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Application of a hybrid algorithm –PSOSA in well test parameter estimation

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ABSTRACT

Estimating the significance parameters, such as skin factor, permeability, wellbore storage coefficient, are the most component of transient pressure analysis. Many optimization algorithms have been applied to parametric estimation and realized the minimum error of well test curve. Although a flexible heuristic particle swarm optimization can hunt optimal solution rapidly, it is difficult to search further in the vicinity of the optimal solution. Hence, to alleviate the local optimum and premature convergence, a global hybrid algorithm referred to as particle swarm simulated annealing is proposed, and proves to have better performance of convergence and accuracy than traditional methods, which are more suitable for parameter estimation.

1. Introduction

Analyzing the reservoir transient pressure is essential to its production, and the large amounts of approaches of interpretation to analysis pressure transient. The emergence of double logarithmic marked the beginning of new well test analysis. Subsequently, some people [1,2] made related improvements to make it more practical than before. Besides, the parameter estimation is an integral part of the well test, and the extensive research has been done. Meanwhile, the unknown parameters, such as skin factor, wellbore storage coefficient etc, are obtained. The objective function most frequently used is residual sum of squares between the actual measured value and the theoretical value, In fact, the optimal fitting is a process that minimizes the objective function and the data mismatch with model. It has been interpreted as the least square method (LS). Accordingly, aiming at the property of nonlinear, the model is transformed into solve the nonlinear least squares problem. In fact, the optimal fitting process minimizes the function value and decreases data mismatch. Then the modified algorithm [3] was represented in parameter identification, which was regarded as accurate and satisfied with practical needs. Total least squares considered the influence of pressure and time domain in process of transient analysis, compared with PSO and differential evolution [4], the nonlinear regression LM method were the worst option to estimate the parameters [5,6]. It largely depends on initial state and easily converged to local optimal [7]. Meanwhile, the development global optimization technique gained popularity in the aspect of parametric estimation [5]. Artificial Neural Network, with the merits of classification and noise insensitive characteristics, was also used to estimate parameters from the well test data, in order to distinguish the real response with noise, the data of pressure derivatives were first pre-treated, the initial estimation of reservoir parameter according to the derivative plot, and then proceed nonlinear regression based on the value obtained, hence, the flow patterns and parameter estimation were accurately identified. Whereas it has many advantages, this method relied on the option to initial configuration [8–10]. Besides, many researchers pay widespread attention to the hybrid of the nonlinear regression and intelligence algorithm for the problem of parameter estimation in well test. The majority of global optimization methods mentioned above were introduced the nonlinear regression. But some were not efficient and subjected to some restrictions, PSO [11] easily get the premature convergence and SA [12] is easy to trap

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local optimal. In recent years, the compound algorithm which consists of various intelligence algorithms, with wide-range applications, have been studied in many fields, their combination have excellent performance [13–15].

However, there almost no one apply it to well test. Thus, the main purpose of this article is to propose the hybrid algorithm about parametric identification in well test. The rest of article is presented as: The objective function of model is obtained at initial part. Then, the method of PSO, SA and their hybrid are presented. The model involved parameters estimation of homogeneous vertical well is suggested by different algorithms and gives conclusions at last.

2. The fitness function in parametric estimation

To compare the superiority of the various optimization algorithms, the computing method of relevant fitness value is consistent, by calculating the least square sum of difference between measured value and theoretical value. The intuitive interpretation of fitting effectiveness is that pressure curve and derivative curve gradually approach or overlap at some extent. The quality of parameter estimation, through consideration of different single algorithms, synthesis algorithm, has a direct influence on the transient pressure performance of well test. As the model is explored, automatic fitting gets simultaneously, a preferable matching will receive at last.

The performance of involved algorithms is decided by following equation.

$$F(\alpha) = \min\sum_{i=1}^{n}\left[p'_{cl}(i) - p'_{th}(i)\right]^2 \tag{1}$$

where $F(\alpha)$ indicates the fitness function and α are vector of unknown parameters that are calculated by optimization technique, p'_{cl} and p'_{th} represent the measured and theoretical values of pressure derivatives, $n = (1, 2...N')$ is the number of data. The reason for selecting derivative is that it is sensitive to parameters.

