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Construction of Optimal Portfolio Using Single Index Model and Constant Correlation Model for the LQ45 Index over the Period 2013 - 2017

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Abstract:
The increasing of shares volume in Indonesia happened from 2013 – 2017. This proves that there is an increasing in interest and awareness of the Indonesian people on investing. Therefore, investors need to do an optimal portfolio analysis to maximize return and minimize the risk. The purpose of this research is to create an optimal portfolio using single index model and constant correlation model. To create an optimal portfolio, the researcher used monthly data of return from 26 stocks that consistently listed in LQ45 from 2013 – 2017. The calculation of optimal portfolio performance formed is done to know which method has better optimal portfolio formation. The performance of each portfolio formed is evaluated by using Sharpe ratio, Treynor ratio, and Jensen ratio. The result of single index model method observed that only 10 stocks out of 26 stocks sampled stocks are qualified to be included in optimal portfolio. While for constant correlation model consist of 6 stocks. Conclusively, the research revealed that optimal portfolio constructed by using single index model has better performance compare with constant correlation model. Sharpe ratio, Treynor ratio, and Jensen ratio are showed the same result performance.

Keywords: *Construction optimal portfolio, single index model, constant correlation model, performance optimal portfolio*

1. Introduction

The increasing of the volume shares sold in IDX showed that there is an increasing of an interest and awareness of the Indonesian people on investing. The purpose of investors investing is to improve the welfare of investors. In this case is monetary welfare[1]. Investors invest in anticipated future returns, but those return rarely can be predicted precisely. There will almost always be risk associated with investments [1]. If investor wants a higher expected return, investor will have to pay a price in terms of accepting higher investment risk and it concludes that as a risk-return trade off.

To reduce the risk received by investors, as stated by Markowitz “not put all eggs in one basketball”. Markowitz stated that risk might be reduced by combining several single securities into portfolio form. The risk can be eliminated as long as the return each security does not correlate positively and perfectly [2]. Keown stated that it is very important for investors to do diversification because it could decrease the portfolio risk without decreasing the expected return [3]. It is very important for investors to do diversification because it could decrease the portfolio risk without decreasing the expected return that might be accepted, which investor need the most.

William Sharpe has been developed method to construct optimal portfolio, namely Single Index Model. Single Index model is based on the observation that the price of a security fluctuates in the direction of the market price index [2]. Another method to determine the optimal portfolio can also use the constant correlation model. The constant correlation model assumes that the correlation for all pairs of securities is the same.

Many researchers have been conducted the research regarding how to construct the optimal portfolio. Dhea Ayu and Irni Ynuita (2015) also Nyokangi C.D (2016) conducted research on constructing optimal portfolio using single index model and constant correlation model. The result revealed that research from Dhea Ayu shows that single index model has better performance while Nyokangi has revealed that constant correlation model is better when considering investment with long time horizon whereas the single index model is better when considering investment with short time horizon.

As mentioned above, after knowing the optimal portfolio formation, it is important to know how well the portfolio perform by doing assessment to the optimal portfolio. The evaluation of portfolio performance consists of Sharpe ratio, Treynor ratio, Jensen ratio. Sharpe ratio is done by dividing the excess return to portfolio variability which is stated by the standard deviation. On the other hand, Treynor ratio uses different divider which divides the excess return with the systematic risk known as beta portfolio. Jensen ratio is a ratio that shows the difference between the level of actual return obtained by a portfolio and the level of expectation if the portfolio is in the capital market line.

Optimal portfolio with higher ratio means the optimal portfolio has better performance than the other optimal portfolio formed.

2. Objective of the Research

The purpose of this research is to construct optimal portfolio by using single index model and constant correlation model with the data used are from index LQ45. The optimal portfolio formed also being evaluated by using Sharpe ratio, Treynor ratio and Jensen ratio to know which method has better performance.

