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N Schofield, S Gatehouse and J Carswell

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Contact Author:
Full name: Neil Schofield
Position title: Manager
Organisation Name: Hellman and Schofield Pty. Limited.
Address: P.O. Box 599, Beecroft NSW, 2119
Phone: 612 9858 3863
Fax: 612 9858 4057
Mobile:
Email: nas@hellscho.com.au

Fundamental Sampling Error and Sampling Precision in Resource Estimation: A Discussion

Neil Schofield[1], Simon Gatehouse[2] and J Carswell[3]

1. Manager, Hellman and Schofield Pty. Ltd, P.O. Box 599, Beecroft, NSW 2119.
Email: nas@hellscho.com.au , MAusIMM.

2. Senior Consultant Geologist, Hellman and Schofield Pty. Ltd. P.O. Box 599, Beecroft NSW, 2119.
Email: simon@hellscho.com.au , MAIG.

3. Mining Geology Manager, Sunrise Dam Gold Mine, WA.
Email: JCarswell@AngloGoldAshanti.com.au , MAusIMM.

ABSTRACT

For the sampling of particulate materials in mineral exploration and mining, the fundamental sampling error (FSE) introduced by Gy is a measure of the constitutional heterogeneity of the sample lot and is expressed as the relative standard deviation of sub-sample grades of a particular mass. The FSE is presently regarded as an important indicator of quality for samples used in resource estimation. Constitutional heterogeneity describes an ideal state of heterogeneity with respect to all constituents of the lot such that the fundamental error among sub-samples of a particular mass is minimised.

In this paper, the term "sampling precision" used in relation to mineral resource sampling usually refers to a statistic based on the assay results drawn from pairs of separate sub-sample splits of multiple sample lots. The pairs of splits are extracted after the lots have been prepared by crushing and pulverising to meet specified criteria of grain size and sub-sample mass. There is presently no generally accepted method of calculating the "sampling precision" from such paired data but some form of the relative standard deviation that is consistent with definition of the FSE is preferred here. The work of Thompson and Howarth in the 1970s on geochemical analytical precision, which has been subjected to extensive examination and criticism by Stanley and Lawie (2007), and that of Francois-Bongarçon since the 1990s forms the background to the discussion in the paper.

A significant aspect of the application of Gy's sampling theory to sampling in gold deposits is the requirement that the fundamental error be maintained at less than +/-20%. The most important reason for this is to ensure that the effects of other sampling errors such as the grouping and segregation, and increment delimitation which can cause local and global biases in sampling are kept as small as possible.

This paper is focused mainly on the quality of sampling in gold deposits and looks at an alternative to the use of a constant sampling precision applied to a collection of lots with varying average grade. A theoretical model for the FSE based on duplicate sampling is presented and the simulated results of the model are compared to practical outcomes of real duplicate pairs drawn from multiple sample lots. The goal of the work is to present a more informative analysis of duplicate sample data from multiple lots which may allow a more informed view of the potential for bias generating errors during sample preparation and processing.

INTRODUCTION

Geologists with some exposure to sample assay data for elements such as gold and copper are aware that chemical analyses of additional splits from the same sample material do not return exactly the same grade as the existing assay for those samples which contain measurable concentrations of the element or mineral of interest (MI). The physical explanation for these differences is;

1. Sample lots are comprised of discrete particles and are therefore, not a continuous medium such as for example, a copper sulphate solution. Different numbers of particles of a particular MI, for example, chalcopyrite, in the sub-sample to be assayed cause different outcomes in the assaying.

2. Measurement error in the assaying process due to minor differences in the way each chemical analysis is performed.

The discussion in this paper is concerned with the first of these two issues which is related to the nature of the sample material and the way that it is prepared for assaying. The second is related to the problems of quality control in laboratory assaying.

The nature of the differences between repeat assays of samples is of significant interest to resource estimators. These differences are one of the sources of both random and systematic errors in resource estimates undertaken by any estimation method, although some methods are more affected than others by these differences.

The sampling theory of particulate materials proposed by Gy and articulated by Pitard (1993) discusses sampling errors under seven categories. This paper is focused on two of these categories: the Fundamental Sampling Error (FSE) and the Grouping and Segregation Errors (GSE). The FSE, which arises as a result of the Constitutional Heterogeneity (CH) within the sample lot, is the only category of sampling error which does not have the potential to generate local or global bias. The GSE, which is related to the Distributional Heterogeneity (DH) within the lot, can result in local and global bias. Local bias, which may occur when the distribution of grades of sub-samples within the lot is not symmetric about the average grade, is caused by local departures from CH within the lot. Global bias is usually related to large scale segregation of particular constituents of the lot, for example gravity settling of more dense particles.

Gy and Pitard propose the use of the Sampling Constant and nomograms to address the problems related to the CH of the sample lot, and the use of correct sampling practice to overcome the effect of in-homogeneities in the lot caused by grouping and segregation of the constituents of the lot. In summary, they recommend that one should never assume that the lot is homogeneous and always adopt sampling practices which mitigate the potential for biasing errors in the lot. Gy and Pitard do not suggest any approach to monitoring the quality of the samples generated by a particular sampling protocol once it is in operation. Specifically, they do not suggest any monitoring process which addresses the GSE. In discussing the problem of minimising the fundamental error, Pitard (1993, p175) comments that, “Accepting too large a fundamental error may not leave much place for other types of errors, promote large grouping and segregation errors, and lead to fluctuations which are difficult to interpret”. In relation to sampling in gold deposits, Pitard (1993, p356) implies that the fundamental error should be kept to less than +/-16%.

In 1973, Thompson and Howarth (TH) presented an experimental approach to monitoring the analytical precision of geochemical data and followed this up with a new approach in 1978. Their approach is based on the relationship of the absolute deviation between the grades of pairs of duplicate sub-samples of the lot and the mean grade of the pairs. There is an underlying assumption of a linear relationship between the median absolute deviation of the pair differences and the mean grade of the duplicate pairs. The slope of a linear regression of the absolute deviation onto the mean grade provides an estimate of the analytical precision. Carswell et al (2009) present a case study of the application of this approach at the Gosowong Gold Mine in Indonesia.

Bongarçon (1998) provides a concise summary of the 1978 contribution of Thompson and Howarth and the essential ideas underlying Gy’s theory. He discusses the complexity of the various factors such as liberation and shape factors that are integral to the application of the theory and indicates that the approach is difficult to implement in practice. Bongarcon suggests that the availability of duplicate data from coarse rejects and duplicate assay grades provides another avenue for investigation and proceeds to develop an approach to extract the desired sampling error variance from the paired information. Importantly, he draws attention to the differences between the nature of the analytical errors and the sampling errors which are the main focus of Gy and Pitard. The variance of analytical errors is likely related to the square of the mean grade and approximately linear for positively skewed lot grades. The variance of FSE is related to the mean grade of the lot through the model of a Poisson distribution and consequently behaves in a linear fashion with respect to increasing lot grade (the Poisson distribution is discussed in more detail below under Constitutional Heterogeneity).

