = 0$. The parameters y_a and x_b are varied until a least-cost solution is found. Finally, the actual audit rate is calculated,

$$p_a(x) = p(x)/(1+\pi)$$

Numerical results for the EF model have been obtained with a C++ computer program developed by the author. With a uniform distribution of the control variable, the differential equations in the EF solution can be solved analytically. The results have been verified in another computer program using the analytical solutions.

An article closely related to the EF model was presented by Newman, Patterson, and Smith (2001). They studied optimal auditing in a general auditing framework with accounting fraud consisting of overstated income. Their model is similar to the EF model, formulated as an equilibrium game and with a distinction between inherently honest and potentially dishonest auditees. Newman et al. arrived at exactly the same solution as Erard and Feinstein, which regrettably is not commented on in their paper.

A recent application of the EF model was presented by Boserup and Pinje (2013). They applied the model on the entire taxpayer population of Denmark, segmented according to the amount of third-party reported income. The objective was to study regressive bias, i.e., the fall in effective tax rate with increasing income. With an information strategy, those who declare a high income will be audited less, thus they will be able to commit a larger share of fraud on self-reported income and thus pay less taxes. The results were that within each income segment, effective tax rates were regressively biased, whereas effective tax rates were increasing with income between segments due to the allocation of more audit resources to high-income segments.

Di Porto et al. (2013) studied a decentralized audit organisation where the auditors are given an incentive in relation to the audit results. The authors observed that an incentive proportional to the amount of detected fraud is sub-optimal since it does not take into account the deterrent effect of the audit strategy. Instead, they presented a model where the auditors maximize the number of audits where fraud is detected. They made the same assumption as Erard and Feinstein using two groups of taxpayers, one inherently "honest" and one "strategic", i.e., potentially dishonest. Like Erard and Feinstein, they solved for the Nash equilibrium.

It is obvious that the auditor's objective function used by Di Porto et al. can never produce a cut-off strategy since such a strategy will never detect fraud in equilibrium. Below the cut-off level, fraud is zero, whereas no audits are made above the cut-off level. The authors observe that a cut-off strategy seems to generate more revenues than the model above. The value of the Di Porto model is that it provides a less sub-optimal strategy for a decentralized audit organisation compared to the case where the auditors maximize the amount of detected fraud.

4. The pre-announced strategy cases

4.1. The sequential game model

The problem is modelled as a sequential game where the first player has the strategy set $\boldsymbol{A}$ and the objective function C to be minimized. The second player, who is informed of the strategy $\boldsymbol{a}$ selected by the first player, has the strategy set $\boldsymbol{B}(\boldsymbol{a})$ and the objective function $E(U)$ to be maximized. The mathematical formulation is

$$\underset{\boldsymbol{A}}{Min}\, C[\boldsymbol{A}, \mathbf{b}^*(\boldsymbol{A})] \text{ where } \boldsymbol{b}^*(\boldsymbol{A}) \text{ maximizes } E\{U[\boldsymbol{B}(\boldsymbol{a})]\}$$

The strategy set $\boldsymbol{A}$ corresponds to the set of audit functions $p_a(x)$ and the strategy set $\boldsymbol{B}(\boldsymbol{a})$ to the set of declared control variable functions $x(y)$ for a given audit function $p_a(x)$, i.e.,

$$\underset{p_a(x)}{Min}\, C[p_a(x), x^*(y)] \text{ where } x^*(y) \text{ maximizes } E\{U[x(y), p_a(x)]\}$$

The optimization of auditee behaviour is exactly the same as in the AN and HN classes, rendering Eq. (16),

$$-p'(x)/(1-p(x)) = 1/(y-x) \tag{16}$$

4.2. The AP case: all auditees dishonest, pre-announced strategy

Reinganum and Wilde (1985) were the first to develop an information strategy model where both taxpayer behaviour and audit strategy were determined. Their paper treats a case with lump-sum taxes. It is assumed that the taxpayers are risk-neutral and that the tax agency maximizes net expected revenue, i.e., tax and penalty revenue less audit cost. The authors studied audit strategies of the *cut-off* type, i.e., where the audit probability is zero for declared incomes above the cut-off level and 100% for incomes below the cut-off level. Their main conclusion was that a cut-off strategy is preferable compared to random auditing.

The problem was treated as a sequential game where the tax agency declares an audit strategy and the taxpayers adapt their behaviour to this strategy. Their model thus belongs to the AP class. Since their model was based on lump-sum taxes, it falls outside the scope of this paper.

Reinganum and Wilde (1986) stated that it would be useful to compare their results for the non-announced case with the case where the tax agency is able to pre-commit, “Unfortunately, the latter problem remains unsolved…” (page 755). The problem was solved by Morton (1993) and Sanchez and Sobel (1993).