3. Theoretical Background

3.1. Optimal Portfolio Using Single Index Model

According to Zubir, single index model is a technique for measuring the return and risk of a stock or portfolio which assumes that stock return movements are only related to market movements [4]. The calculation of the optimal portfolio using single index model is calculated based on a number that can determine whether a security can be included in the optimal portfolio or not. This number is called excess return to beta, which can be calculated with the following equation:

$$ERB_r = \frac{E(R_i) - (R_{Br})}{\beta_i}$$

To know which securities are qualified to enter optimal portfolio selection, cut-off rate point is needed. The securities that will be included in the optimal portfolio are securities that have an ERB value greater than or equal to each Ci value. While securities with ERB value with their Ci value are not included in the construction of an optimal portfolio. The formula to determine Ci value is as follows:

$$C_i = \frac{\sigma_M^2 \sum_{j=1}^{i} \frac{[E(R_i) - R_{fr}] . \beta_i}{\sigma_{ei}^2}}{1 + \sigma_M^2 \sum_{j=1}^{i} \frac{\beta_i^2}{\sigma_{ei}^2}}$$

Where C* is the last value of Ci where the ERB value is still greater than the value of Ci. After knowing the stock qualified to be included in portfolio, it is important to know the proportion of the investment to each stock. The proportion formula is written below:

$$w_i = \frac{Z_i}{\Sigma Z_j}$$

Where:

$$Z_i = \frac{\beta_i}{\sigma_{ei}}(ERB_i - C*)$$

Expected return and risk portfolio single index model is calculated by using the formula:

$$E(R_p) = \alpha_p + \beta_p . E(R_M)$$

$$\sigma_p^2 = \beta_p . \sigma_M^2 + \sigma_{ei}^2$$

3.2. Optimal Portfolio Using Constant Correlation Model

According to Elton, constant correlation model is accepted as the best way to forecast correlation coefficient that assumes the correlation between all pairs of securities is same. The formation of optimal portfolio in constant correlation model is having the same procedure with single index model, the difference is all securities ranked by their Excess Return to Standard deviation (ERS) [4]. The formula of excess return standard deviation is written as:

$$ERS_i : \frac{(R_i - R_F)}{\sigma_i}$$

The formula to set the cut-off point is written as follows:

$$C_i = \frac{\rho}{1 - \rho + i\rho} \sum_{j=1}^{i} \frac{R_j - R_F}{\sigma_j}$$

Where ρ is coefficient correlation that assumed constant for all securities with the formula:

$$\rho = \sum_{i=1}^{N} . \sum_{i=1}^{N} \rho_{ij}$$

The proportion of the optimal portfolio chosen for each security is calculated as following:

$$w_i = \frac{Z_i}{\Sigma Z_j}$$

Where:

$$Z_i = \frac{1}{(1-\rho)\sigma_i}\left[\frac{R_{i-} R_F}{\sigma_i} - C^*\right]$$

Expected return and risk portfolio single index model is calculated by using the following formula:

$$E_{Rp} = \Sigma w_i . E(R_i)$$

$$\sigma_p^2 = \Sigma w_i^2 \sigma_i^2 + \Sigma\Sigma w_i . w_j . \sigma_{ij}$$

Where σ is covariance between each stock. Covariance between each stock is calculated by using the stock with positive ERS value.

4. Research Method

Based on existing theory and method, this research uses quantitative method. This study is also a comparative study because it compares the object with different method. The sample of research is 26 stock that consistently listed on the LQ45 Index over the period 2013 – 2017. The research used secondary data such as adjusted closing monthly closing price of each stock, adjusted closing price of IHSG as the market index and SBI rate as the proxy of risk-free rate.