In the discussion which follows, the impact of analytical errors which is generally monitored through the use of standards, blanks and replicate determinations, is assumed to be insignificant when compared to that of the sampling errors.

RELATIVE STANDARD DEVIATION OF DUPLICATE SUBSAMPLES

Notwithstanding the usefulness of Gy's sampling theory to the problem of establishing an appropriate sampling protocol in any particular mining or mineral exploration situation, the authors support the view of Bongarcon that duplicate data from various sources that is gathered as part of the ongoing sampling process provide a valuable resource from which information about the quality of the sampling and assaying process can be monitored.

Current industry practice of analysing duplicate data is heavily focused on generating a statistic that estimates the magnitude of the sampling error in the form of a precision or relative deviation ie expressed as a percentage variation around a particular sample grade. In the elementary case of a single lot of mass ***M*** with mean grade ***a*** and a particular sub-sample mass ***m***, the precision ***p(m)*** of the sub-sampling process is expressed as the relative standard deviation ***s(m)/a*** where ***s(m)*** is the standard deviation of the sub-sample grades: $p_1(m) = \frac{s(m)}{a}$.

An alternate expression for the precision can be made in terms of the differences ***d(m)*** between pairs of sub-samples of mass ***m***: $p_2(m) = \frac{1}{\sqrt{2}} \frac{s\{d(m)\}}{a}$ where the variance of the differences between pairs of samples is twice that of variance among the sample grades. Such expressions generate a minimum value for the precision if the grades of the sub-samples can be considered statistically independent of each other – a condition which characterises CH within the lot. The existence of grouping and segregation problems within the lot will always cause the value of ***p(m)*** to be greater than that related only to the CH within the lot.

THE FUNDAMENTAL SAMPLING ERROR AND THE GROUPING AND SEGREGATION ERROR

Figures 1(a) and (b) illustrate the essential differences between a sample lot (a) for which ***p(m)*** would correctly describe the CH of the lot, and sample lot (b) for which the ***p(m)*** would be significantly affected by DH within the lot.

Figure 1(a) presents an impression of a lot comprised of some 10000 sample particles of equal size distributed on a regular grid in the horizontal plane; for example a ground pulp containing gold particles in a siliceous host flattened onto a table. The darker particles represent the MI (eg gold particles) and comprise ten percent of the total particles. The two smaller and darker squares overlain on the lot are intended to present the idea of a pair of duplicate samples taken from the lot. The spatial distribution of the darker particles within the larger square is approximately uniform and consistent with a state of CH in the lot. It is important to appreciate that there is no unique state of CH for a particular lot. Many different arrangements of the lighter and darker particles would be consistent with a state of CH within the lot.

Figure 1(b) presents an image with the same proportions of darker and lighter particles as 1(a) but the distribution of the darker particles is affected by grouping and segregation processes such as gravity and grain size segregation which leads to a distinctly non-uniform pattern of the darker particles (at the scale of observation) and a significant departure from a state of CH. Many different arrangements of lighter and darker particles would be consistent with similar states of grouping and segregation within the lot.

Figure 2 presents a spatial statistic which describes the difference between the distribution of the darker particles on these two plots in the form of directional indicator variograms for which the darker particles are coded as 1 and the lighter particles are coded as 0. A nugget effect sample variogram (near parallel to the horizontal axis) reflects the lack of spatial structure in the distribution of darker particles in Figure 1(a) ie an absence of DH. Conversely, a structured indicator variogram indicates the presence of DH in the lot.

Considering the pairs of smaller squares in Figures 1(a) and 1(b) and assuming a constant grade of zero for the lighter coloured particles and 1 for the grade of the darker particles, it is evident that either of the expressions for relative precision given in the previous section will evaluate to significantly different values of relative precision in 1(a) and 1(b). It is also evident that the heterogeneities within the lots are a function of the scale of observation: smaller squares will generate higher relative precision in each case.

Table 1 shows the details of the two precision measures presented above for a number of different sized squares. Both measures of precision are much larger for Figure 1(b) reflecting the much greater DH. The table also shows that both measures of precision are a function of the scale of sampling: large squares generate lower precisions. It is not difficult to foresee that for a sufficiently large lot with a DH similar to that of Figure 1(b) and a sufficiently large sub-sample square (much larger than the lot), the precisions would approach those of the lot in Figure 1(a).

In practice, the view of the CH and DH in the lots shown in Figure 1 is not available to the sampler. In response to the ubiquitous problems of grouping segregation within sample lots, correct sampling practice requires that the lot is never assumed to be homogeneous and the sampling of the lot should always use practices which attempt to minimise the effects of grouping and segregation eg techniques which partition the lot and recombine the partitions multiple times eg a Jones riffle splitter. As far as the authors are aware, there are no analytical tools available which allow the isolation of the contribution of GSE in the lot from the assessment of the FSE based on the analysis of duplicate pairs drawn from the lots. It is usually only in the case where the average grades of the duplicate pairs, taken over a reasonable number of samples, differ significantly, that one begins to worry about the sampling process.

THE THOMPSON AND HOWARTH APPROACH

The work of TH in this area was focused on exploration geochemical sampling but there is no good reason to suppose that the approach could not be applied to data used for the estimation of mineral resources. Carswell et al (2009) present an application of this approach to grade control sampling at the Gosowong Gold mine in Halmahera, Indonesia and provide a description of the method for determining relative imprecision of the sampling. Essentially, the measure of the precision relies on the slope of a regression line fitted to the plot of the absolute differences between duplicate pairs and their average grades. The paired data are typically combined into small groups of 11 in increasing order of average grade to allow the calculation of the median absolute difference between duplicates. Figure 3 presents an example from Carswell et al.

Underlying the derivation of a sampling precision of TH that may be applied to a group of lots with varying grades is the assumption that the relative precision (slope of the regression line) can be considered constant over this range. In Figure 3, the relative precision at a 68% confidence interval (or one standard deviation) is shown as 33.1%, significantly larger than the recommended 20% for gold sampling.

Other approaches such as plots of the “Half Absolute Relative Difference” (HARD) and “Rank Half Absolute Relative Difference” are discussed in the literature. Stanley and Lawie (2007) discuss the application of five “common” measures of relative error calculated from duplicate pairs used in a variety of geologic applications. They conclude that only the root mean square coefficient of variation calculated from the individual coefficients of variation produces unbiased estimates of measurement error. Stanley and Lawie further conclude that the results of the TH precision measure differ substantially from the root mean square coefficient of variation and recommend that TH results should not be used to determine the magnitudes of component errors introduced during geochemical sampling, preparation and assaying.

