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An extended GRA method for MCDM with interval-valued triangular fuzzy assessments and unknown weights ☆

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ABSTRACT

Multiple criteria decision making (MCDM) is the process of ranking the feasible alternatives and selecting the best one by considering multiple criteria. Owing to the complexity, fuzziness and uncertainties of the objective things, the criterion values often take the form of linguistic variables, which can be expressed in interval-valued triangular fuzzy numbers. The purpose of this paper is to develop an extended grey relational analysis (GRA) method for solving MCDM problems with interval-valued triangular fuzzy numbers and unknown information on criterion weights. In order to determine the criterion weights, some optimization models based on the basic idea of traditional GRA method are established. Then, calculation steps of extended GRA method for MCDM are given. Finally, a numerical example is shown to verify the developed method and to demonstrate its practicality and feasibility.

1. Introduction

Multiple criteria decision making (MCDM) problem is a well-known branch of decision theory. It has been found in real life decision situations (Zhang, Fan, & Liu, 2010; Ho, Xu, & Dey, 2010; Cheng, 2008; Chang & Wang, 2009; Yu & Hu, 2010). A MCDM problem consists of ranking the feasible alternatives and selecting the most desirable one(s) by considering multiple criteria, which are frequently in conflict with each other. In the real world, due to the complexity, fuzziness and uncertainties of the objective things, the criteria involved in the decision making problem may not be appropriate to express them by exact numerical values. It is more suitable to describe them by means of linguistic variables. Hence, linguistic information has frequently been applied to MCDM problems.

Since linguistic variables are not directly mathematically operable, to cope with this difficulty, each linguistic variable is associated with a fuzzy number characterizing the meaning of each generic verbal term (Chou & Chang, 2008). In some existing literatures (Xu & Chen, 2007; Tuzkaya, Gülsün, Kahraman, & Özgen, 2010; Mahdavi, Mahdavi-Amiri, Heidarzade, & Nourifar, 2008; Wang & Lee, 2007; Wang & Chen, 2008; Chen, Wang, Chen, & Lee, 2010), linguistic variables are converted to triangular fuzzy numbers in the decision making process. The reason (Lam, Tao, & Lam, 2010) of using triangular fuzzy number is because of its intuitive, easy to use, computational simplicity, useful in promoting representation and information processing in a fuzzy environment. However, in some actual decision making process, it is difficult to determine the specific values for lower and upper bounds of triangular fuzzy number. If the values' range is relatively easy to determine, then it is appropriate to define lower and upper bounds values as an interval. Thus, it can not only avoid the loss of information, but also express the decision making information more precisely. Therefore, linguistic variables were converted to interval-valued triangular fuzzy numbers in some literatures. Ashtiani, Haghighirad, Makui, and Montazer (2009) presented a fuzzy TOPSIS method for handing MCDM problems, in which all the information on criterion values and criterion weights is represented as linguistic variables, expressed in interval-valued triangular fuzzy numbers. Vahdani, Hadipour, Sadaghiani, and Amiri (2010) considered the same problem and developed a fuzzy VIKOR method. The above two literatures mainly focus on the group MCDM problems with known information on criterion weights under interval-valued triangular fuzzy environment. However, because of time pressure and the expert's limited expertise about the problem domain, the information on criterion weights in the process of MCDM is sometimes completely unknown. The situation leads to the problem how to derive the criterion weights, which is an interesting and important research topic. Therefore, based on the above two

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literatures, this paper investigates the MCDM problem with unknown weights.

Grey relational analysis (GRA) (Deng, 1989; Deng, 2002) is a helpful tool in MCDM problems that was originally proposed by Deng. It has been successfully applied in solving a variety of MCDM problems. GRA is an impact evaluation model that can measure the correlation between series and belongs to the category of the data analytic method or geometric method. Usually, researchers will set the target series based on the objective of the studied problem as the reference series. Hence, the purpose of grey relational analysis method is to measure the relation between the reference series and comparison series.