5. Result and Discussion

5.1. Single Index Model

The following table is the calculation result for optimal portfolio construction using single index model:

Stock Code	E(R)	β	α	ΣEI	ERB	A	B	ΣA	ΣB	C	Z	W	W.β	W.α	W.σei
BBNI	-0.0151	-0.2093	0.0953	0.1667	0.0965	0.0257	0.2629	0.0257	0.2629	0.0000	-0.1075	-0.0170	0.0036	-0.0016	-0.0028
INCO	0.0131	0.2779	0.0258	0.3007	0.0270	0.0071	0.2569	0.0328	0.5198	0.0000	0.0149	0.0024	0.0007	0.0001	0.0007
PTBA	0.0243	0.7684	0.0229	0.0371	0.0241	0.3915	15.9087	0.4243	16.4284	0.0005	0.2731	0.0432	0.0332	0.0010	0.0016
UNVR	0.0203	0.6157	0.0225	0.0146	0.0237	0.6298	25.9771	1.0541	42.4056	0.0012	0.5401	0.0855	0.0526	0.0019	0.0012
TLKM	0.0212	0.7133	0.0205	0.0071	0.0217	1.5924	71.8457	2.6464	114.2513	0.0028	1.0854	0.1718	0.1226	0.0035	0.0012
UNTR	0.0152	0.5570	0.0160	0.0149	0.0171	0.3657	20.7778	3.0121	135.0291	0.0031	0.2333	0.0369	0.0206	0.0006	0.0006
BBRI	0.0306	1.6540	0.0137	0.0017	0.0149	24.5111	1607.9295	27.5232	1742.9586	0.0107	3.9305	0.6222	1.0291	0.0085	0.0011
BUMI	0.0193	1.1140	0.0110	0.0901	0.0122	0.1725	13.7675	27.6958	1756.7261	0.0107	0.0167	0.0026	0.0029	0.0000	0.0002
ICBP	0.0185	1.1105	0.0104	0.0025	0.0116	5.8928	495.8006	33.5886	2252.5266	0.0109	0.3189	0.0505	0.0561	0.0005	0.0001
GGRM	0.0125	0.6296	0.0099	0.0105	0.0111	0.4332	37.9128	34.0218	2290.4394	0.0109	0.0122	0.0019	0.0012	0.0000	0.0000
ADRO	0.0109	0.5138	0.0092	0.0476	0.0104	0.0598	5.5435	34.0816	2295.9829	0.0109					
AKRA	0.0133	0.7444	0.0092	0.0110	0.0104	0.5376	50.1994	34.6192	2346.1823	0.0109					
BBCA	0.0163	1.1164	0.0084	0.0011	0.0096	10.6209	1084.4120	45.2401	3430.5943	0.0106					
BMRI	0.0158	1.6114	0.0051	0.0005	0.0063	33.4419	5153.9224	78.6820	8584.5167	0.0084					
KLBF	0.0109	1.0899	0.0037	0.0023	0.0049	2.6182	516.2077	81.3002	9100.7243	0.0082					
INDF	0.0085	0.9785	0.0019	0.0039	0.0031	0.8025	248.1356	82.1027	9348.8599	0.0081					
INTP	0.0073	1.2503	0.0003	0.0037	0.0015	0.6586	421.0853	82.7614	9769.9452	0.0078					
JSMR	0.0063	0.9211	-0.0002	0.0045	0.0009	0.2005	188.9284	82.9619	9958.8736	0.0077					
ASII	0.0066	1.3351	-0.0003	0.0012	0.0009	1.3747	1444.1965	84.3366	11403.0702	0.0069					
CPIN	0.0041	1.6867	-0.0020	0.0039	-0.0008	-0.5610	730.2870	83.7756	12133.3572	0.0065					
ASRI	-0.0046	2.1260	-0.0059	0.0020	-0.0047	-10.6753	2273.2369	73.1003	14406.5941	0.0048					
SMGR	-0.0028	1.3873	-0.0070	0.0015	-0.0058	-7.7182	1317.2182	65.3820	15723.8124	0.0039					
LPKR	-0.0057	1.2376	-0.0101	0.0068	-0.0089	-2.0039	224.2561	63.3781	15948.0685	0.0038					
PGAS	-0.0079	1.2005	-0.0121	0.0067	-0.0109	-2.3914	216.4465	60.9867	16164.5149	0.0036					
LSIP	0.0143	-0.2002	-0.0449	0.4704	-0.0437	-0.0038	0.0852	60.9829	16164.6002	0.0036					
AALI	0.0024	0.0517	-0.0582	3.9167	-0.0570	0.0000	0.0007	60.9829	16164.6008	0.0036					