There is an apparent weakness with the approach of TH in relation to mineral resource sampling. In the relatively simple case where the MI in the lot is confined to a single mineral eg chalcopyrite, and the size of the sub-sample is assumed large enough that at least twenty particles of MI of uniform particle size are expected to occur in the sub-sample, the statistical distribution of the particle counts of MI in the sub-sample can be assumed to be Gaussian (bell shaped) with a standard deviation equal to the square root of the mean particle count (20 or greater). This means that for the Poisson distribution used to model the sampling of particulate materials, the relative precision of the particle count of MI varies as the inverse of the square root of the mean particle count (Thomas 1981, Pitard 2003) and consequently decreases with increasing particle counts of MI and sub-sample grade (see Constitutional Heterogeneity below for more details).

THE ANALYSIS OF DUPLICATE DATA

Common ways of presenting duplicate data are through the use of scatter plots and quantile-quantile plots. It is possible to gain some feeling for the TH precision from the scatter plot and the linear correlation of the duplicate data; for example to achieve a TH sampling precision of less than 20%, the linear correlation between the pairs of duplicates usually has to be around 0.95 or higher. As mentioned above, the TH precision is based on a plot of absolute differences between the duplicates against the average grade of the duplicates (Figure 3). The quantile-quantile (QQ) plot provides a very convenient method of comparing the shapes of the histograms of two different data sets, particularly when histograms are expected to be very similar as is the case with pairs of duplicate data. The QQ plot of duplicate data should fall close to a straight line with a slope of one through the origin.

Figures 4, 5 and 6 present a scatterplot, absolute difference plot and QQ plot of some 160 duplicate data from RC grade control sampling in a gold deposit. The data are assay duplicates ie they have been extracted from the pulverised material prepared for extraction of the assay aliquot. The scatterplot shows a close grouping in the range up to 1.0 g/t but a broader scatter above this level and a tendency to higher grades in D2. The mean grades of the duplicates differ by around 10 percent with D2 being higher in grade and the linear correlation of 0.85 suggests a TH precision (at one standard deviation) of greater than 20%. The departure of the QQ plot from the diagonal above 1 g/t is consistent with the pattern in the scatterplot and indicates the histograms of D1 and D2 differ significantly in the top ten percent of the grades. The plot of absolute differences in Figure 5, like the scatterplot, shows a close grouping of points near the origin and a wide scatter of around ten sample pairs with much larger absolute deviations. The TH precision at one standard deviation for these data is 35%.

Experience in analysing duplicate data generated from sample processing from many deposits suggests that the absolute difference between some of the duplicate pairs may be the result of both CH and DH in the samples giving rise to GSE in addition to the FSE. This is sometimes reinforced by the differences between the mean grades of the duplicates and the divergence of their cumulative histograms in the upper tail. Without engaging in a difficult debate about which measure of duplicate precision is the most appropriate, can it be demonstrated that these outcomes are not consistent with an assumption that a state of CH has been achieved in the lots from which these duplicates were extracted? Precision measures notwithstanding, if a state of CH cannot be demonstrated, biased sampling is certain.

CONSTITUTIONAL HETEROGENEITY

The Binomial – Poisson Distributions in Sampling

Consider a sample lot comprised of particles of the same size but of two different mineral types, one being the MI. The proportion of MI particles in the lot by volume is p. If a sub-sample of n particles is selected from the lot, the Binomial distribution describes the probability that r particles of type MI will occur among the n particles selected. For example, if the proportion of MI particles in the lot $p=$ 0.1 and a sub-sample composed of 200 particles is taken, there is a 9.4 percent chance that the sub-sample will contain 20 particles of MI. For a Binomial random variable, the mean is np and the variance is $np(1-p)$. For this example, the mean of the

Binomial random variable is 20 and the variance is 18. The relative standard deviation is 0.21 and the precision at one standard deviation is 21%.

Consider now a similar lot of two component minerals but with two important differences: the proportion of MI particles is now much smaller at 0.0002 and the number of particles comprising the sub-sample is much larger at around 100 000 or more. Under these conditions, the probability of obtaining ***r*** particles of MI in the sub-sample is best described by the Poisson probability distribution – a single parameter distribution for which the mean and variance are equal. The Poisson distribution appears as a limiting case of the Binomial distribution when the probability of selecting an MI particle in a single particle trial is very small and the number of trials or particles in the sub-sample is very large. The mean and the variance of the Poisson random variable are simply the expected number of particles of MI that will occur in a sub-sample of a certain size. In practice, the mean is usually calculated based on the mass of the sub-sample but to persist with the example above, if the sub-sample contains 100 000 particles and the proportion of MI particles in the lot is 0.0002, then the expected number of particles of MI in the sub-sample is 20. The relative standard deviation of a Poisson random variable with a mean of 20 is 0.22 – very similar to the binomial case above. The Poisson distribution becomes increasingly positively skewed as the mean decreases below 20. Figure 7 shows the probability distributions of two Poisson random variables with means of 5 and 20. As the mean of the Poisson random variable increases above 20, the Poisson distribution can be closely approximated by a Gaussian distribution with the same mean and variance.

A Computational Model and Constitutional Heterogeneity Plots

The spatial distribution of the darker particles in Figure 1(a) was generated using a very simple Monte Carlo simulation in which the number of gold particles in the sub-sample is assumed to follow a Poisson distribution. The spatial pattern in Figure 1(b) has a much more complex mathematical description and is consequently more difficult to generate. It turns out that although a constitutionally heterogeneous pattern of the MI is very difficult to achieve and maintain in physical reality, it is comparatively simple to generate reasonable simulations of such patterns mathematically and simulate duplicate sampling from these patterns (Bratley, Fox and Schrage, 1987). The process can be designed to allow for a range of particles sizes of the MI as well as variations in the size of the lot and the size of the duplicate sub-samples extracted from the lot.

Figures 8 and 9 present absolute difference and scatterplots respectively for the duplicate data shown in Figures 5 and 4 as well as for simulated duplicates generated by the model of CH described above. The parameters of the model include a sub-sample mass of 50 g for fire assay, drawn from a 2 kg lot with 95% passing 100 µm. The grain size distribution of the gold grains in the lot is assumed to have a right angled triangular shape with the proportion of finer grains increasing to a maximum of around 20 percent at the smallest grain size of 5 µm. The maximum grain size is 110 µm. The simulated duplicates were generated from the cumulative histogram of the mean grades of the data duplicates.