In this paper, we propose an extended fuzzy GRA method to solve MCDM problems, in which the criterion values are in the form of linguistic variables expressed in interval-valued triangular fuzzy numbers and the information on criterion weights is unknown. In order to determine the criterion weights, some optimization models based on the basic idea of traditional GRA are established. Then, calculation steps of extended GRA method for MCDM with interval-valued triangular fuzzy assessments are given to rank the alternatives and select the desirable one. To do that, the rest of this paper is organized as follows. In Section 2, we briefly introduce the GRA method. Section 3 illustrates triangular fuzzy sets. Section 4 describes a developed GRA method to solve interval-valued triangular fuzzy MCDM problems with unknown weights. Section 5 investigates a numerical example including an application to select a system analysis engineer for a software company to illustrate the applicability of the proposed method. Finally, the conclusion is given in Section 6.

2. Grey relational analysis method

Suppose a multiple criteria decision making problem having m non-inferior alternatives $A_1, A_2, \ldots, A_m$ and n criteria $C_1, C_2, \ldots, C_n$. Each alternative is evaluated with respect to the n criteria. All the evaluate values/ratings are assigned to alternatives with respect to decision matrix denoted by $X(=(x_{ij})_{m\times n})$.

The GRA procedure consists of the following steps:

Step 1. Calculate the normalized decision matrix. The normalized value r_{ij} is calculated as:

$$r_{ij} = \frac{x_{ij}}{\max(x_{ij})}, \quad i=1,2,\ldots,m; \quad j=1,2,\ldots,n, \quad \text{for } j\in I, \tag{1}$$

$$r_{ij} = \frac{\min(x_{ij})}{x_{ij}}, \quad i=1,2,\ldots,m; \quad j=1,2,\ldots,n, \quad \text{for } j\in J. \tag{2}$$

where I is the set of benefit criteria, and J is the set of cost criteria.

Step 2. Determine the reference series R_0.

$$R_0 = \{r_{01}, r_{02}, \ldots, r_{0n}\}. \tag{3}$$

where $r_{0j} = max_j r_{ij}$, $j = 1, 2, \ldots, n$.

Step 3. Establish the distance matrix. The distance δ_{ij} between the reference value and each comparison value is given as

$$\delta_{ij} = r_{0j} - r_{ij}. \tag{4}$$

Then the distance matrix Δ can be obtained as:

$$\Delta = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ \vdots & \vdots & & \vdots \\ \delta_{m1} & \delta_{m2} & \cdots & \delta_{mn} \end{bmatrix}. \tag{5}$$

Step 4. Calculate the grey relational coefficient. The grey relational coefficient ξ_{ij} is defined as:

$$\xi_{ij} = \frac{\delta_{min} + \zeta\delta_{max}}{\delta_{ij} + \zeta\delta_{max}}, \quad i=1,2,\ldots,m; \quad j=1,2,\ldots,n. \tag{6}$$

where δ_{max} and δ_{min} are the maximum and minimum of $\delta_{ij}(i = 1,\ldots,m; j = 1,\ldots,n)$, respectively, and ζ is the distinguishing coefficient between 0 and 1. Usually, suppose that ζ is 0.5.

Step 5. Estimate the grey relational grade γ_i by the relation

$$\gamma_i = \sum_{j=1}^{n} w_j \xi_{ij}, i = 1,2,\ldots,m. \tag{7}$$

where w_j is the weight of the jth criterion, and $w_j \geqslant 0, \sum_{j=1}^{n} w_j = 1$.

Step 6. Rank the alternatives in accordance with the value of grey relational grade, the bigger the value γ_i, the better the alternative A_i is.

3. Triangular fuzzy sets

In the following, we briefly introduce some basic definitions of triangular fuzzy sets. Definition 1 is the definition of triangular fuzzy number. Definition 2 shows the operational laws of triangular fuzzy numbers. Definition 3 gives the distance measure of triangular fuzzy numbers. These basic definitions will be used in next section, especially, the distance measure will be applied in Step 3 of Section 4.