3. The basic content of the three algorithms

3.1. Particle swarm optimization and simulated annealing

The algorithm simulates the birds feeding behavior in nature and regards a bird as a particle, which is considered as a possible solution to parametric estimation problem of the well test. Also, it is a random searching algorithm based on group cooperation between individuals and groups by establishing information sharing mechanism, and search target quickly. The main updating formulations are offered as follows:

$$v_j^{n+1} = w\cdot v_j^n + c_1\cdot rand\left(gbest_j^n - p_j^n\right) + c_2\cdot rand\left(zbest_j^n - p_j^n\right) \tag{2}$$

$$p_j^{n+1} = p_j^n + v_j^n \tag{3}$$

Among two equations, all vectors are used on the condition of *nth* iteration$n = \{0, 1, 2\cdots N-1\}$, and $j \in \{1, 2, 3\}$ are the dimension of the solution space. p_j^n and v_j^n are the position and velocity of random particle respectively; $zbest_j^n$ represents the global optimal position in searching process so far, $gbest_j^n$ is the individual optimal position of p_j^n; w is the inertial weight which maintain the balance between global and local search. c_1 and $c_2$are the constant named as acceleration coefficient, we consider $w = 1.2$ and$c_1 = c_2 = 2$in this article [16]. The *rand* can generate a random variable with the range of $[0, 1]$ uniformly. The capability of PSO is to find the best particle (solution) in the searching space. According to practical situation of model to set initial value, the initial position p_j^n that is considered to be both as local and global optimal value, respectively as$gbest_j^n$ $zbest_j^n$, which randomly are distributed from Eqs. (2) and (3). They continuously update the current velocity and position of the particle by Eqs. (2) and (3). Since these particles have the feature with memorability. Consequently, communication between individuals can better determine the local optimal ($gbest_j^n$) in the searching process. If the current position p_j^n is superior to$gbest_j^n$, it employs the p_j^n instead of the$gbest_j^n$, otherwise, $gbest_j^n$ represents the personal best value. Furthermore, the comparison of fitness function value from different local optimums can access to obtain the global optimal position ($zbest_j^n$).Then the method proceed a new searching, eventually, the algorithm will continue to iterate until the termination criterion is satisfied.

The SA algorithm [17] is illuminated by the practical process which simulates the physical quenching in thermodynamics. Increased the internal temperature, solid internal particle are changed into the disordered state. Conversely, when gradually cooling it, the condition of particles from disorder into order, has implemented equilibrium at each different temperatures. Ultimately, all material (particles) reach the ground state at room temperature which be deemed to attain the optimal condition. It is important that SA has a high local search capability. The two significant concepts are applied to SA algorithm as follows:

Boltzmann probability distribution and cooling schedule [18].

$$P \propto \exp(-\Delta E/kT) \tag{4}$$

$$T = \alpha\cdot T(0 < \alpha < 1) \tag{5}$$

Eq. (4) is used to select the configuration to freely exit the local value, and accepts worst solution as the optimal value with high probability to keep algorithm running. Where $P$is the probability that the particles tend to be equilibrium at temperature$T$; $E$is the energy of particles, ΔE is the energy differential among the fitness function value of two particles, calculated by Eq. (1). The k represents the Boltzmann constant and consider $k = 1$ in this article. Data production, selection, acceptation or discard are three main process of SA.

In this section, the basic mathematical background of SA have been only established, the other detail steps have a better illustration in next.

3.2. The hybrid algorithm PSOSA

The novel algorithm is introduced to solve the parametric estimation in well test analysis. One side, although the PSO can be easily attained the optimal solution with great speed, it is hard to search the vicinity of optimal solution further. There may appear the circumstance in which calculation enters a period of stasis and falls to the local minimal. Then the ability of SA to search the global optimal might be poor. Hence, this paper presents the algorithm described as simulated annealing particle swarm (PSOSA), which has the capable of intensifying the local search ability and increasing the global search ability. The main idea of PSOSA is to utilize the fast speed of global search capability with PSO and the accuracy of local convergence with SA, which integrate the two kinds of merits at the parametric estimation process.