The cloud of simulated duplicates in Figures 8 and 9 is intended to provide a reasonable indication of the spread of duplicate outcomes that would arise from a constitutionally heterogeneous lot and it is clear that in each case, the cloud envelops almost all of the data outcomes. Both plots show four pairs of data duplicates falling on the edge of, or just outside the cloud envelope. These four data pairs are also the main cause of the difference between the mean grades of the data duplicates. The overall analysis indicates that the assay results of these samples should be regarded with suspicion and more generally, that the sampling protocol is generating around three percent of assay grades which may be affected by sampling errors that potentially cause biased outcomes.

Importance of Constitutional Heterogeneity in Resource Estimation

As Gy and Pitard correctly state, the FSE which results from the CH of the lot, is the only component of sampling error that is random, by which they mean that it has an average of zero and does not cause a bias in estimation of the lot grade.

In geostatistical terms, the variance of all sampling errors in the sample grades is represented in the nugget effect on the sample variogram and variogram model (Journel and Huijbregts, 1978). The variogram model is the continuity function used to generate the sample weights in the various kriging (spatial estimation) methods provided by geostatistical theory. However, it is important to realise that sampling errors other than the FSE may affect the sample variogram at lags other than the nugget effect of zero lag. An incorrect sampling protocol is likely to induce dependences among the grades of samples in the process stream that are unrelated to their spatial context and consequently have the potential to cause bias in the kriging estimates.

Block kriging methods are spatial averaging techniques which generate an estimate of the block grade as a weighted average of the sample grades within the local neighbourhood of the block. The impact of truly random errors in the sample grades is strongly mitigated by the averaging process. The nugget effect associated with kriging estimates of block grade is generally considered to be negligible provided a sufficient number of samples are used to estimate the block grade.

If the sampling errors in a particular data set include a significant non-random component related to GSE or other sampling problems, locally biased estimates with unquantifiable estimation errors will inevitably arise.

Further Examples of Constitutional Heterogeneity Plots

Gold and Chalcopyrite Mineralisation (Field Duplicates)

Figures 10 and 11 show the absolute difference plot and scatterplot of gold duplicate data from the first split of samples taken from RC grade control drilling in a gold-copper deposit which exhibits both supergene as well as primary copper mineralisation. In the primary mineralisation, the gold is fine grained and intimately associated with chalcopyrite. The maximum gold grade in the samples is around 15 g/t and the coefficient of variation of gold grade is typically around 1 to 1.5. The simulated duplicates shown in each plot have been generated using similar parameters of grain size distribution with a maximum gold particle size of 220 µm. A sub-sample mass of 500 g was assumed.

In Figures 10 and 11, almost all of the data duplicates plot well within the cloud of simulated duplicates with a couple of outcomes closer to the edge of the cloud but none falling outside the cloud. The TH precision at one standard deviation for the field duplicates is +/-23% compared to +/-35% for the assay duplicates shown in Figure 8. These observations are consistent with this particular mode of occurrence of fine gold in chalcopyrite and indicate that conditions approaching CH with respect to gold may be achieved with a relatively uncomplicated sampling protocol.

Free and Sulphide associated High Grade Gold (Assay duplicates)

Figures 12 and 13 show the plot of absolute differences and the scatterplot for assay duplicates from a deposit which exhibits high grade gold in excess of 700 g/t in two metre composites and coefficients of variation in the range of 6 to 10.The sampling conditions are similar to those described for Figures 8 and 9. The TH precision at one standard deviation is 46%.

On these plots, a large number of data duplicates fall well outside the range of variation consistent with a state of CH in the lot. The plots suggest strongly that CH is not being achieved for the bulk of the samples being processed. Experiments with different maximum particles sizes suggests that a maximum gold particle size of around 900 µm is required to create a cloud of simulated duplicates large enough to envelope the sample duplicates in Figures 12 and 13. These simulations are shown in Figures 14 and 15. The CH model indicates that an assay sub-sample of around 500 g is required to achieve a pattern of simulated duplicates similar to that shown in Figures 12 and 13 ie the use of a screened fire assay of around 500 g rather than a 50 g fire assay.

OBSERVATIONS AND CONCLUSIONS

The simulation of duplicate outcomes reflecting CH conditions from the histogram of the average grades of duplicate samples drawn from lots provides a broader basis on which to study the potential problems that may be occurring in a sampling processing protocol, and a basis that is consistent with the framework of sampling theory provided by Gy in a limited way. By comparison, the limitations of a single calculated relative precision such as the TH precision are obvious. There is little doubt that an experienced practitioner might come to similar conclusions about the quality of a sampling protocol based only on the scatterplot of duplicate grades but there is little opportunity to gain greater insights into the nature of the sampling problems and formulate suitable remedies.

The simulation and sampling trials that form the basis of this paper suggest the following observations;

1. The process could be enhanced with more detailed information about grain size distributions in the sample lot. Empirical observations suggest that smaller grain sizes are more abundant than coarser grain sizes but there may be situations in which this assumption does not apply well.

2. Plots of absolute difference versus average grade for many duplicate data sets indicate behaviour near the origin that is inconsistent with the assumption of a similar pattern of grain size distributions in all samples. Lower grade samples often show behaviour consistent with higher proportions of finer particles of the MI.

3. There is a reasonable consistency between the simulated outcomes based on known parameters of particle size and sub-sample size over a wide range of these parameters eg for maximum particle sizes of the MI up to 1mm and sub-sample sizes up to 500 g.

4. The requirement that the relative standard deviation of accumulated sampling errors for gold sampling not exceed +/-20% appears sound. However, in the practice of analysing duplicate data from gold deposits using the TH approach, a comparable result is rarely achieved.

The model of CH presented above assumes that the MI occurs as free spherical particles of MI rather than in composite form with other minerals. The net effect of this assumption is that the spread of the simulated duplicates on the absolute difference plots and scatterplots will be larger than would be generated by composite particles containing the MI.

Considering the absolute difference plots and scatterplots of sample data and simulated data presented above, the fact that a particular point (duplicate pair) falls within the cloud of simulated values does not guarantee that the lot from which they were drawn is free of GSE and other bias generating problems – only that the effect of potentially bias generating problems are small enough that they cannot be distinguished from the deviations related only to the CH.

It is important that the result is understood to apply to the entire collection of lots from which the duplicates were drawn and relies on the initial decision of the analyst to pool these data in order to derive a statistic or generate a plot that is representative of the entire population. If a significant proportion (>20%) of the total population of duplicate pairs plot outside the cloud, the sampling protocol is performing poorly and needs urgent attention (Example 2, Figure 12).

DATA QUALITY AND RESOURCE CLASSIFICATION

Mines which pay insufficient attention to data quality issues are frequently plagued by production and reconciliation problems. Measurement errors in the location and in the grades of samples contribute directly to the estimation error of the block grades in resource models ie the difference between the estimated grade and the actual grade of the block, whatever the estimation method used.