Definition 1. (Fenton & Wang, 2006) A triangular fuzzy number $\tilde{a}$ is defined by a triple (a_1, a_2, a_3). The membership function is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a_1)/(a_2-a_1), & \text{if } a_1 \leqslant x \leqslant a_2, \\ (a_3-x)/(a_3-a_2), & \text{if } a_2 \leqslant x \leqslant a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

The triangular fuzzy number is based on a three-value judgment: the minimum possible value a_1, the most possible value a_2 and the maximum possible value a_3.

Definition 2. (Yu & Hu, 2010) Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, where $a_1 \leqslant a_2 \leqslant a_3$ and $b_1 \leqslant b_2 \leqslant b_3$, the basic operations of triangular fuzzy numbers are defined as follows:

(1) Addition:
$\tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
(2) Multiplication:
$\tilde{a} \otimes \tilde{b} = (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = (a_1 b_1, a_2 b_2, a_3 b_3)$.
(3) Division: $\tilde{a}/\tilde{b} = (a_1, a_2, a_3)/(b_1, b_2, b_3) = (a_1/b_1, a_2/b_2, a_3/b_3)$.

Definition 3. (Chen, 2000) Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, the distance between $\tilde{a}$ and $\tilde{b}$ can be calculated as:

$$\delta(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2]}. \tag{9}$$

4. An extended GRA method for MCDM with unknown weights

In fuzzy MCDM problems, performance rating values are usually characterized by fuzzy numbers. In this paper, criteria values are considered as linguistic variables. The concept of a linguistic variable is very useful in dealing with situations that are too complex or too ill-defined to be amenable for description in conventional quantitative expressions. These linguistic variables can be expressed as interval-valued triangular fuzzy numbers given in Table 1.

Consider a MCDM problem, let $A = \{A_1, A_2, \ldots, A_m\}$ be a finite set of feasible alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a finite set of criteria. The weight vector of the criteria $w = (w_1, w_2, \ldots, w_n)$ is unknown, but it satisfies $w_j \geqslant 0, j = 1, 2, \ldots, n, \sum_{j=1}^{n} w_j = 1$. Suppose that the performance of alternative A_i with respect to criterion C_j is denoted as $\widetilde{x}_{ij}$, then $\widetilde{X} = [\widetilde{x}_{ij}]_{m\times n}$ is a fuzzy decision matrix. As illustrated in Fig. 1, $\widetilde{x}_{ij}$ is expressed in interval-valued triangular fuzzy number:

$$\tilde{x} = \begin{cases} (x_1, x_2, x_3), \\ (x_1', x_2, x_3'). \end{cases}$$

Then $\tilde{x}$ can be also demonstrated as $\tilde{x} = [(x_1, x_1'); x_2; (x_3', x_3)]$.

Below, we develop an extended GRA method for MCDM with interval-valued triangular fuzzy assessments and unknown weights, which can be described as follows:

Step 1. Calculate the normalized decision matrix $\widetilde{R}$. Given $\tilde{x}_{ij} = \left[\left(a_{ij}, a_{ij}'\right); b_{ij}; \left(c_{ij}', c_{ij}\right)\right]$, the normalized performance rating can be calculated as:

$$\tilde{r}_{ij} = \left[\left(\frac{a_{ij}}{c_j^+}, \frac{a_{ij}'}{c_j^+}\right); \frac{b_{ij}}{c_j^+}; \left(\frac{c_{ij}'}{c_j^+}, \frac{c_{ij}}{c_j^+}\right)\right], \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n, \quad \text{for } j \in I. \tag{10}$$

$$\tilde{r}_{ij} = \left[\left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{c_{ij}'}\right); \frac{a_j^-}{b_{ij}}; \left(\frac{a_j^-}{a_{ij}'}, \frac{a_j^-}{a_{ij}}\right)\right], \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n, \quad \text{for } j \in J. \tag{11}$$

where $c_j^+ = max_i\{c_{ij}, i = 1, \ldots, m\}$ and $a_j^- = min_i\{a_{ij}, i = 1, \ldots, m\}$. Here, we simply denote $\tilde{r}_{ij} = \left[\left(g_{ij}, g_{ij}'\right); h_{ij}; \left(l_{ij}, l_{ij}'\right)\right]$. Hence, the normalized decision matrix $\widetilde{R} = [\tilde{r}_{ij}]_{m\times n}$ can be obtained.