The major steps as shown below:

① Get the global optimal solution through the PSO algorithm;

②The result value obtained above as the initial value to conduct subsequent steps;

③According to the judge Metropolis criterion to select the best

particle;

④ There may arise two cases: One is that continues to SA and proceed ⑤; The another is that the program return to the modified PSO algorithms (PSO2) to recalculate the global optimum once again and then repeats ②-④;

⑤To determine whether the solution satisfies the convergence condition. If condition is not meet, the current value is turned by the method of random disturbance, and then the algorithm repeats ③-④, or else the best solution is obtained.

The steps of novel algorithm have specific interpretation as shown in Figs. 1–3.

4. The concrete realization of three algorithms with model

4.1. Physical model of the homogeneous vertical well

The model that considers storage coefficient and skin effect in homogeneous vertical well is presented.

The dimensionless mathematical model that account for storage coefficient and skin effect with homogeneous vertical well present as follows:

$$\begin{cases} \dfrac{\partial^2 p_D}{\partial r_D^2} + \dfrac{1}{r_D}\dfrac{\partial p_D}{\partial r_D} = \dfrac{\partial p_D}{\partial r_D} \\ p_D(r_D, 0) = p_D(\infty, 0) = 0 \\ p_{wD} = \left(p_D - S\dfrac{\partial p_D}{\partial r_D}\right)\Big|_{r_D=1} = 1 \\ C_D\dfrac{dp_{wD}}{dt_D} - \left(r_D\dfrac{\partial p_D}{\partial r_D}\right)\Big|_{r_D=1} = 1 \end{cases} \tag{6}$$

We get the dimensionless form of wellbore pressure.

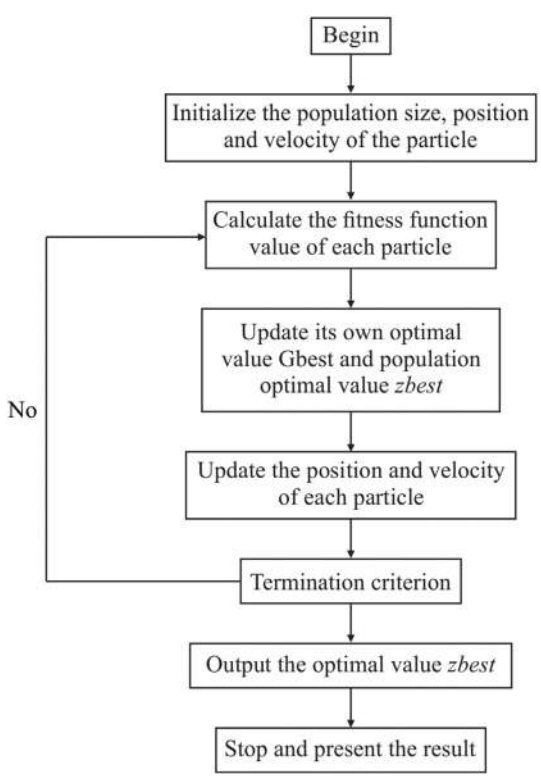

Fig. 1. The PSO algorithm.

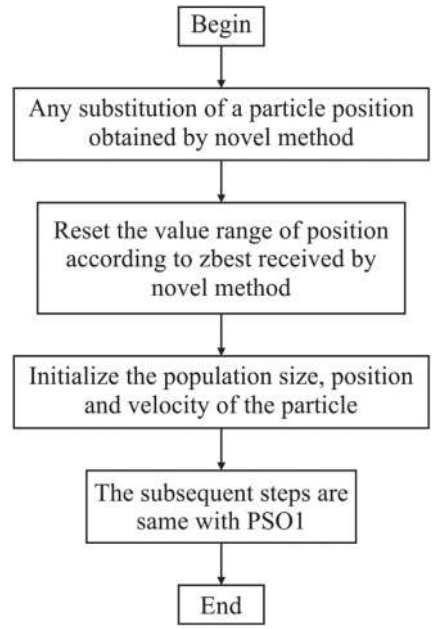

Fig. 2. The PSO2 algorithm.

$$\overline{p}_{wD} = \frac{K_0\left(\sqrt{f(z)}\right)}{z\left\{zK_0\left(\sqrt{f(z)}\right) + \left[\sqrt{f(z)}\,K_1\sqrt{f(z)}\right]\right\}} \tag{7}$$

Among$f(z) = z$, where the modified Bessel functions of second kind, zero order is $K_0(x)$and $K_1(x)$ is the modified Bessel function of second kind, first order.$\overline{p}_{wD}$ is the dimensionless pressure in Laplace space; z is the Laplace transformation variable [19]. In this example, the computation of Eq. (7) needs $k, S$and C, which are unknown variables to be solved by optimal algorithms.