In the current practice of resource estimation, a range of precision measures such as TH precision are used along with other quality control measurements such as assays of standards and blanks to characterise the quality of sample data used for resource estimation. This paper links the behaviour of duplicate data more directly to the physical states of heterogeneity in the sample lots proposed in Gy's sampling theory.

Whether one adopts a measure of precision or the CH plots described in this paper to summarise the quality of sampling from a collection of lots, there is currently no useful framework for dealing with the issues of sampling quality with reference to the classification of mineral resource estimates. There is a general expectation among exploration and mine geologists that provided some effort is made to establish a "reasonable" sampling protocol for a mineral project, as well as collect and analyse some duplicate sample data, resource estimates will ultimately achieve a "Measured" status when an appropriate drill hole spacing has been reached. Problems with sample quality indicated by the analysis of duplicates for example, are generally assumed to have no significant impact on the classification of mineral resource estimates.

The sampling conditions for a particular mineral deposit are in large part a function of the prevailing geological and physical properties of the mineralisation and it is unreasonable to expect that the limited set of equipment available for sample extraction and processing will reduce samples of enormous geologic variety to anything approaching a uniform state. As a consequence, it is unreasonable to expect that similar levels of data quality will be achieved across a wide range of mineralisation types using similar processing tools. Some deposits and sampling circumstances seem to defy reasonable efforts to be tamed.

The Australasian Joint Ore Reserves Committee (JORC) resists being prescriptive on certain issues and prefers to place the onus of responsibility for resource classification onto the competent person. However, the time has come to incorporate a minimum set of guidelines for reporting of sample data quality information under JORC. Such guidelines should specify minimum rates for the incorporation of standards, field duplicates, assay duplicates and blanks in data that are used for reporting of mineral resource estimates. A range of relatively simple presentation options for these data should also be recommended.

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FIGURE CAPTIONS

Fig 1 – Spatial distribution of particles in two lots. The darker particles represent the mineral of interest.

Fig 2 - Sample indicator variograms of the spatial distribution of darker particles in Figure 1.

Fig 3 – Thompson Howarth plot of underground face sampling duplicate pairs, 2008 (After Carswell *et al* 2009). Each point on the plot represents a grouping of eleven duplicate samples.

Fig 4 – Scatterplot of assay duplicates from reverse circulation grade control sampling, gold deposit.

Fig 5 - Absolute Difference plot of assay duplicates from RC grade control sampling, gold deposit.

Fig 6 – Quantile – Quantile plot of assay duplicates from RC grade control sampling, gold deposit.

Fig 7 – Probability distributions of two Poisson random variables with different means.

Fig 8 – Absolute difference plot of assay duplicates (D) (Figure 5) and simulated duplicates (S).

Fig 9 – Scatterplot of Assay duplicates (D) (Figure 4) and simulated duplicates (S).

Fig 10 – Example 1: Absolute difference plot of field duplicates (D) and simulated duplicates (S).

Fig 11 – Example 1: Scatterplot of Field duplicates (D) and simulated duplicates (S).

Fig 12 – Example 2: Absolute difference plot of assay duplicates (D, 50 g) and simulated duplicates (S, 50 g).

Fig 13 – Example 2: Scatterplot of Assay duplicates (D, 50 g) and simulated duplicates (S, 50 g).

Fig 14 – Example 2: Absolute difference plot of assay duplicates (D, 50 g) and simulated duplicates (S, 500 g).

Fig 15 – Example 2: Scatterplot of Assay duplicates (D, 50 g) and simulated duplicates (S, 500 g).

TABLE CAPTIONS

Table 1 – Relative precision of grades within squares of different sizes by two different methods.

FIGURES

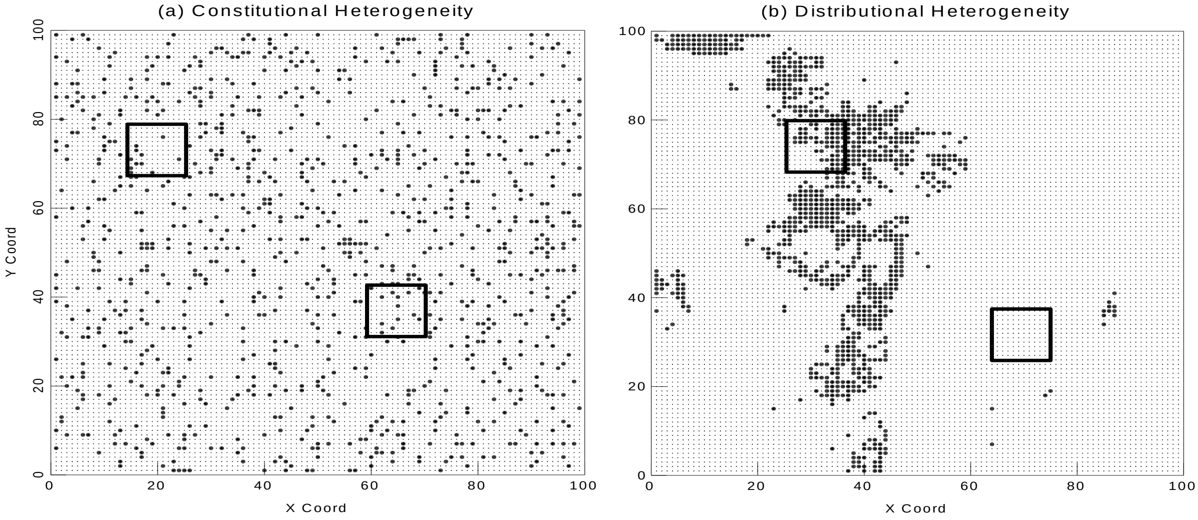

Fig 1 – 2D Plots of the spatial distribution of particles in two lots. The darker particles represent the mineral of interest.

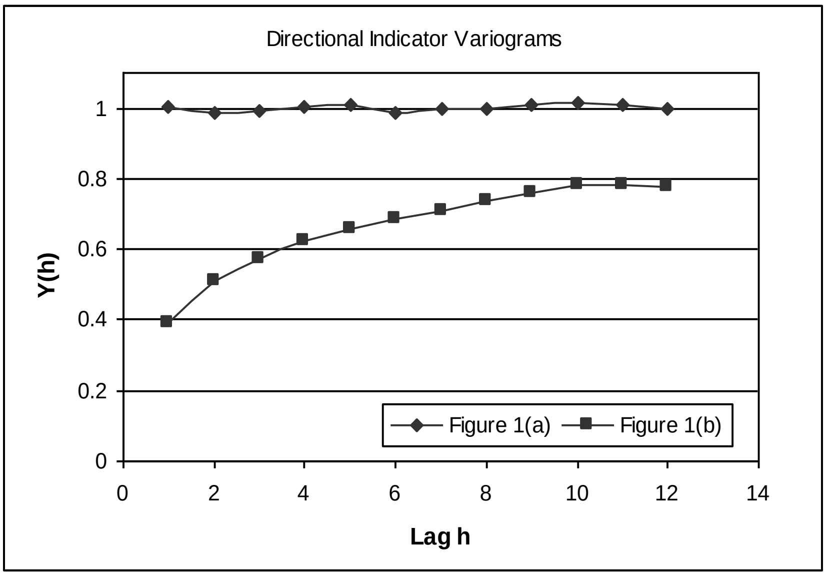

Fig 2 - Sample indicator variograms of the spatial distribution of darker particles in Figure 1.