Step 2. Determine the reference series. The reference series can be defined as:

$$R_0 = (r_{01}, r_{02}, \ldots, r_{0n}) = ([(1,1); 1; (1,1)], [(1,1); 1; (1,1)], \ldots, [(1,1); 1; (1,1)]). \tag{12}$$

Step 3. Calculate the distance between the reference value and each comparison value. The distance between the reference value and each comparison value can be calculated using Definition 3 as follows:

$$\begin{aligned} \delta_{ij}^{(1)} &= \sqrt{\frac{1}{3}\left[\left(g_{ij}' - 1\right)^2 + (h_{ij} - 1)^2 + \left(l_{ij}' - 1\right)^2\right]}, \\ \delta_{ij}^{(2)} &= \sqrt{\frac{1}{3}[(g_{ij} - 1)^2 + (h_{ij} - 1)^2 + (l_{ij} - 1)^2]}. \end{aligned} \tag{13}$$

Eq. (13) is employed to determine the distance between the reference value with comparison value in interval value $\bar{\delta}_{ij} = \left[\delta_{ij}^{(1)}, \delta_{ij}^{(2)}\right]$. In this way we lose less information than just converting immediately to a crisp value. At the same time, we can obtain their maximum $\delta_{\max}^{(1)}, \delta_{\max}^{(2)}$ and their minimum $\delta_{\min}^{(1)}, \delta_{\min}^{(2)}$, where $\delta_{\max}^{(1)} = \max_{i,j}\delta_{ij}^{(1)}, \delta_{\max}^{(2)} = \max_{i,j}\delta_{ij}^{(2)}, \delta_{\min}^{(1)} = \min_{i,j}\delta_{ij}^{(1)}, \delta_{\min}^{(2)} = \min_{i,j}\delta_{ij}^{(2)}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

Table 1
Definitions of linguistic variables for the ratings.

Linguistic variables	Interval-valued triangular fuzzy numbers
Very poor (VP)	[(0,0); 0; (1,1.5)]
Poor (P)	[(0,0.5); 1; (2.5,3.5)]
Medium poor (MP)	[(0,1.5); 3; (4.5,5.5)]
Medium (M)	[(2.5,3.5); 5; (6.5,7.5)]
Medium good (MG)	[(4.5,5.5); 7; (8,9.5)]
Good (G)	[(5.5,7.5); 9; (9.5,10)]
Very good (VG)	[(8.5,9.5); 10; (10,10)]

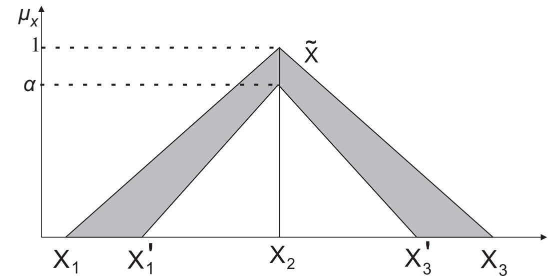

Fig. 1. An interval-valued triangular fuzzy number.

Step 4. Calculate the grey relational coefficient. Refer to Eq. (6) for obtaining the grey relational coefficient:

$$\begin{aligned} \xi_{ij}^{(1)} &= \frac{\delta_{min}^{(1)} + \zeta\delta_{max}^{(1)}}{\delta_{ij}^{(1)} + \zeta\delta_{max}^{(1)}}, \\ \xi_{ij}^{(2)} &= \frac{\delta_{min}^{(2)} + \zeta\delta_{max}^{(2)}}{\delta_{ij}^{(2)} + \zeta\delta_{max}^{(2)}}, \quad i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \end{aligned} \tag{14}$$

Here, suppose that ζ is 0.5.