Morton (1993) studied optimal auditing of a firm where the manager can underreport the profit to the owner and the owner can establish the true profit through a costly audit. The owner maximizes expected profit less audit cost. Upon detection, a penalty is charged, proportional to the underreported profit with a proportionality factor π. The problem was treated as a sequential game with perfect information.

As in section 3, the current paper eliminates the penalty rate by normalizing costs and audit rate. The notation below is different from the Morton paper.

Morton observed that the existence of an optimal solution $p(x)$ and $x(y)$ implies that a function $P(y) = p(x(y))$ exists which is non-increasing in y. The total cost can be expressed as an integral

$$C = \int P(y)m(y)dy$$

where

$$m(y) = 1 - F(y) - cf(y)$$

Since $P(y)$ is non-increasing in y it is easily proven that C is minimized for any function $m(y)$ by a cut-off strategy,

$P(y) = 1$ for $y < Y$
$P(y) = 0$ for $y \geq Y$

This is a normalized version of Eq. (4) in the Morton paper.

Sanchez and Sobel (1993) studied optimal auditing as well as optimal tax structure. They assumed risk-neutral taxpayers and a general tax function $\tau(y)$ where y is the assessed income. Like Morton, they assumed a penalty rate π and treated the problem as a sequential game with perfect information.

They used the hazard function

$$g(y) = (1 - F(y))/F'(y)$$

where $F(y)$ is the cumulative distribution of true income. Important results, expressed in a normalized setting, were

- If the auditor maximizes tax revenue less audit costs, a cut-off strategy is optimal (Proposition 3 in the Sanchez & Sobel paper). This is the same result as in Morton (1993).
- If $\tau'(y)g(y)$ is strictly decreasing and the auditor has a budget constraint, the optimal audit strategy is a cut-off strategy (Proposition 2 (a)).
- With a budget constraint, the optimal strategy consists of a maximum of three normalized audit probability levels, i.e., 1, an intermediate level and the zero level (Corollary 1).

4.3. The HP case: some honest auditees, pre-announced strategy

Since the HP case has not been treated in the literature, a new model is presented here. Like the cases above, the model uses the relative audit frequency function

$$p(x) = (1 + \pi)\, p_a(x)$$

declining with declared income x. The auditee population is assumed to consist of two groups, one honest group with the share Q that always declares true income and one potentially dishonest group with the share 1-Q that optimizes its utility, i.e., selects the optimal declared income x for a given true income y. The share of honest auditees is exogenous in the model.

The expected loss of taxable income per taxpayer is the same as in the AP case,

$$L = (1 - Q)\int (y - x)[1 - p(x)]f(y)dy \tag{17}$$

where $f(y)$ is the true income frequency function.

The auditor wishes to find the functions $x(y)$ and $p(x)$ which minimize $C = L + cD$. Since the maximization of U with respect to $x(y)$ results in Eq. (3), the auditor faces an infinite-dimensional optimization problem with the variables $x(y)$ and $p(x)$,

$\underset{p(x)}{Min}\, C[p(x), x(y)]$ where

$$C = (1 - Q)\int [y - x(y)]\{1 - p[x(y)]\}f(y)dy + c(1 - Q)\int p[x(y)]f(y)dy + \\ + cQ\int p(x)f(x)dx \tag{18}$$

under the constraint

$$-p'(x)/(1 - p(x)) = 1/(y - x) \tag{19}$$

The constraint is valid for all values of x in the interior where $0<p<1$.

In the Appendix (Eq. A.7), it is shown that the optimum relation between declared income x and true income y satisfies

$$x'' = x'\left\{\frac{2}{y - x} - \left[1 + \frac{x'Qf(x)}{f(y)(1 - Q)}\right]\frac{cx'}{(y - x)^2} + \frac{f'(y)}{f(y)}\right\} \tag{20}$$

This second-order differential equation is integrated forward numerically with the initial values $x(y_a) = y_{min}$ and $x'(y_a) = \alpha$. Eq. (3) is thereafter integrated backwards starting with the initial value $p(x_b) = 0$. The parameters y_a, α and x_b are varied until a least-cost strategy is obtained. Finally, the actual audit rate is calculated,

$$p_a(x) = p(x)/(1 + \pi)$$

Numerical results for the HP case have been obtained using a C++ computer program developed by the author. At present, this program can only accommodate a uniform control variable distribution.

4.4. A simulation method for near-optimal solutions

Near-optimal audit strategies can be obtained by limiting the study to a class of normalized $p(x)$ functions in which the best function is selected. Experiments have been carried out with some different classes of functions.