4.2. The concrete realization of PSO algorithm

The iteration number is thirty, the performance of the PSO reflect the false appearance that attains the result as we expected, as shown below in Fig. 4(a) and (b). The randomness of searching region based on that, it needs repeat many times. Fig. 4 is result performance chosen from the multiple runs. Fig. 4(a) illustrate original image, Fig. 4(b) how the magnified image in the interval$t_D/C_D \in [10^0, 10^4]$. The green points present the pressure (p_i) based on measured data and red points present the pressure derivative obtained from measured data; The red and black lines indicate the fitting curve of pressure and pressure derivative separately in view of optimized parameters obtained. The Fig. 4 describes the fitting effect after the PSO which can get the optimized parameters in well test analysis. Actually, we observe they have weak performance, especially Fig. 4(b). It clearly shows that most points of measured value curve are not close to theoretical curve. The relation of iterations to fitness value got by Eq. (1) is studied in Fig. 5, the increase iteration from thirty to thousand, the corresponding fitness value decrease. Therefore, the more repeat, the better optimized parameters are achieved.

4.3. The concrete realization of SA algorithm

Although the SA algorithm possesses the merit that is able to search the local region optimal quickly, it has a deficiency that

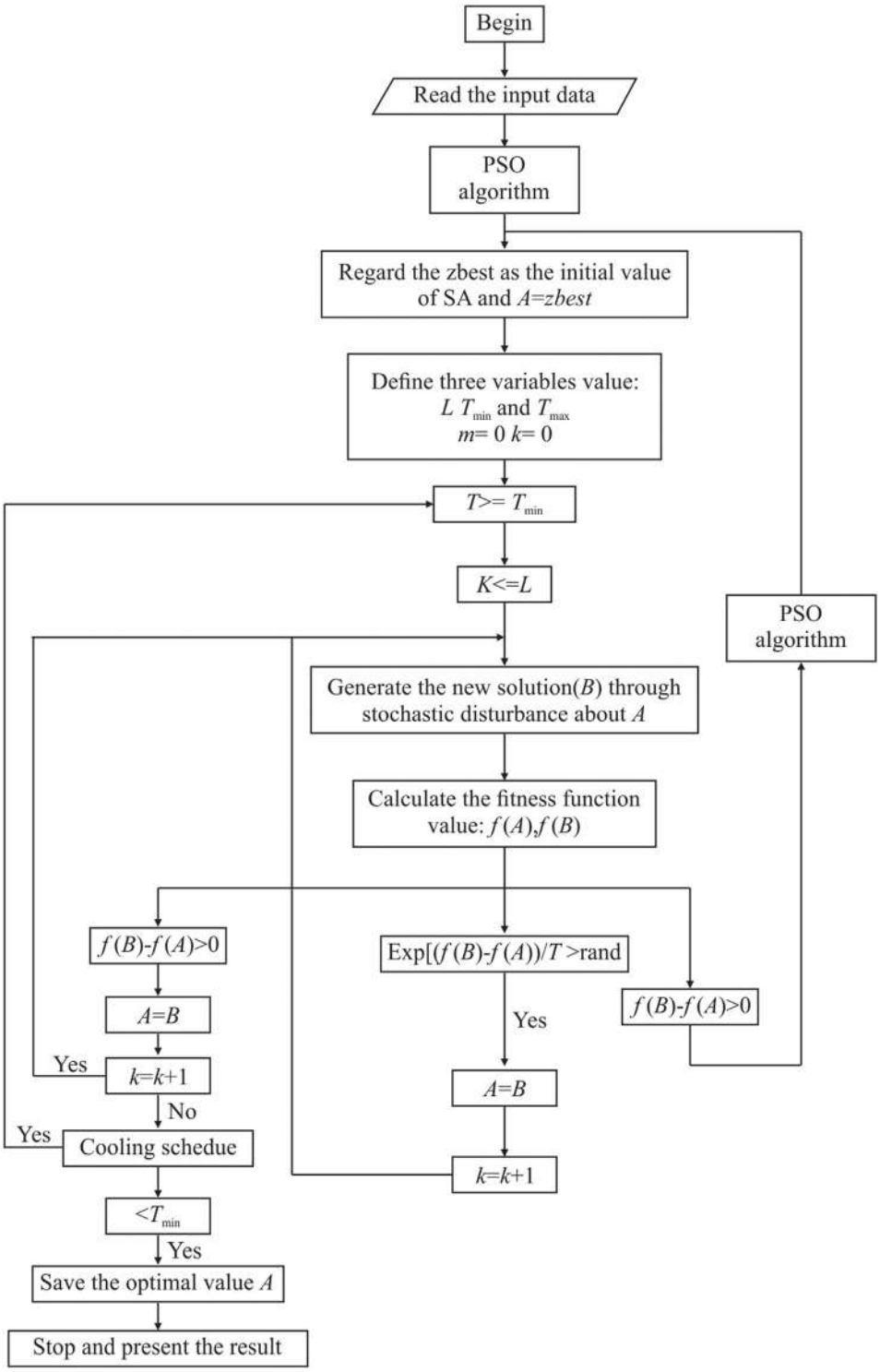

Fig. 3. The specific steps of PSOSA algorithm.

difficult to search the global optimal value. At the respect of parametric estimation, the worst values are considered as optimized parameters obtained in well test process, so the fitted curve which completely not accord with practical situation. Fig. 6 reflects the matched curve of relevant data and reveals the performance of the fitted curve is exceedingly low when L = 500 and 100. We can view that the red and blue matched curves of Fig. 6 are both not close to black, which are real values from the homogeneous vertical well, it is not a wise choice to estimate parameters in this model. Obviously, comparing the degree of different curves closeness, the PSO is superior to SA.

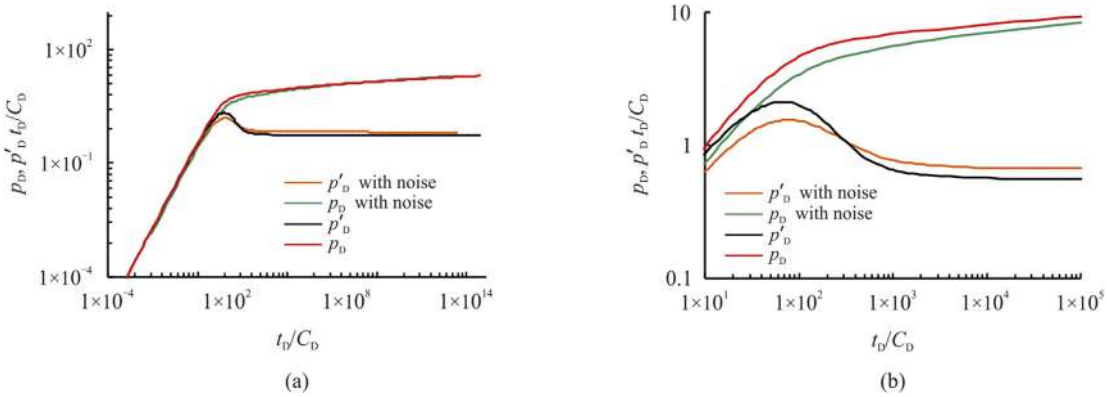

Fig. 4. The PSO is used to estimate parameters with homogeneous vertical well.