Step 5. Estimate the grey relational grade. Refer to Eq. (7) for giving the grey relational grade:

$$\begin{aligned} \gamma_i^{(1)} &= \sum_{j=1}^{n} w_j \xi_{ij}^{(1)}, \\ \gamma_i^{(2)} &= \sum_{j=1}^{n} w_j \xi_{ij}^{(2)}, \quad i = 1, 2, \ldots, m. \end{aligned} \tag{15}$$

The basic principle of the GRA method (Wei, 2010) is that the chosen alternative should have the "largest degree of grey relation" from the reference solution. Obviously, for the weight vector given, the larger the values $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$, the better the alternative A_i is. But the information on criterion weights is unknown. So, in order to get the values $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$, we must first calculate the weight information. For this purpose, we can establish the following multiple objective optimization model to obtain the weight information:

$$(M-1)\begin{cases} \max \gamma_i^{(1)} = \sum_{j=1}^{n} w_j \xi_{ij}^{(1)}, \quad i = 1, 2, \ldots, m, \\ \max \gamma_i^{(2)} = \sum_{j=1}^{n} w_j \xi_{ij}^{(2)}, \quad i = 1, 2, \ldots, m, \\ \text{s.t.}: \sum_{j=1}^{n} w_j^2 = 1, w_j \geqslant 0, j = 1, 2, \ldots, n. \end{cases}$$

Since each alternative is non-inferior, so there exists no preference relation on all the alternatives. Therefore, we can aggregate the above multiple objective optimization model with equal weights into the following single-objective optimization model:

$$(M-2)\begin{cases} \max \gamma = \sum_{i=1}^{m}\sum_{j=1}^{n} w_j\left(\xi_{ij}^{(1)} + \xi_{ij}^{(2)}\right), \\ \text{s.t.}: \sum_{j=1}^{n} w_j^2 = 1, w_j \geqslant 0, j = 1, 2, \ldots, n. \end{cases}$$

To solve the above model, referring to (Wu & Chen, 2007), we construct the Lagrange function of the constrained optimization problem (M-2):

Table 2
Decision maker's preference information based on each criterion.

	C_1	C_2	C_3	C_4	C_5
A_1	G	MG	G	MG	M
A_2	G	MG	G	G	MG
A_3	MG	M	MG	VG	MG

$$L(w,\lambda)=\sum_{i=1}^{m}\sum_{j=1}^{n}w_j\left(\xi_{ij}^{(1)}+\xi_{ij}^{(2)}\right)+\frac{1}{2}\lambda\left(\sum_{j=1}^{n}w_j^2-1\right) \tag{16}$$

where λ is the Lagrange multiplier, and it is a real number.

Differentiating Eq. (16) with respect to $w_j(j = 1, 2,\ldots,n)$ and λ, and setting these partial derivatives equal to zero, we obtain the following set of equations:

$$\begin{cases}\frac{\partial L}{\partial w_j}=\sum\limits_{i=1}^{m}\left(\xi_{ij}^{(1)}+\xi_{ij}^{(2)}\right)+\lambda w_j=0\\ \frac{\partial L}{\partial \lambda}=\sum\limits_{j=1}^{n}w_j^2-1=0\end{cases} \tag{17}$$

By solving Eq. (17), we get a simple and exact formula for determining the criteria weights as follows:

$$w_j^{(*)}=\frac{\sum_{i=1}^{m}\left(\xi_{ij}^{(1)}+\xi_{ij}^{(2)}\right)}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\xi_{ij}^{(1)}+\xi_{ij}^{(2)}\right)}}. \tag{18}$$

By normalizing $w_j^{(*)}(j=1,2,\ldots,n)$ be a unit, we have

$$w_j=\frac{\sum_{i=1}^{m}\left(\xi_{ij}^{(1)}+\xi_{ij}^{(2)}\right)}{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\xi_{ij}^{(1)}+\xi_{ij}^{(2)}\right)}. \tag{19}$$

The weight vector of criteria is $w = (w_1,w_2,\ldots,w_n)$. Then, we can get $\gamma_i^{(1)}$ and $\gamma_i^{(2)}(i=1,2,\ldots,m)$ by Eq. (15). That is to say, the grey relational grade between the reference series and comparison series is an interval value $\bar{\gamma}_i=\left[\gamma_i^{(1)},\gamma_i^{(2)}\right](i=1,2,\ldots,m)$.