For tax auditing, the method uses a class of exponential functions with two parameters a and b where a indicates the slope of the curve and b is the declared income where $p(x)$ hits the zero level.

$p(x) = 1 - e^{a(x-b)}$ for $x \leq b$

$p(x) = 0$ for $x > b$

The reason for this choice of audit function is that experimentation with the EF model often leads to functions with such a shape. In addition, the solutions presented in Reinganum and Wilde (1986) belong to this class of functions.

This solution method is a simple simulation model, imitating the assumed auditee behaviour. The true income interval (y_{min}, y_{max}) is divided into a large number of segments. For each segment, the optimum declared income x is determined by maximization of the expected utility of the auditee.

$$U = (y - x)(1 - p(x))$$

The expected tax loss is calculated and the process is repeated with different values of the parameters a and x_b until an optimum is reached. The optimization method used is very simple, using a two-dimensional grid which is extended in case the best solution is on the boundary and reduced in case the best solution is in the interior. Finally, the actual audit rate is calculated,

$$p_a(x) = p(x)/(1 + \pi)$$

The simulation method is implemented in the computer code AUDSIM developed by the author. The input data required is the distribution of the true control variable and the share of honest persons. AUDSIM can accommodate distributions with piecewise linear frequency functions with up to 25 breaking points. The code is written in C++ and is available from the author.

Experiments have been carried out using AUDSIM with a slightly more complex class of audit functions with two additional parameters pc and d, i.e.,

$$p(x) = pc\left(1 - e^{a\left(x - b - d(x-b)^2\right)}\right) \quad \text{for } x \le b$$

$$p(x) = 0 \quad \text{for } x > b$$

In case $p(x) > 1$, $p(x)$ is reduced to 1. The parameter d is only permitted to take on values such that $p(x)$ is monotonic.

The improvement achieved with two additional parameters has been very small in the numerical examples studied.

4.5. Extensions of the simulation method

The simulation method, originally developed for tax fraud, can easily be adapted to accommodate concave utility functions, benefit and accounting fraud, partially discrete true control variable distributions, various forms of social penalty and a mix of fraud and errors.

In benefit and accounting fraud, the fraud direction is often reversed. For instance, sickness benefits are generally controlled by the number of sickness days, rendering higher benefits for a longer sickness duration. For benefit and accounting fraud where the auditee fraudulently overstates a variable x, the class of exponential audit frequency functions is increasing with x, i.e.,

$$p(x) = 1 - e^{-a(x-b)} \text{ for } x \ge b$$

$$p(x) = 0 \quad \text{for } x < b$$

Examples of such audit frequency functions are shown in sections 5.3 and 5.4.

In cases where fraud is committed by overstating the control variable, this variable may be restricted to non-negative values, for instance the amount of benefit claims. It may then be necessary to introduce a discrete part of the distribution of the true control variable at value zero since part of the fraud may be committed by persons with no true benefit claims.

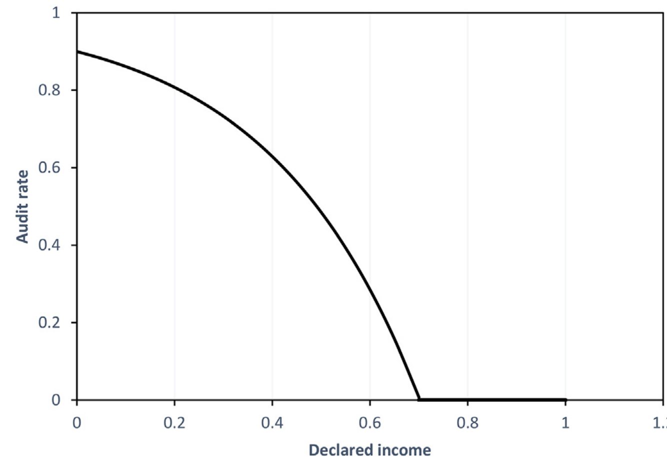

Fig. 1. Typical audit rate function for the Reinganum and Wilde model.

With the simulation method, the utility function of the auditee can easily be modified, affecting the optimum fraud amount. One example is a concave utility function, another example is the introduction of a social penalty.

A simple form of errors, i.e., unintentional misstatements, has been studied in the form of a fixed probability of an error in a fixed amount among the inherently honest auditees. This has the effect of reducing the efficiency of the audit method since the auditees making errors do not adapt their behaviour to the audit strategy.

5. Numerical examples

5.1. The Reinganum and Wilde model

Fig. 1 below shows a typical tax audit rate function for the RW model, with true income in the interval between zero and 1 and no penalty. The audit rate is decreasing to zero such that auditees with declared income above a certain level will not be audited at all.