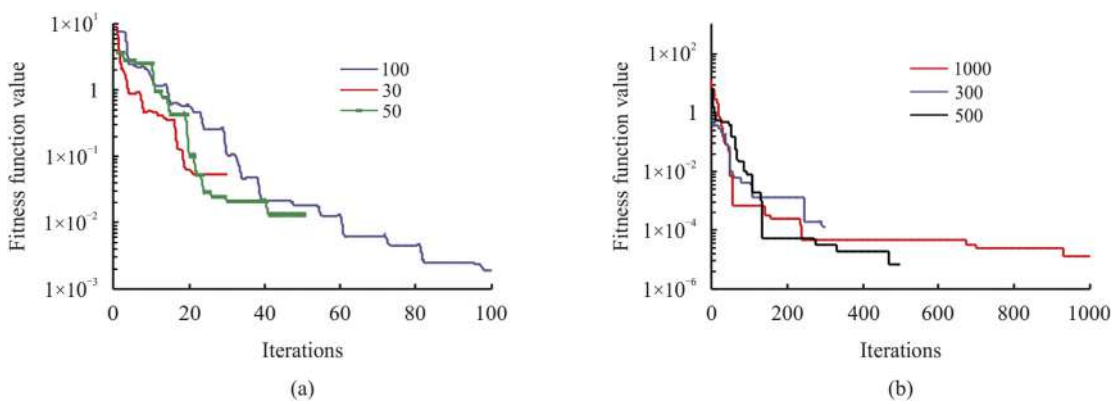

Fig. 5. The fitness function value with different iterations of PSO.

4.4. The concrete realization of novel hybrid PSOSA algorithm

It employs the novel hybrid algorithm discussed in detail in Fig. 3. When the iteration of particles is less than thirty, Fig. 4 shows that the PSO not have fine prospect in this model. Besides, although the SA has inferior performance, the superiority of local searching can be taken full consideration. So, the major work is introduced the hybrid algorithm in parametric estimation process. The vicinity of global optimum is required to be search further in proceeding. Therefore, make stochastic disturbance which obey the uniform distribution with regard to global optimum. In order to verify the reliability of PSOSA, it should be repeated it again, without changing specific parameter settings, such as$L$,$T_{\max}$, $\alpha \in (0,1)$and so on, The different emergence of fitted curves and optimized parameters, which interpret the comprehension of well test analysis with difficulty. Some situations that randomly appear are not necessary in searching process. Accordingly, the series of data analysis are demanded to repeat selection in this phase, at the stochastic disturbance, each iteration runs hundred times and get hundred fitness function values. The minimum value is selected as the true stochastic disturbance to continue the subsequent steps. Furthermore, it is also the flexible algorithm that can adjust parameter settings through the practical condition at the optimization process. After tuning parameter settings continuously, such$T_{\max}$,$T_{\min}$ and$\alpha \in (0,1)$, it can get the desired effectiveness of fitted curves. This approach can flexibly control the parameter selection to achieve optimal solution based on practical requirements. In addition to adjusting $T_{\max}$ and αseparately, the remaining parameters keep constant. The Fig. 7 represents the PSOSA to precede automatic fitting based on optimized parameters which are closest to true k, $S$and$C$in well test analysis. Clearly, the vital procedure of PSOSA is that making the global optimal value gained from PSO as initialization on SA.

The similarity of Figs. 4(b) and 7(b) -(c) is amplifying images and the scope of enlarged interval, the Fig. 7(c) is more accurate, which demonstrate the hybrid PSOSA has preferable effectiveness on the

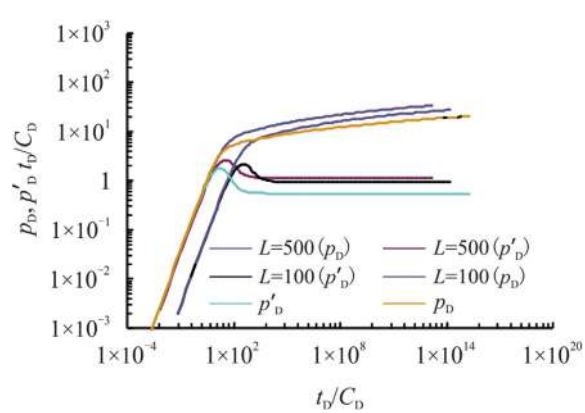

Fig. 6. The SA is used to estimate parameters with homogeneous vertical well.