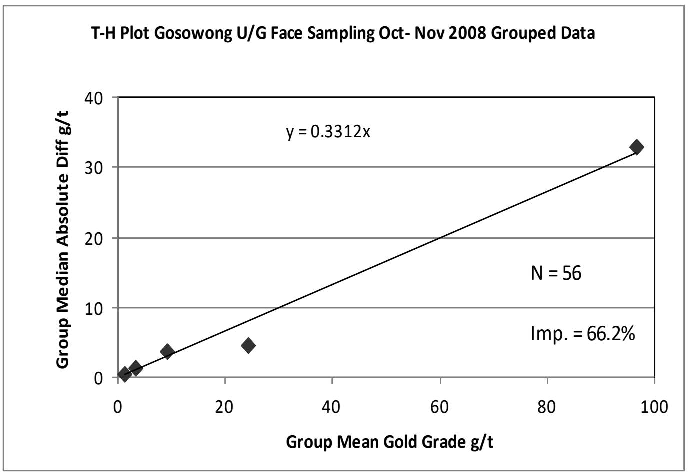

Fig 3 – Thompson Howarth plot of underground face sampling duplicate pairs, 2008 (After Carswell et al, 2009). Each point on the plot represents a grouping of eleven duplicate samples.

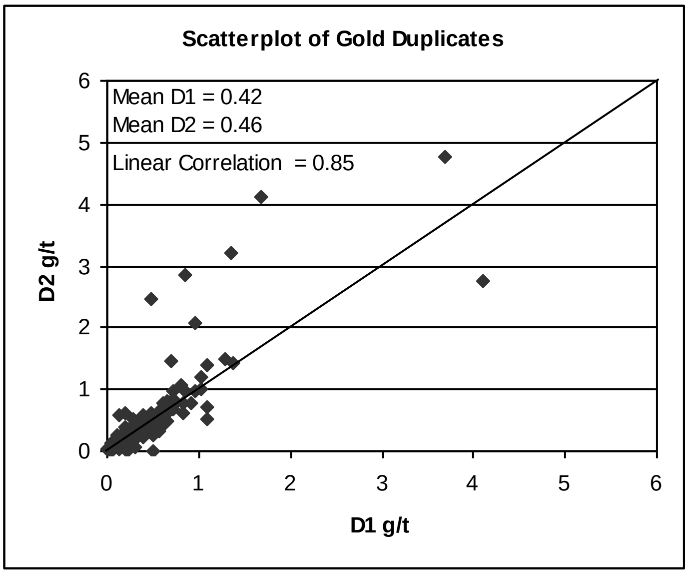

Fig 4 – Scatterplot of assay duplicates from RC grade control sampling, gold deposit.

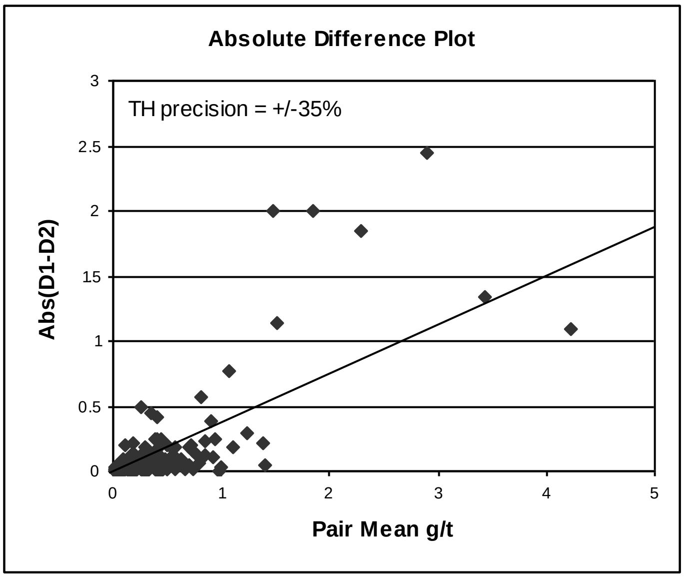

Fig 5 - Absolute Difference plot of assay duplicates from RC grade control sampling, gold deposit.

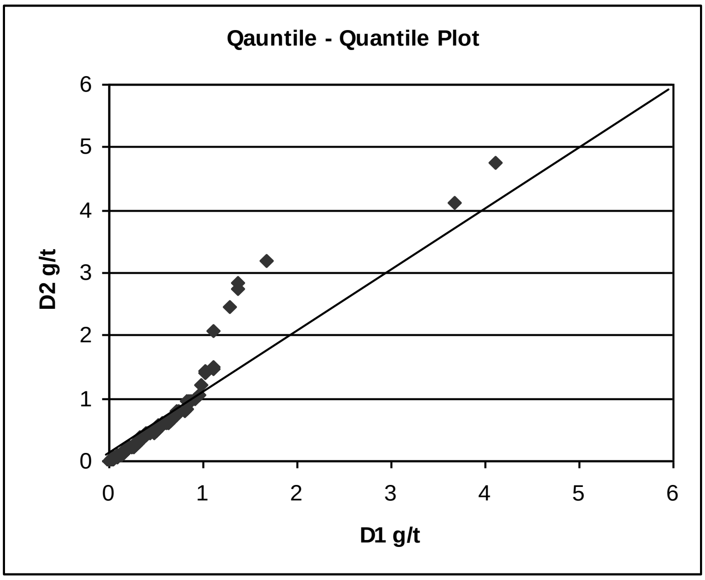

Fig 6 – Quantile – Quantile plot of assay duplicates from RC grade control sampling, gold deposit.