Fig. 2 shows that for a case with a uniform true income distribution, the sequential game solution can lower costs by up to almost 20% compared to the RW equilibrium solution. The results are obtained using the simulation method with two parameters. The simulation method generates solutions with very large values of the parameter a, i.e. with

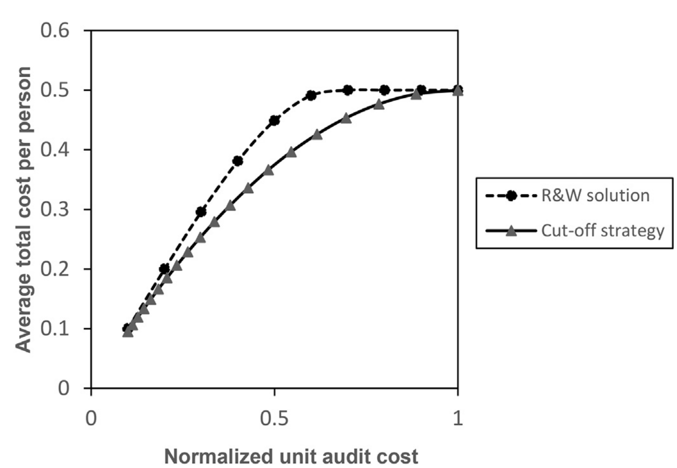

Fig. 2. Average total cost per person as a function of the unit audit cost with no honest taxpayers (Q = 0) and true income uniformly distributed between 0 and 1.

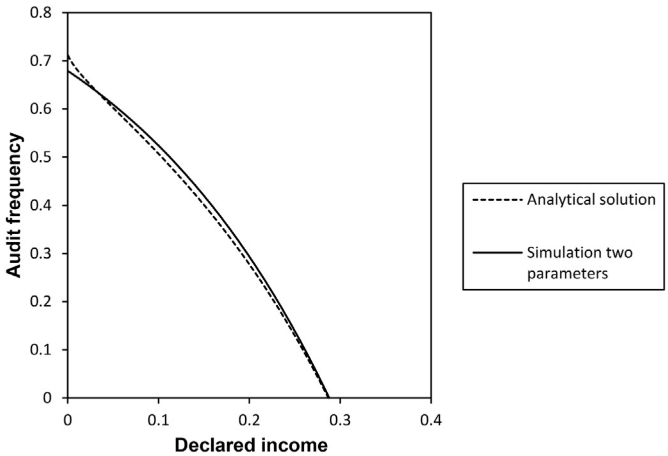

Fig. 3. Audit frequency as a function of declared income, unit audit cost c = 0.4, fraction of inherently honest taxpayers Q = 0.5.

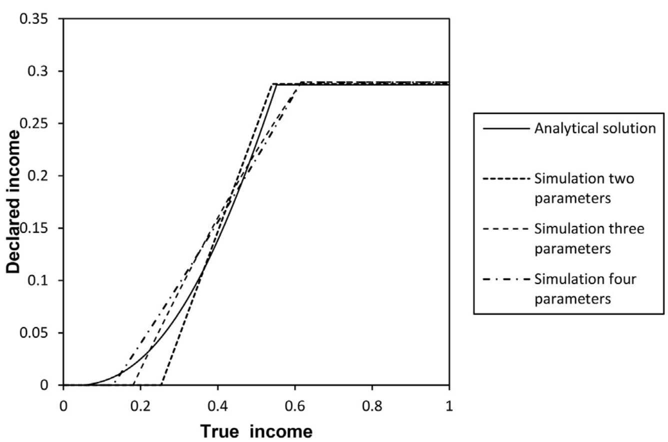

Fig. 4. Optimal declared income as a function of true income, unit audit cost c = 0.4, fraction of inherently honest taxpayers Q = 0.5.

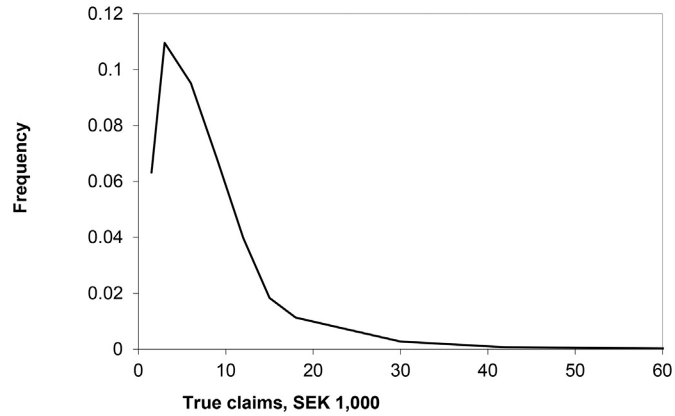

Fig. 5. An example of a true claims distribution for benefit fraud, excluding the discrete probability at zero true claims.