Step 6. Rank the alternatives.

"Alternative A_s being not inferior to A_t" is denoted by $A_s \succeq A_t$. The likelihood of $A_s \succeq A_t$ is defined and measured by that of $\bar{\gamma}_s \succeq \bar{\gamma}_t$, where $\bar{\gamma}_s$ and $\bar{\gamma}_t$ are corresponding grey relational grade interval numbers of alternatives A_s and A_t in A, respectively. Using the concept of likelihood for interval numbers (Li, Wang, Liu, & Shan, 2009), the likelihood of $A_s \succeq A_t$ for alternatives A_s and A_t in A can be determined as follows:

$$\begin{aligned}p(A_s\succeq A_t)&=p(\bar{\gamma}_s\geqslant\bar{\gamma}_t)\\&=max\left\{1-max\left\{\frac{\gamma_t^{(2)}-\gamma_s^{(1)}}{L(\bar{\gamma}_s)+L(\bar{\gamma}_t)},0\right\},0\right\},\end{aligned} \tag{20}$$

where $\bar{\gamma}_s=\left[\gamma_s^{(1)},\gamma_s^{(2)}\right],\bar{\gamma}_t=\left[\gamma_t^{(1)},\gamma_t^{(2)}\right],L(\bar{\gamma}_s)=\gamma_s^{(2)}-\gamma_s^{(1)}$ and $L(\bar{\gamma}_t)=\gamma_t^{(2)}-\gamma_t^{(1)}$.

Thus, the likelihood matrix can be obtained and expressed as follows:

$$P=(p_{st})_{m\times m}=\begin{bmatrix}p_{11}&p_{12}&\cdots&p_{1m}\\p_{21}&p_{22}&\cdots&p_{2m}\\\vdots&\vdots&&\vdots\\p_{m1}&p_{m2}&\cdots&p_{mm}\end{bmatrix}. \tag{21}$$

where $p_{st} = p(A_s \succeq A_t)(s,t = 1,2,\ldots,m)$ for alternatives A_s and A_t in A.

As the matrix P is a fuzzy complementary judgement matrix, optimal degrees of membership for alternatives $A_i(i = 1,2,,\ldots,m)$ can be defined as follows (Li et al., 2009):

$$V_i=\frac{1}{m(m-1)}\left(\sum_{r=1}^{m}p_{ir}+\frac{m}{2}-1\right). \tag{22}$$

Thus, a sort vector $V = (V_1,V_2,\ldots,V_m)$ of the alternatives can be obtained.

Rank all the alternatives $A_i(i = 1,2,\ldots,m)$ and select the best one(s) in accordance with the value $V_i(i = 1,2,\ldots,m)$. The bigger the value V_i, the better the alternative A_i is.

5. Numerical example

In this section, a numerical example (adapted from (Chen, 2000)) is presented to illustrate the use of the proposed method. Suppose that a software company desires to hire a system analysis engineer. After initial screening, three candidates (i.e., alternatives) A_1, A_2 and A_3 remain for further evaluation. In order to select the most suitable candidate, the decision maker takes into account the following five criteria: (1) emotional steadiness (C_1); (2) oral communication skill (C_2); (3) personality (C_3); (4) past experience (C_4); (5) self-confidence (C_5).

Assume that the decision maker provide his/her preference information on candidates with regard to criteria by using a linguistic variable, as depicted in Table 2.

To select the most suitable system analysis engineer, a brief description of the resolution process is given below.

First, we convert the linguistic variables shown in Table 2 to interval-valued triangular fuzzy numbers, as depicted in Table 3.

By using Eq. (10), the interval-valued triangular fuzzy decision matrix is normalized. Table 4 depicts the normalized decision matrix.

Second, using Formula (13), the distance of each candidate from the reference series is calculated. The result is depicted in Table 5. As demonstrated, the obtained distance is determined as an interval value.