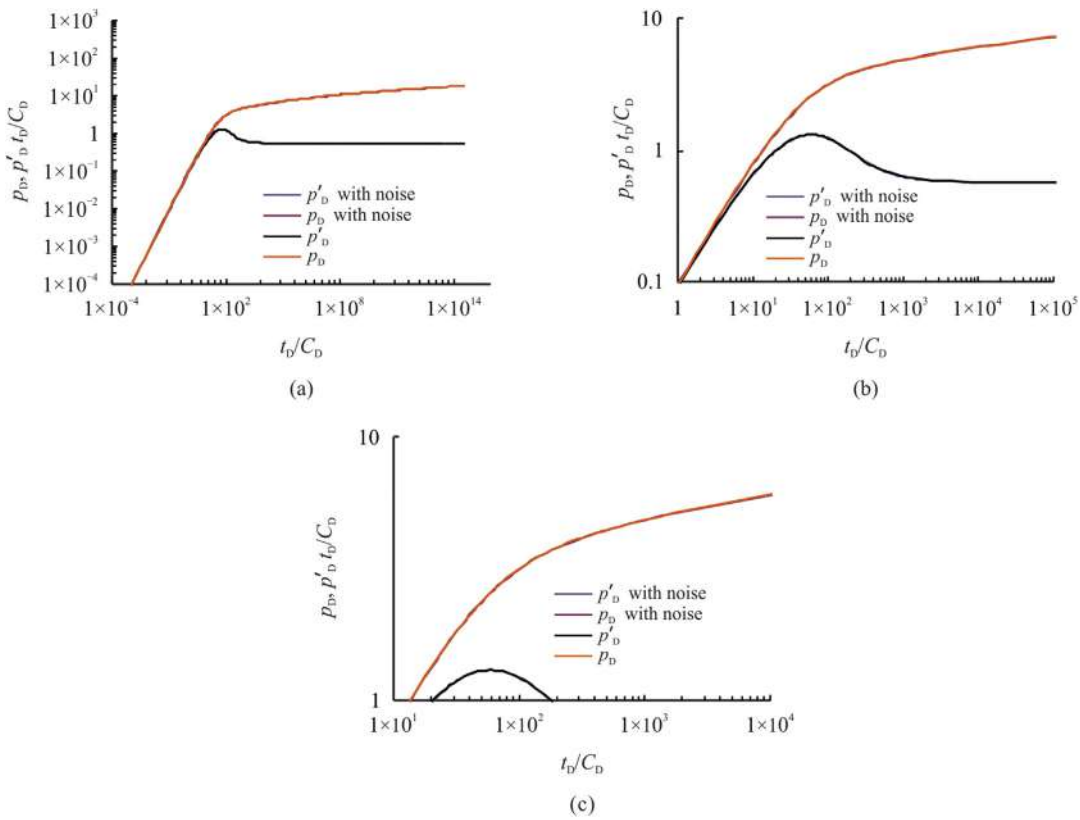

Fig. 7. The novel algorithm is studied to estimate parameters with same model.

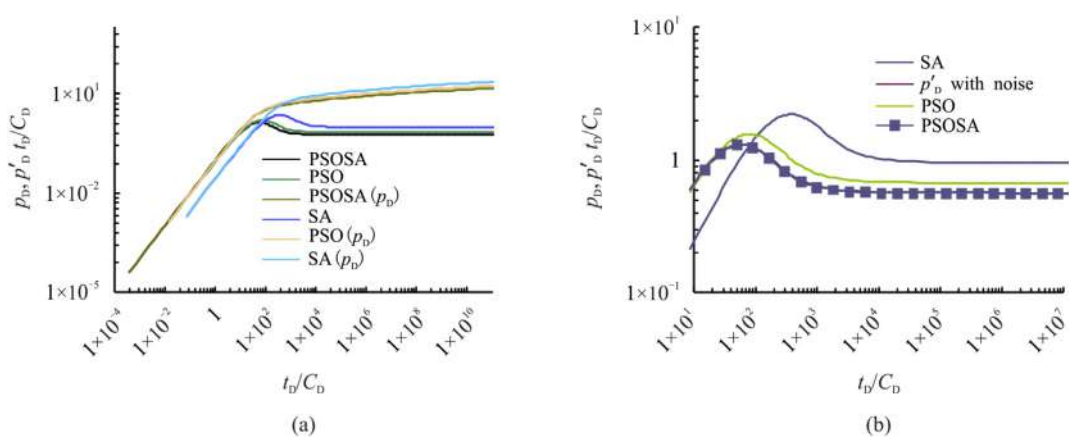

Fig. 8. Comparison of fitting performance with three kinds of algorithms.

basis of same iteration at fitting process. Focus on it, the influential of performance with different $T_{max}$by comparing with various f_{min} designed by Eq. (1) is showed in Table 3. From the Fig. 8, we can draw that the smallest fitness function value, the excellent optimized parameters and fitted curves are achieved in the process of well test analysis. Table 1 represents the known parameters of the article.