Table 1: The Calculation Result of Optimal Portfolio Construction Using Single Index Model

The average value of SBI rate over the period 2013 -2017 is 0.00535 and Expected return market is 0.006655. From the table above, it can be clearly seen that there are only 10 stocks out of 26 stocks that have ERB value more than Ci point. It means that only 10 stocks that qualified to be included in the construction of optimal portfolio using single index model. The proportion of each stock included in optimal portfolio using single index model are described in the following chart:

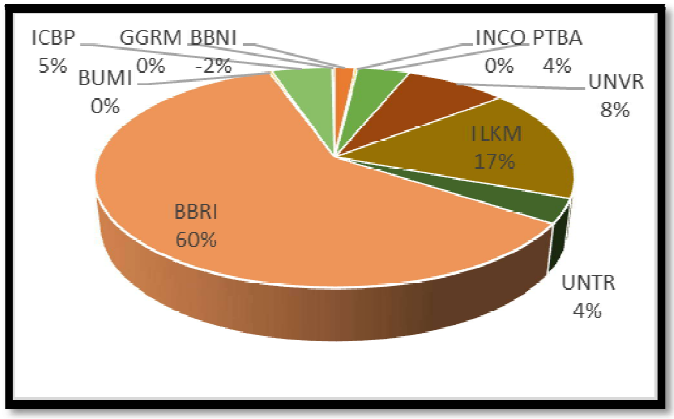

Figure 1: The Proportion of Investment for Each Stock

From the chart above, the highest proportion of stock to be invested is owned by BBRI with the amount of weight is 60%. The lowest proportion is owned by GGRM with 0.193%. The negative value in the proportion of BBNI means that BBNI has negative dependency of market price fluctuations because of value of beta owned by BBNI has negative value.

The expected return of optimal portfolio constructed using single index model has given 2.33% based on monthly calculation. While for the risk of optimal portfolio has given 0.211%. The risk of optimal portfolio calculated is lower than all risks which are owned by all the stocks that consistently listed in LQ45 index over the period 2013 – 2017. This means

that the optimal portfolio formed is well diversified because it can reduce the risk without eliminating the return. Beside the expected return and risk portfolio, this optimal portfolio produce beta more than 1 and included in aggressive stock which means that optimal portfolio constructed using single index model has higher sensitivity with the market price fluctuations. If market price increase 1%, this portfolio will increase 1.322% and when market goes down for 1%, this portfolio will be decreased as well with the amount of beta. The calculation result for expected return portfolio, risk portfolio and beta portfolio are illustrated on the following table.

Return Portfolio	Risk Portfolio		Beta Portfolio
	Std. Dev	Variance	
0.023379164	0.045995308	0.002115568	1.322572

Table 2: Expected Return, Risk and Beta Portfolio

5.2. Constant Correlation Model

The table below is the calculation result for optimal portfolio using constant correlation model:

Stock Code	E(R)	Excess Return	Std. Dev	ERS	ΣERS	ρ/1-ρ - ρ_i	VARIANCE	C
BBRI	0.030563926	0.024704527	0.08951	0.275997052	0.275997	0.23864083	0.00801206	0.065864166
TLKM	0.021159612	0.015456952	0.06535	0.236517185	0.512514	0.192663462	0.00427093	0.098742767
BBCA	0.016283855	0.010662457	0.05433	0.19625957	0.708774	0.161540508	0.00295157	0.114495681
ICBP	0.018549103	0.012889951	0.06781	0.190087313	0.898861	0.139074365	0.00459829	0.125008539
UNVR	0.020276896	0.014588948	0.07796	0.187134279	1.085995	0.122094192	0.00607773	0.132593731
BMRI	0.015805578	0.010192152	0.06669	0.152834656	1.23883	0.108809219	0.00444721	0.13479613
UNTR	0.015153229	0.009550675	0.07128	0.133981458	1.372812	0.098131596	0.00508135	0.134716185
PTBA	0.024259707	0.018505378	0.15162	0.122053923	1.494865	0.089362328	0.02298754	0.133584656
GGRM	0.012544606	0.006985529	0.06848	0.102009326	1.596875	0.082031778	0.00468942	0.130994476
AKRA	0.013321588	0.007749562	0.08297	0.093407089	1.690282	0.075812725	0.00688327	0.128144873
KLBF	0.01087816	0.005346858	0.06485	0.082444522	1.772726	0.070470188	0.00420604	0.124924361
LSIP	0.01434604	0.008756939	0.13866	0.063154547	1.835881	0.065831061	0.01922629	0.120857989
INCO	0.013067674	0.007499879	0.15399	0.048702326	1.884583	0.061765005	0.02371423	0.116401294
ADRO	0.010894146	0.005362577	0.11444	0.046860106	1.931443	0.058172011	0.01309606	0.112355943
INDF	0.008514456	0.003022548	0.07004	0.043153036	1.974596	0.054974059	0.00490596	0.108551578
BUMI	0.019309147	0.013637328	0.33946	0.040173554	2.01477	0.052109394	0.11523331	0.104988441
INTP	0.007305678	0.001833917	0.0882	0.020792753	2.035563	0.049528494	0.0077792	0.100818355
ASII	0.006620933	0.001160585	0.06616	0.017540839	2.053104	0.047191186	0.00437777	0.096888391
JSMR	0.006327585	0.000872125	0.06996	0.01246666	2.06557	0.045064537	0.00489392	0.093083964
CPIN	0.00405417	-0.001363399	0.12119	-0.011250178	2.05432		0.01468681	0.490244634
AALI	0.002447222	-0.002943565	0.10315	-0.028536532	2.025783		0.01064008	0.483434653
ASRI	-0.00463401	-0.009906778	0.12067	-0.08210034	1.943683		0.01456045	0.463842159
LPKR	-0.00570887	-0.010963721	0.11168	-0.098167764	1.845515		0.01247321	0.440415323
SMGR	-0.00277888	-0.008082564	0.07189	-0.112431333	1.733084		0.00516801	0.413584616
PGAS	-0.00791295	-0.013131064	0.10716	-0.122537226	1.610547		0.01148322	0.384342231
BBNI	-0.01510286	-0.020201144	0.08649	-0.233575554	1.376971		0.00747992	0.328601567

Table 3: The Calculation Result of Optimal Portfolio Using Constant Correlation Model

The calculation of coefficient correlation (ρ) is done by averaging the matrix of coefficient correlation of the stock with ERS with positive value which is only 19 stocks that calculated on the matrix. The value of ρ is 0.2386. Based on the calculation, the stocks that have ERS value more than its Ci rate is only 6 stocks out of 19 stocks. This means that only 6 stocks that qualified to be included in optimal portfolio construction using constant correlation model. The C* point for constant correlation model is the point where the last ERS value has higher value than Ci, is equal to 0.1347. The proportion of each stock that included in optimal portfolio for constant correlation model is illustrated on the chart below:

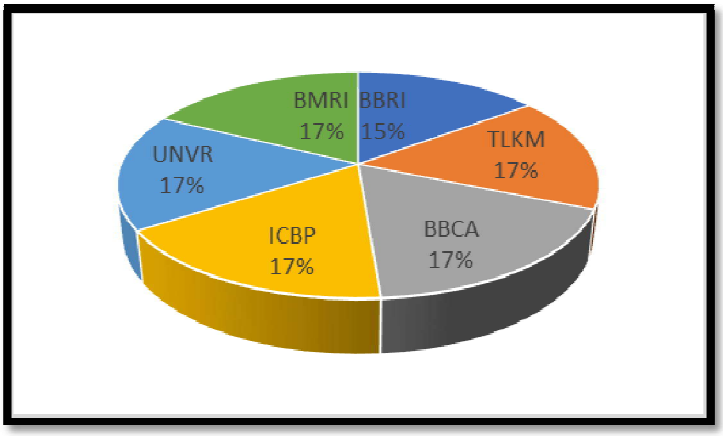

Figure 2: The Proportion of Investment for Each Stock

From the chart above it can be clearly seen that the proportion of each stock is having similarity. The highest proportion is owned by BMRI with 17.53% and followed by BBCA 17.44%. While the lowest proportion is owned by BBRI with 14.77% weight.