Table 3
The interval-valued triangular fuzzy decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	[(5.5,7.5);9;(9.5,10)]	[(4.5,5.5);7;(8,9.5)]	[(5.5,7.5);9;(9.5,10)]	[(4.5,5.5);7;(8,9.5)]	[(2.5,3.5);5;(6.5,7.5)]
A_2	[(5.5,7.5);9;(9.5,10)]	[(4.5,5.5);7;(8,9.5)]	[(5.5,7.5);9;(9.5,10)]	[(5.5,7.5);9;(9.5,10)]	[(4.5,5.5);7;(8,9.5)]
A_3	[(4.5,5.5);7;(8,9.5)]	[(2.5,3.5);5;(6.5,7.5)]	[(4.5,5.5);7;(8,9.5)]	[(8.5,9.5);10;(10,10)]	[(4.5,5.5);7;(8,9.5)]

Table 4
Normalized decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	[(0.55,0.75);0.9;(0.95,1)]	[(0.45,0.55);0.7;(0.8,0.95)]	[(0.55,0.75);0.9;(0.95,1)]	[(0.45,0.55);0.7;(0.8,0.95)]	[(0.25,0.35);0.5;(0.65,0.75)]
A_2	[(0.55,0.75);0.9;(0.95,1)]	[(0.45,0.55);0.7;(0.8,0.95)]	[(0.55,0.75);0.9;(0.95,1)]	[(0.55,0.75);0.9;(0.95,1)]	[(0.45,0.55);0.7;(0.8,0.95)]
A_3	[(0.45,0.55);0.7;(0.8,0.95)]	[(0.25,0.35);0.5;(0.65,0.75)]	[(0.45,0.55);0.7;(0.8,0.95)]	[(0.85,0.95);1;(1,1)]	[(0.45,0.55);0.7;(0.8,0.95)]

Table 5
The distance from the reference series.

	C_1	C_2	C_3	C_4	C_5	$\left[\min_i\delta_{ij}^{(1)}, \min_i\delta_{ij}^{(2)}\right]$	$\left[\max_i\delta_{ij}^{(1)}, \max_i\delta_{ij}^{(2)}\right]$
$\left[\delta_{1j}^{(1)}, \delta_{1j}^{(2)}\right]$	[0.1555,0.2677]	[0.3136,0.3797]	[0.1555,0.2677]	[0.3136,0.3797]	[0.4950,0.5583]	[0.1555,0.2677]	[0.4950,0.5583]
$\left[\delta_{2j}^{(1)}, \delta_{2j}^{(2)}\right]$	[0.1555,0.2677]	[0.3136,0.3797]	[0.1555,0.2677]	[0.1555,0.2677]	[0.3136,0.3797]	[0.1555,0.2677]	[0.3136,0.3797]
$\left[\delta_{3j}^{(1)}, \delta_{3j}^{(2)}\right]$	[0.3136,0.3797]	[0.4950,0.5583]	[0.3136,0.3797]	[0.0289,0.0866]	[0.3136,0.3797]	[0.0289,0.0866]	[0.4950,0.5583]
$\left[\delta_{min}^{(1)}, \delta_{min}^{(2)}\right]$						[0.0289,0.0866]	
$\left[\delta_{max}^{(1)}, \delta_{max}^{(2)}\right]$							[0.4950,0.5583]

Table 6
The grey relational coefficient matrix $\left(\xi_{ij}^{(1)}\right)_{3\times5}$.

	C_1	C_2	C_3	C_4	C_5
$\xi_{1j}^{(1)}$	0.6859	0.4926	0.6859	0.4926	0.3723
$\xi_{2j}^{(1)}$	0.6859	0.4926	0.6859	0.6859	0.4926
$\xi_{3j}^{(1)}$	0.4926	0.3723	0.4926	1.0000	0.4926

Third, applying Eq. (14), the grey relational coefficient is obtained and the results are depicted in Tables 6 and 7.