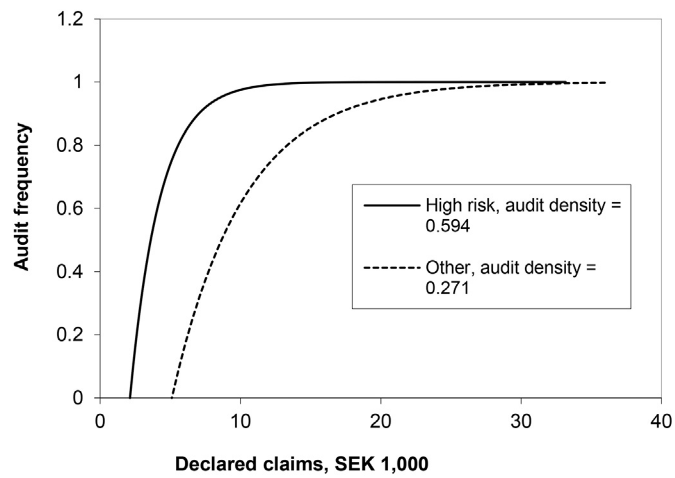

Fig. 6. Audit frequency functions for two population segments at audit unit cost SEK 2000 (Swedish Krona).

very steep slopes, asymptotically resembling the cut-off strategy. The results by Morton (1993) are thus confirmed, i.e., that the cut-off strategy is optimal in the sequential game with perfect information and no inherently honest auditees.

5.2. The simulation method versus the analytical solution

This example refers to a tax audit with a uniform distribution of true income between 0 and 1, a unit audit cost of 0.4, no penalty and 50% inherently honest taxpayers. In Fig. 3, an example of the audit frequency function $p(x)$ is shown for the two solution methods for the sequential game. An indication of the quality of the simple simulation method with two parameters is that the difference in total cost between the two methods is less than 0.2%.

The corresponding relation between true and declared income is shown in Fig. 4 for the analytical solution and simulation with two, three and four parameters. The improvement in total cost with three parameters (a, b and d) compared to two parameters (a and b) is 0.2%, and the additional improvement with the fourth parameter (pc) is only 0.04%.

5.3. Benefit fraud

This example is taken from Appelgren (2017), a case study of Temporary Parental Benefits (TPB) in Sweden based on data collected by Engström and Hesselius (2007). TPB is available to caregivers (parents or other persons) who lose income when taking care of sick children.

The control variable is the amount of benefit claims during a specific time period, and the audit rate is increasing with the amount claimed. The distribution of true claims and the fraction of honest auditees is derived from a set of audits performed in an earlier study. It is important to note that the true claims distribution has a significant portion, about 50%, of potentially fraudulent auditees with zero true claims. A typical distribution is shown in Fig. 5.

Fig. 6 shows near-optimal audit rates for two different risk segments of the studied population. With no penalty upon discovery, the audit rates tend asymptotically to 100% for high values of declared claims.

5.4. Accounting fraud

This example is taken from Newman et al. (2001), which models accounting fraud where a fraudulent auditee overstates company earnings. The auditor uses his prior beliefs about the probability that the auditee may commit fraud and the distribution of true company earnings. As stated in section 3.3, Newman et al. treat the problem as a game without a pre-announced strategy. The authors present numerical results for a case with an exponential earnings distribution, $f(y) = 0.001\ exp\ (-0.001y)$. They do not state all the parameters used in their numerical examples, but it can be deduced from their figures

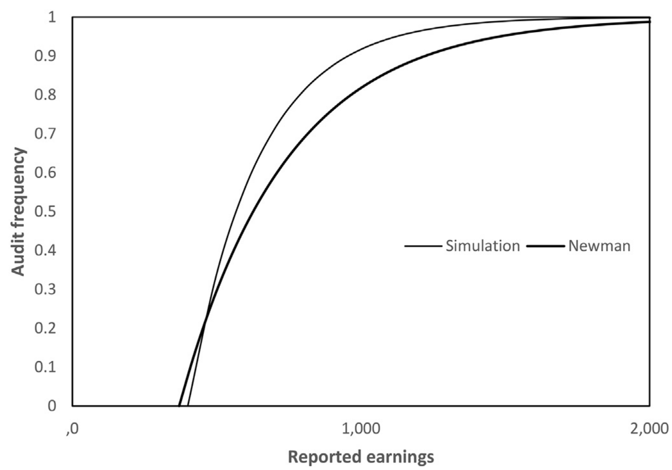

Fig. 7. Near-optimal audit frequency functions with (Simulation) and without (Newman et al.) a pre-announced audit strategy, with exponential earnings distribution, audit unit cost of 120 and 25% potentially fraudulent auditees.