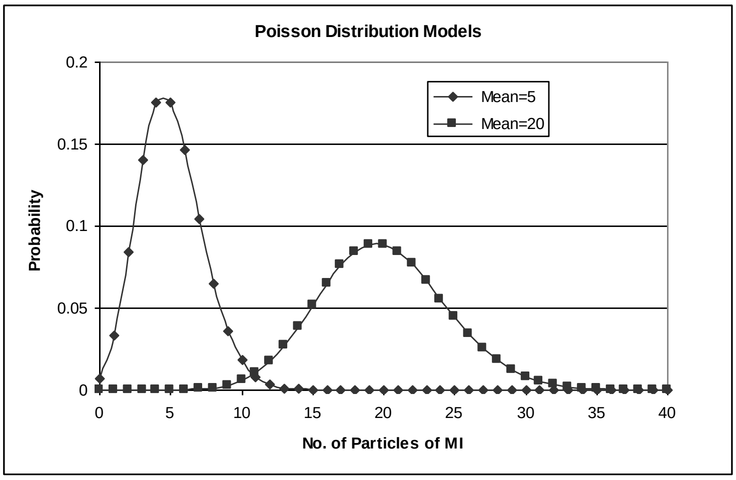

Fig 7 – Probability distributions of two Poisson random variables with different means.

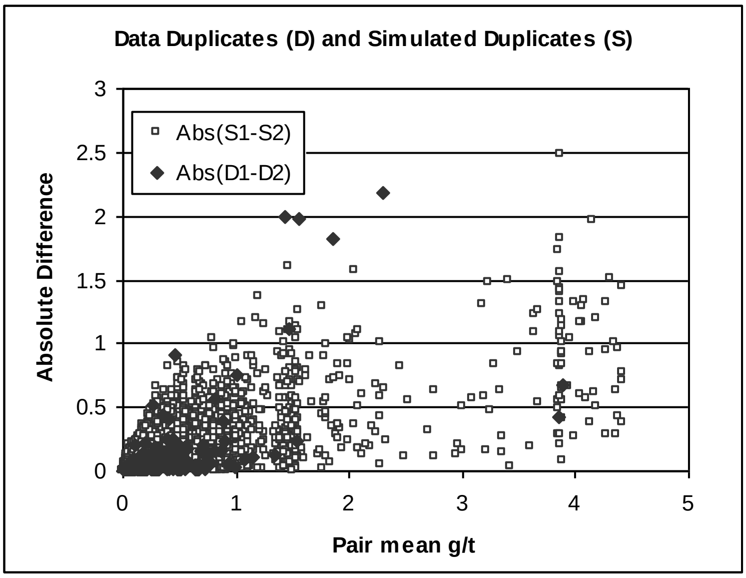

Fig 8 – Absolute difference plot of assay duplicates (D) (Figure 5) and simulated duplicates (S).

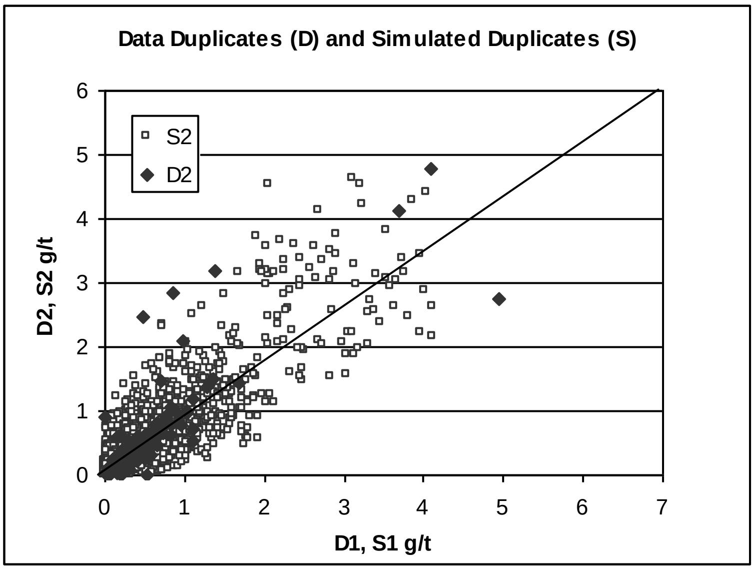

Fig 9 – Scatterplot of Assay duplicates (D) (Figure 4) and simulated duplicates (S).

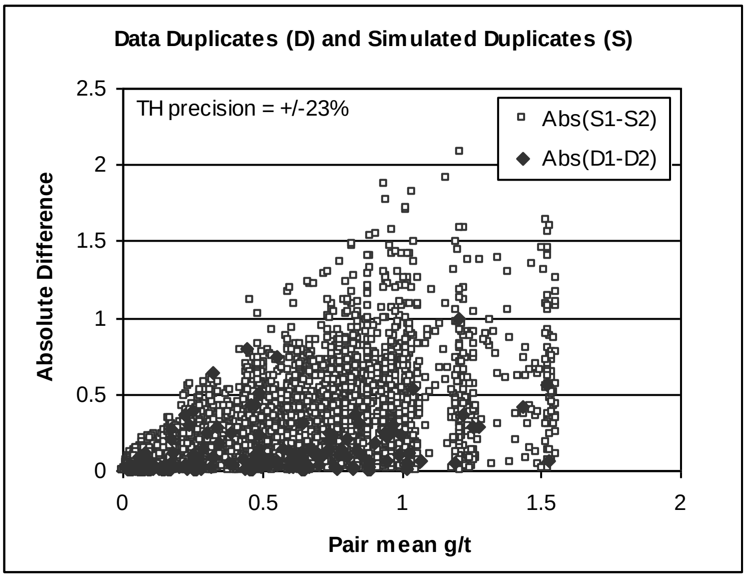

Fig 10 – Example1: Absolute difference plot of field duplicates (D) and simulated duplicates (S).

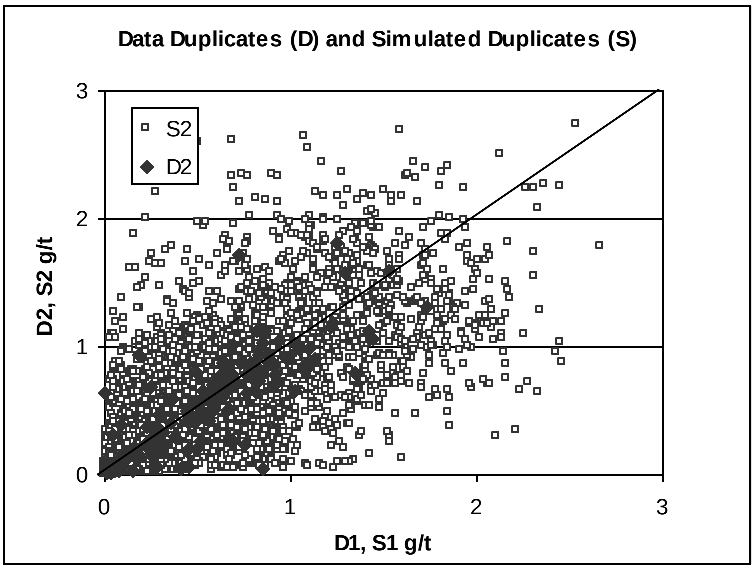

Fig 11 – Example 1: Scatterplot of Field duplicates (D) and simulated duplicates (S).