From Table 2, under the same temperature, along with the αincreasing, the number of iterations is increased correspondingly. Based on optimal value obtained by PSO, it should be searched further in smaller areas. Through heating and cooling, the resulted value can jump out of the local optimal, by this way the values of different neighborhood can be optimized continuously, which get the accurate optimized parameters. In general, at the same initial temperature (T_{max}), when$\alpha$is larger, the process search more fully, the better value can be obtained.

Distinctly, Table 3 represents the specific parameters values of

Table 1
Parameter values used in physical model.

	q	B	μ	c_t	r_w	φ	h
value	$\frac{10}{24\times60\times60}$	1.639	9×10^{-4}	$\frac{0.00463}{10^6}$	0.1	0.1	10

Table 2
The f_{min} obtained from the PSOSA with various parameter settings.

T_{max}	50	200	500	1000	5000
$f_{min}(\alpha_1 = 0.90)$	9.444e-05	6.868e-05	1.014e-04	1.960e-04	1.096e-04
m_1	40	53	62	68	83
$f_{min}(\alpha_2 = 0.95)$	1.398e-05	3.253e-05	2.574e-06	7.538e-05	5.339e-07
m_2	81	108	126	140	171
$f_{min}(\alpha_3 = 0.99)$	2.641e-06	9.849e-06	2.299e-06	1.549e-06	4.717e-07
m_3	412	550	641	710	870

Table 3
The specific parameter settings with three kinds of algorithm.

	n	N	T_{max}	T_{min}	L	α	m	f_{min}
PSO	40	10	/	/	/	/	/	0.3743
SA	/	/	200	0.8	1000	0.9	40	7.93233
PSOSA	40	10	200	0.8	1000	0.9	40	9.4442e-05

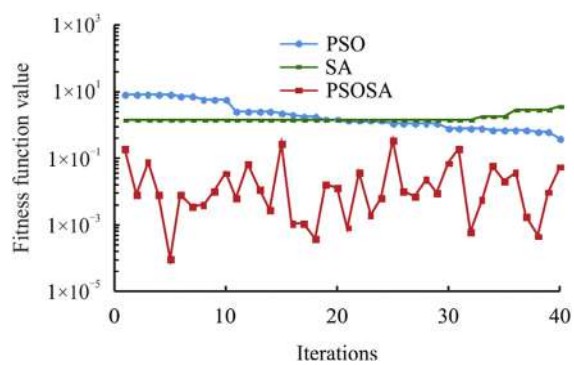

Fig. 9. Comparison of fitting effectiveness with three kinds of algorithms.

three algorithms. The Fig. 8, based on Table 3, describes that the hybrid algorithm has better performance compared with single. We focus on fitness function values and show the smaller of these have better effectiveness, which has extensively stability in well test analysis.

From Fig. 9, with increase the iterations, the fitness function values obtained by novel method are less than the other on the whole. It elaborate that the composited algorithm has higher efficiency. Furthermore, from the above analysis, we can draw that getting the superiority of PSOSA by comparing both fitting curves performance and the effectiveness of fitness function values in three algorithms.

5. Conclusions

The purpose of this paper is to provide a hybrid algorithm to apply parametric estimation in well test analysis. Through comparing the application of three global optimization algorithms, we observe that varying performance with different values of the parameters is received. The applicability of algorithm is judged by its efficiency and performance. Hence, we conclude that the hybrid algorithm is demonstrated best, the PSO shows medium and the SA is worst, which are all based on the model of homogeneous vertical well to precede parametric estimation.

Acknowledgments

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Nomenclature

T_{max}	Initial temperature
T_{min}	Final temperature
f_{min}	Minimum fitness function value
n	Iteration number
N'	Data points
N	Population size
m	Iteration number of novel algorithm
L	Markov Chain length
k	Permeability, m^2
C	Wellbore storage coefficient, m^3/Pa
S	Skin factor, dimensionless
p_i	Initial pressure of reservoir, Pa
q	Production rate of homogeneous vertical well, m^3/s
B	Oil volume factor, dimensionless
μ	Oil viscosity, Pa•s
C_t	Compressibility of model, 1/Pa
r_w	Radius of well, m
ϕ	Porosity, dimensionless
h	Formation thickness, m
p	Pressure, Pa

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