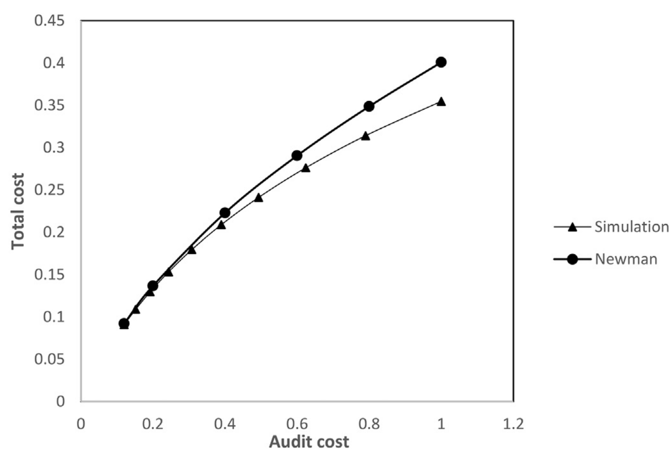

Fig. 8. Total cost (audit cost plus fraud loss) with and without pre-announced audit strategies, with exponential earnings distribution, 25% fraudulent auditees and audit unit cost of 120.

that the fraction of potentially fraudulent auditees is approximately 25% and that the audit cost is approximately 120.

These same data have been use in an application of the simulation model for the sequential game with an announced audit strategy. Fig. 7 shows the optimal audit frequency functions in the Newman et al. model (i.e., non-announced strategy) and in the sequential game simulation model (i.e., pre-announced strategy).

In Fig. 8, the total cost, i.e., audit cost plus fraud loss, is presented as a function of audit cost. The total cost is lower when the auditor can pre-commit to the audit strategy, making it advantageous for the auditor to announce the audit strategy in advance in order to create a sequential game. This is in line with the general result that the leading player in a sequential game with perfect information comes out better compared to the corresponding player in a simultaneous-move game.

6. Discussion and conclusions

This paper has provided both an analytical solution and a simple approximation method for producing near-optimal audit strategies in a strategic audit setting with some inherently honest auditees and a pre-announced strategy.

6.1. Segmentation strategy

Segmentation of the auditee population should be used whenever adequate data is available. Segmentation has the effect that high fraud segments will be audited at the critical audit level $1/(1+\pi)$ whereas the low fraud segment will be left unaudited.

Segmentation preferably can be combined with the use of an information strategy in which the auditor announces the audit strategy with sufficient notice for the auditee to incorporate this information into his/her fraud decision. In Appelgren (2017), the population was split into two segments, a smaller high risk segment with about 17% fraudulent persons and a larger segment with about 8% fraudsters. The sequential game simulation model was applied on both segments as well as on the total population. As expected, segmentation resulted in a lower total cost.

6.2. All auditees potentially dishonest versus some inherently honest auditees

The cases with all dishonest auditees (AN and AP) are included here mainly for historical reasons. They are less realistic since they generate much higher levels of fraud compared to what is observed empirically, as noted by Erard and Feinstein (1994a). Therefore, we have concentrated on the cases with some honest auditees (HN and HP).

With the introduction of a social penalty motivating some auditees to abstain from fraud, new models with all potentially dishonest auditees may become more attractive.

6.3. Non-announced strategy versus pre-announced strategy

There are numerous situations where a pre-announced strategy is not feasible. In audits of financial statements, it seems unlikely that an auditor would want to declare an audit strategy in advance. This explains the extensive use of non-announced strategies in the strategic audit literature in the accounting field. There may also be regulatory restrictions or professional standards that prevent auditors from pre-announcing the audit strategy. The discussion below is therefore limited to situations where pre-announced strategies are possible.

In some papers from the accounting literature, prior distributions of important variables are assumed to be known, for instance in Shibano (1990, page 116), Matsumura and Tucker (1992, page 755), Bloomfield (1995, pages 72–73), and Caplan (1999, page 105). The assumption of such distributions means that there is a population of auditees from which such distributions can be measured, unless it is stated that those distributions are subjectively assessed by the auditor as in Newman et al. (2001).

The existence of such populations seems more likely for some types of audits, such as expense account auditing in large organizations, than for other audit types, such as financial statement audits. When such populations exist, it is advantageous to use pre-announced audit strategies if they are permitted.