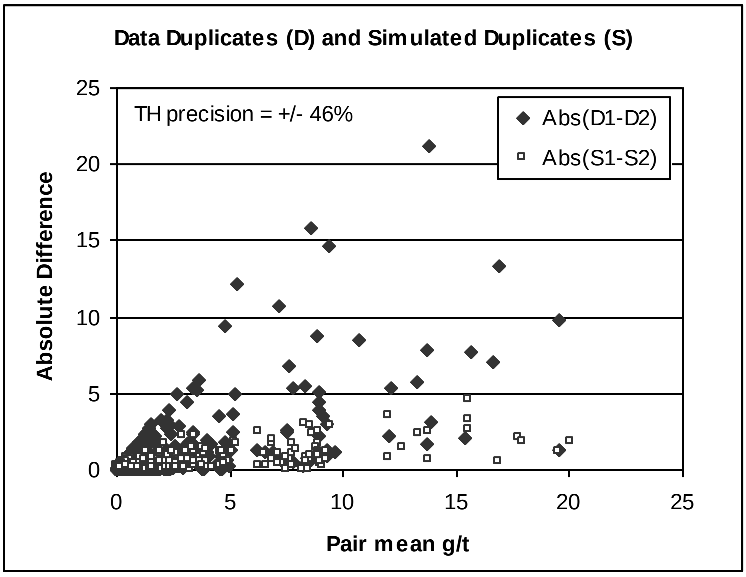

Fig 12 – Example 2: Absolute difference plot of assay duplicates (D, 50 g) and simulated duplicates (S, 50 g).

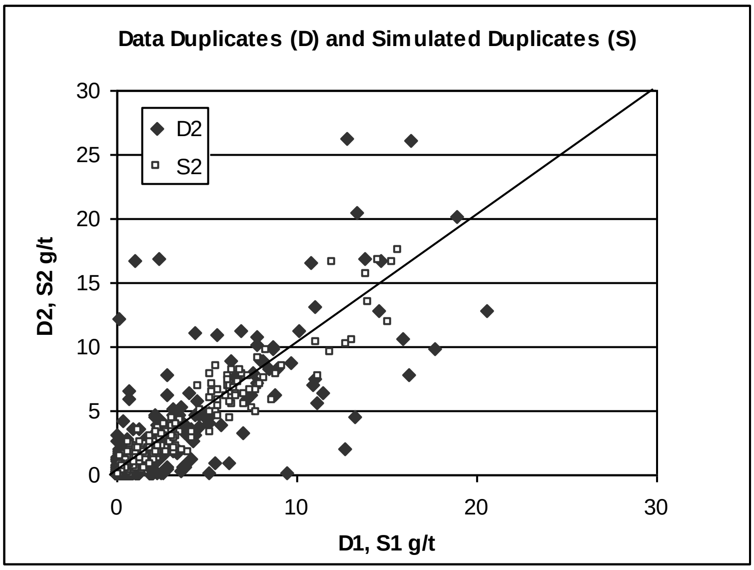

Fig 13 – Example 2: Scatterplot of Assay duplicates (D, 50 g) and simulated duplicates (S, 50 g).

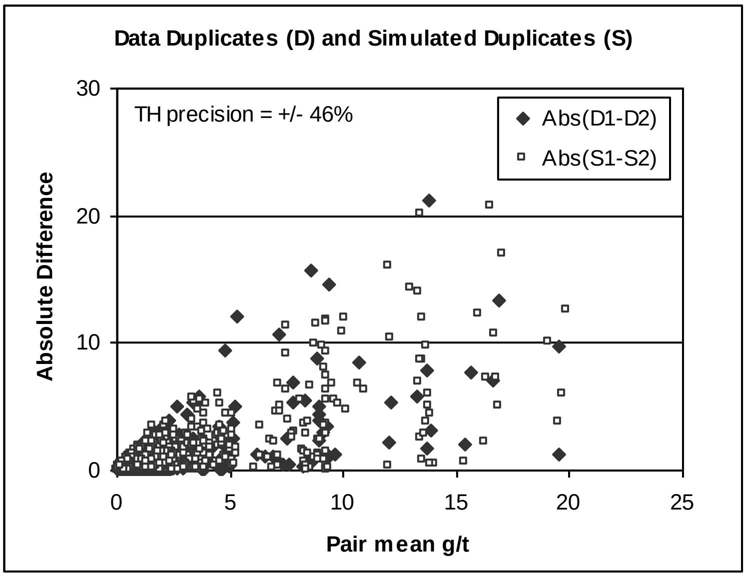

Fig 14 – Example 2: Absolute difference plot of assay duplicates (D, 50 g) and simulated duplicates (S, 500 g).

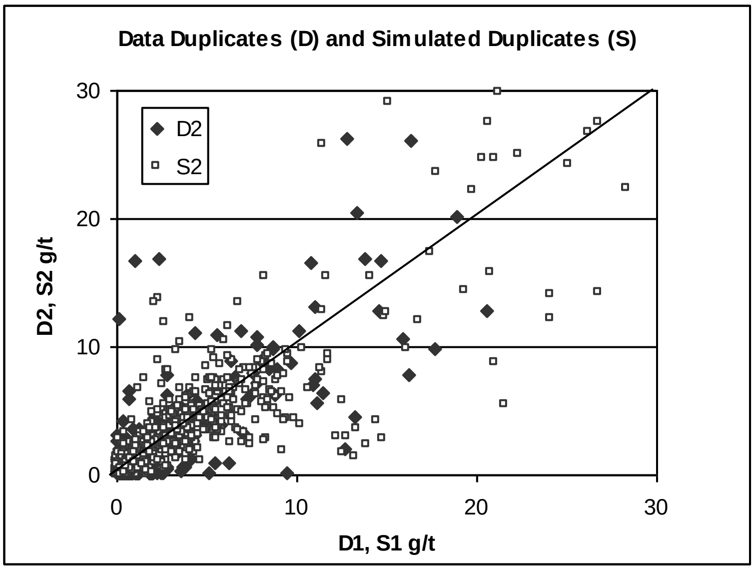

Fig 15 – Example 2: Scatterplot of Assay duplicates (D, 50 g) and simulated duplicates (S, 500 g).

TABLES

Square Size in Points	Precision Measures			
	Figure 1(a)		Figure 1(b)	
	$p_1(m)$	$p_2(m)$	$p_1(m)$	$p_2(m)$
3	0.76	0.76	2.16	2.12
6	0.48	0.48	1.95	1.90
9	0.31	0.31	1.77	1.73

Table 1 – Relative precision of grades within squares of different sizes by two different methods.