The expected return portfolio for optimal portfolio construction using constant correlation model has valued 2.01% and it's lower than the expected return of single index model. The result of the risk portfolio for constant correlation model is 0.20% and it is lower than all the risk of each stocks and it means that the optimal portfolio is well diversified. This portfolio also included in aggressive portfolio because the value of the beta is more than 1. The following table is the result calculation of the expected return portfolio, risk portfolio and beta portfolio.

Return Portfolio	**Risk Portfolio**		**Beta Portfolio**
	Std. Dev	Variance	
0.020166	0.04579	0.002097	1.131

Table 4: Expected Return, Risk, and Beta Portfolio

5.3. The Portfolio Performance Comparison

The following table is the comparison result of the portfolio performance:

Sharpe Ratio		**Treynor Ratio**		**Jensen Ratio**	
SIM	CCM	SIM	CCM	SIM	CCM
0.391978328	0.3235	0.0136319	0.013097863	0.016302996	0.013342093

Table 5: The Portfolio Performance Comparison

Based on the table above, it showed that portfolio optimal with single index model has better performance in Sharpe ratio, Treynor ratio, and Jensen ratio compare with constant correlation model.

6. Conclusion

This study aims to construct optimal portfolio by using single index model and constant correlation model. Optimal portfolio performance formed also assessed with Sharpe ratio, Treynor ratio, Jensen ratio to know which optimal portfolio formed has better performance. Based on the research that has been done, then obtained the following conclusions:

Optimal portfolio construction using single index model construct a portfolio consisting of 10 stocks. The combination of the stocks is BBRI with the weight 62.20%, TLKM with the weigh 17.147%, UNVR with the weight 8.530%; ICBP 5.112%; PTBA 4.313%; UNTR 3.688%; BUMI 0.265%; INCO 0.235%; GGRM 0.203%; BBNI 1.697%.

Optimal portfolio construction using constant correlation model construct a portfolio consisting of 6 stocks. The combination of the stocks is TLKM with the weight 16.53%; BBRI with the weight 14.78%; BBCA 17.44%; ICBP 17.02%; UNVR 16.7%; BMRI 17.53%.

Optimal portfolio construction using single index model generate expected return 2.338% and portfolio risk 0.0211% with beta portfolio 1.32. It means, if market price increase 1%, optimal portfolio with single index model will have a higher increase in 1.32%. While optimal portfolio construction using constant correlation, model generate expected return 2.016% and portfolio risk 0.0209% with beta 1.32.

The calculation of optimal portfolio performance obtains results:

- Based on Sharpe ratio, optimal portfolio constructed using single index model has better performance with score 0.3919 and constant correlation with 0.3235. It means that single index model can gives compensation return for each return higher than constant correlation model gives.
- Based on Treynor ratio, optimal portfolio constructed using single index model has better performance by providing the compensation return of beta portfolio in the amount of 0.0136, whereas constant correlation model only can give the amount of return 0.01309.
- Based on Jensen ratio, optimal portfolio constructed by using single index model has better performance. This portfolio has a higher return than the expected return if the portfolio is in capital market line in the amount of 0.01630. While optimal portfolio formed by using constant correlation model has Jensen ratio with the value 0.01334.

7. Other Recommendations

Future researchers are hoped to use another model for constructing optimal portfolio such as Markowitz Model, CAPM and Stochastic Model also possible, so that the result can be compared with the result of this research. Another choice is similar research can be conducted, but try to use another object such as Sri Kehati Index, JII, Kompas 100. Different years also can be used, researcher can add 2018 to the research to find out whether the increase of the value of rupiah affects the stock price which then affects the optimal portfolio construction.

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