The strategies studied in this paper are based on utility-maximizing auditees. There are arguments for and against modelling pre-announced audit strategies in this setting, An argument for a pre-announced strategy is the general result that the utility for the first player in a sequential game with perfect information is higher or equal to the utility of the same player in a simultaneous-move game.

An argument against pre-announced strategies is that auditors may be unwilling to disclose their audit strategy since they feel that this would provide the auditees with too much information. This may be especially true in cases where the perception of the auditees is that the audit density is higher than the actual level. Therefore, models of pre-announced strategies may have limited external validity. It is probable, however, that productive results can be obtained by informing the auditees of the general shape of the audit rate function, such as “the audits will be concentrated on taxpayers declaring the lowest incomes" or "the audits will be concentrated on auditees claiming the highest claim amounts".

6.4. Analytical solution versus simulation method

In the pre-announced strategy case, the simulation model is a straightforward method that generates near-optimal audit strategies. Variations of the model are easily implemented, for instance with auditees having concave utility functions, with partially discrete distributions of the true value of the control variable, and with a mix of fraud and errors.

A prerequisite for all models described in this paper is that the probability distribution of the true value of the control variable is known. Since the accuracy of an empirically-derived distribution is limited, there is no need for very accurate calculations of optimal audit rate functions supporting the use of a simple method generating near-optimal solutions.

The results from the tax audit example in section 5.2 indicate that the simple two-parameter approach is only marginally inferior to the more complex model with three or four parameters. This is not necessarily true for all true-income distributions.

6.5. Social penalty

A disadvantage with the sequential game model as well as with the Erard and Feinstein model is that dishonest auditees will always cheat, i.e., there is no set of parameters that will induce the auditee to abstain from fraud. This is due to the penalty being proportional to the fraud amount in the model. In line with Alm (2013), future research should include the construction of models where the proportion of honest taxpayers is calculated endogenously, possibly with the introduction of a social penalty that is not strictly proportional to the fraud amount.

Social penalty can be introduced as a deterministic or stochastic variable, dependent or independent of the fraud amount, which is added when fraud is detected. It has the effect that an auditee may abstain from fraud, in contrast to the models where the share of potentially fraudulent auditees is determined exogenously. Thus, the introduction of an endogenous social penalty can replace the exogenous assumption of some inherently honest auditees. An example of social penalty is found in Dionne et al. (2009), which introduces a moral cost with a known probability distribution.

In addition to the introduction of a social penalty, Alm (2013) also promoted the development of models which consider group behaviour.

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Declaration of Competing Interest

None

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Appendix. Analytical solution of the sequential game model

With $q(x) = 1 - p(x)$, Eq. (3) can be written as

$$q'(x) = q(x)/(y-x) \tag{A.1}$$

This constraint can be included in the objective function via a multiplier function $m(x)$. The objective function then becomes

$$\begin{aligned} C = {} & (1-Q)\int(y-x(y))q(x(y))f(y)dy \\ & + c(1-Q)\int(1-q(x(y)))f(y)dy + +cQ\int(1-q(x))f(x)dx \\ & + \int m(y)(q'-q/(y-x))dy \end{aligned} \tag{A.2}$$

For an optimum in the interior, derivatives with respect to x and q have to be zero.

$$dC/dx = -(1-Q)q(x)f(y) - q(x)m(y)/(y-x)^2 = 0$$

which renders

$$m(y) = -(y-x)^2(1-Q)f(y) \tag{A.3}$$

Before differentiation with respect to q, q' can be eliminated in Eq. (A.2) by means of partial integration

$$\begin{aligned} \int m(y)q'dy &= \int m(y)(dq/dy)/(dx/dy)\,dy \\ &= m(y)q/x' - \int q\,(d(m(y)/x')/dy)\,dy \end{aligned}$$

Thus,

$$\int m(y)q'dy = constant - \int q(x)\left[m'(y)/x' - m(y)x''/x'^2\right]dy \tag{A.4}$$

Differentiation of Eq. (A.2) with respect to q using Eq. (A.4) renders

$$\begin{aligned} dC/dq = {} & (1-Q)(y-x-c)f(y) - cQf(x)x' \\ & + mx''/(x')^2 - m'/x' - m/(y-x) = 0 \end{aligned} \tag{A.5}$$

First, $m'(y)$ is determined from Eq. (A.3),

$$m'(y) = -2(1-Q)(y-x)(1-x')f(y) - (1-Q)(y-x)^2 f'(y) \tag{A.6}$$

Next, the functions $m(y)$ and $m'(y)$ are eliminated from Eq. (A.5),

$$\begin{aligned} & (1-Q)(y-x-c)f(y)x'^2 - cQf(x)x'^3 + (y-x)^2(1-Q)f(y) \\ & \times\left[x'^2/(y-x) - x''\right] \end{aligned}$$

$$+2(1-Q)(1-x')x'f(y) + (1-Q)x'(y-x)^2 f'(y) = 0$$

which can be formulated as a differential equation

$$x'' = x'\left\{\frac{2}{y-x} - \left[1 + \frac{x'Qf(x)}{f(y)(1-Q)}\right]\frac{cx'}{(y-x)^2} + \frac{f'(y)}{f(y)}\right\} \tag{A.7}$$

References

Allingham, M. G., & Sandmo, A. (1972). Income tax evasion: A theoretical analysis. *Journal of Public Economics*, *1*, 323–338.