Then, according to Eq. (19), we get the following weights:

$$w_1 = 0.2120;\ w_2 = 0.1639;\ w_3 = 0.2120;\ w_4 = 0.2482;\ w_5 = 0.1639.$$

Bring the obtained weights into Eq. (15), the grey relational grade is gotten:

$$\gamma_1^{(1)} = 0.5548,\ \gamma_2^{(1)} = 0.6225,\ \gamma_3^{(1)} = 0.5988,$$
$$\gamma_1^{(2)} = 0.5839,\ \gamma_2^{(2)} = 0.6315,\ \gamma_3^{(2)} = 0.6461.$$

That is, the grey relational grades of the candidates are interval numbers:

$$\bar{\gamma}_1 = [0.5548, 0.5839],\ \bar{\gamma}_2 = [0.6225, 0.6315],\ \bar{\gamma}_3 = [0.5988, 0.6461].$$

Afterwards, according to Eq. (20), likelihood of $A_1 \succeq A_2$ can be calculated as follows:

$$\begin{aligned} p(A_1 \succeq A_2) &= p(\bar{\gamma}_1 \geqslant \bar{\gamma}_2) = max\left\{1 - max\left\{\frac{\gamma_2^{(2)} - \gamma_1^{(1)}}{L(\bar{\gamma}_1) + L(\bar{\gamma}_2)}, 0\right\}, 0\right\} \\ &= max\left\{1 - max\left\{\frac{0.6315 - 0.5548}{(0.5839 - 0.5548) + (0.6315 - 0.6225)}, 0\right\}, 0\right\} \\ &= 0. \end{aligned} \tag{23}$$

Similarly, likelihoods of other pair-wise comparisons of candidates $A_i (i = 1,2,3)$ can be obtained. Thus, the likelihood matrix can be expressed as follows:

$$P = \begin{bmatrix} 0.5 & 0 & 0 \\ 1 & 0.5 & 0.5808 \\ 1 & 0.4192 & 0.5 \end{bmatrix}. \tag{24}$$

Finally, using Eq. (22), optimal degrees of membership for candidates $A_i (i = 1,2,3)$ can be calculated as follows:

Table 7
The grey relational coefficient matrix $\left(\xi_{ij}^{(2)}\right)_{3\times5}$.

	C_1	C_2	C_3	C_4	C_5
$\xi_{1j}^{(2)}$	0.6688	0.5551	0.6688	0.5551	0.4367
$\xi_{2j}^{(2)}$	0.6688	0.5551	0.6688	0.6688	0.5551
$\xi_{3j}^{(2)}$	0.5551	0.4367	0.5551	1.0000	0.5551

$$V_1 = 0.1667;\ V_2 = 0.4301;\ V_3 = 0.4032.$$

Since $V_2 > V_3 > V_1$, then $A_2 \succ A_3 \succ A_1$. Therefore, the most suitable candidate is A_2.

6. Conclusion

The traditional grey relational analysis (GRA) methods are generally suitable for dealing with the MCDM problems in which the information taking the form of numerical values, and yet they will fail in dealing with MCDM problems in which the information taking the form of interval-valued triangular fuzzy numbers. In this paper, we have investigated the MCDM problem with unknown criterion weights, in which the criterion values are interval-valued triangular fuzzy numbers. An extended GRA method is proposed. To determine the criterion weights, we establish the multiple objective optimization models based on the basic idea of traditional GRA. Then, by linear equal weighted method, the multiple objective optimization models can be transformed into a single-objective programming model. By solving the single-objective programming model, we can get the criterion weights. Next, based on the traditional GRA method, calculation steps for solving interval-valued triangular fuzzy MCDM problem with unknown criterion weights are given. At the same time, the grey relational grade between the reference series and comparison series are calculated. Because the grey relational grade is an interval value, we use the likelihood degree of comparing interval numbers to rank the alternatives and select the desirable one. Finally, a practical personnel selection problem is given to verify the developed approach. The proposed method has a clear logic and a simple computation process and has less loss of information than other literatures. It is also important for developing and enriching theories and methods for MCDM. Furthermore, we can extend the developed models and procedures to deal with the MCDM with interval-valued trapezoidal fuzzy numbers. In future research, our work will focus on the application of the proposed method in the similar decision problems, such as project evaluation, factory location, bridge risk assessment, and material selection.

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