Alm, J. (2013). Expanding the theory of tax compliance from individual to group motivations. *Working paper 1309*. Department of Economics, Tulane University.

Appelgren, L. (2008). The effect of audit strategy information on tax compliance - an empirical study. *eJournal of Tax Research*, *6*(1), 67–81.

Appelgren, L. (2017). Optimal auditing of social benefit fraud – A case study. *Empirical Economics*, *56*(1), 203–231. https://doi.org/10.1007/s00181-017-1356-9.

Bloomfield, R. (1995). Strategic dependence and inherent risk assessments. *The Accounting Review*, *70*(1), 71–90.

Boserup, S. H., & Pinje, J. V. (2013). Tax evasion, information reporting and the regressive bias prediction. https://www.econ.ku.dk/boserup/papers/regressivebias_aug2013.pdf.

Caplan, D. (1999). Internal controls and the detection of management fraud. *Journal of Accounting Research*, *37*(1), 101–117.

Di Porto, E., Persico, N., & Sahuguet, N. (2013). Decentralized deterrence, with an application to labor tax auditing. *American Economic Journal: Microeconomics*, *5*(1), 35–62.

Dionne, G., Giuliano, F., & Picard, P. (2009). Optimal auditing with scoring: Theory and applications to insurance fraud. *Management Science*, *55*(1), 58–70. https://doi.org/10.1287/mnsc.1080.0905.

Engström, P., & Hesselius, P. (2007). The information method – Theory and application. *Working Paper 2007:17*. The Institute for: Evaluation of Labour Market and Education Policy, Uppsala.

Erard, B., & Feinstein, J. S. (1994a). The role of moral sentiments and audit perceptions in tax compliance. *Public Finance/Finances Publiques*, *49*(Supplement), 70–89.

Erard, B., & Feinstein, J. S. (1994b). Honesty and evasion in the tax compliance game. *RAND Journal of Economics*, *25*, 1–19.

Hasseldine, J., Hite, P., James, S., & Toumi, M. (2007). Persuasive communications: Tax compliance enforcement strategies for sole proprietors. *Contemporary Accounting Research*, *24*, 171–194.

Kleven, H. J., Knudsen, M. B., Kreiner, C. T., & Saez, E. (2011). Unwilling or unable to cheat? Evidence from a tax audit experiment in Denmark. *Econometrica*, *79*, 651–692.

Matsumura, E. M., & Tucker, R. R. (1992). Fraud detection: A theoretical foundation. *The Accounting Review*, *67*(4), 753–782.

Morton, S. (1993). Strategic auditing for fraud. *The Accounting Review*, *68*, 825–839.

Newman, P., Rhoades, S., & Smith, R. (1996). Allocating audit resources to detect fraud. *Review of Accounting Studies*, *1*, 161–182.

Newman, P. D., Patterson, E., & Smith, R. (2001). The influence of potentially fraudulent reports on audit risk assessment and planning. *The Accounting Review*, *76*, 59–80.

Patterson, E., & Noel, J. (2003). Audit strategies and multiple fraud opportunities of misreporting and defalcation. *Contemporary Accounting Research*, *20*(3), 519–549.

Reinganum, J. F., & Wilde, L. L. (1985). Income tax compliance in a principal-agent framework. *Journal of Public Economics*, *26*, 1–18.

Reinganum, J. F., & Wilde, L. L. (1986). Equilibrium verification and reporting policies in a model of tax compliance. *International Economic Review*, *27*, 739–760.

Sanchez, I., & Sobel, J. (1993). Hierarchical design and enforcement of income tax policies. *Journal of Public Economics*, *50*, 345–369.

Shibano, T. (1990). Assessing audit risk from errors and irregularities. *Journal of Accounting Research*, *28*(Supplement), 110–140.

Slemrod, J., Blumenthal, M., & Christian, C. (2001). Taxpayer response to an increased probability of audit: Evidence from a controlled experiment in Minnesota. *Journal of Public Economics*, *79*, 455–483.