

## Research Article

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M. Yetkin\* and O. Bilginer

# On the application of nature-inspired grey wolf optimizer algorithm in geodesy

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**Abstract:** Nowadays, solving hard optimization problems using metaheuristic algorithms has attracted bountiful attention. Generally, these algorithms are inspired by natural metaphors. A novel metaheuristic algorithm, namely Grey Wolf Optimization (GWO), might be applied in the solution of geodetic optimization problems. The GWO algorithm is based on the intelligent behaviors of grey wolves and a population based stochastic optimization method. One great advantage of GWO is that there are fewer control parameters to adjust. The algorithm mimics the leadership hierarchy and hunting mechanism of grey wolves in nature. In the present paper, the GWO algorithm is applied in the calibration of an Electronic Distance Measurement (EDM) instrument using the Least Squares (LS) principle for the first time. Furthermore, a robust parameter estimator called the Least Trimmed Absolute Value (LTAV) is applied to a leveling network for the first time. The GWO algorithm is used as a computing tool in the implementation of robust estimation. The results obtained by GWO are compared with the results of the ordinary LS method. The results reveal that the use of GWO may provide efficient results compared to the classical approach.

**Keywords:** Calibration; Least Trimmed Absolute Value Estimator; Natural Computing; Stochastic Optimization; Swarm Intelligence

## 1 Introduction

Optimization may be defined as the search for decision variables in a defined search space minimizing or maximizing an objective function. Unfortunately, the objective function may have many local optima that are not global. Standard optimization techniques may fail in yielding the global optimum. Therefore, global optimization

techniques are generally preferred (Horst and Tuy, 1996). Global optimization techniques are divided into two categories: deterministic algorithms and metaheuristic algorithms (stochastic algorithms).

Metaheuristic methods based on nature observations have been extensively employed to solve numerous complex optimization problems in different engineering fields. Furthermore, metaheuristic algorithms are generally faster in finding good quality solution than deterministic ones (Gogna and Tayal, 2013).

There are certain important features of a metaheuristic algorithm: simplicity in the design and implementation; the algorithm can be easily programmed using any programming language, flexibility; the metaheuristic algorithms can be used to solve very different optimization problems, derivative-free optimization; search for optimality is carried out stochastically, trapping into local optima avoidance; metaheuristic algorithms might produce global or at least near global optimum solutions, solving problems faster; even a large problem may be solved within a reasonable amount of time, and inspiration by natural phenomena. The interested reader is referred to Talbi (2009) for more information about metaheuristics.

Most of the nature-inspired metaheuristic algorithms emulate animal behaviors. For example, particle swarm optimization algorithm (Kennedy and Eberhart, 1995), ant colony optimization algorithm (Dorigo and Stützle, 2004), artificial bee colony algorithm (Karaboga, 2005), cat swarm optimization algorithm (Chu et al., 2006), shuffled frog leaping algorithm (Eusuff et al., 2006), bat algorithm (Yang, 2010), cuckoo search algorithm (Yang, 2010), firefly algorithm (Yang, 2010) and GWO algorithm (Mirjalili et al., 2014). On the other hand, some algorithms mimic different natural phenomena. For example, simulated annealing method mimics the annealing process in Metallurgy (Kirkpatrick et al., 1983) and harmony search is a music-inspired metaheuristic algorithm (Geem et al., 2001).

The GWO algorithm is a swarm intelligence method that simulates the leadership hierarchy and hunting behavior of grey wolves (timber wolves or *canis lupus*). The population includes four types of grey wolves, i.e., alpha,

\*Corresponding Author: M. Yetkin: Department of Geomatics Engineering, Faculty of Engineering and Architecture, Izmir Katip Celebi University, Izmir, Turkey; E-mail: mevlut.yetkin@ikcu.edu.tr

O. Bilginer: Department of Geomatics Engineering, Faculty of Engineering and Architecture, Izmir Katip Celebi University, Izmir, Turkey

beta, delta, and omega in order to imitate the leadership hierarchy. Additionally, the three basic steps of hunting are utilized: searching for prey, encircling prey, and attacking prey. In this algorithm, initially a random population of candidate solutions (wolves) is created and this population is iteratively improved to reach quality solutions to the problem being solved (Mirjalili, et al., 2014; Mirjalili, 2015; Saremi et al., 2015).

Recently, metaheuristic optimization methods have been employed for solving geodetic optimization problems. For example, geodetic network optimization (Berné and Baselga, 2004; Yetkin et al., 2009; Baselga 2011; Yetkin et al., 2011; Yetkin, 2013), robust parameter estimation (Baselga, 2007; Baselga and García-Asenjo 2008a; Baselga and García-Asenjo 2008b; Baselga and García-Asenjo 2008c; Koch et al., 2017; Koch et al., 2019 Yetkin and Berber, 2013; Yetkin and Berber, 2014; Yetkin, 2018), GPS network adjustment (Civicioglu et al., 2019) and GPS positioning (Baselga, 2010).

In the present paper, the GWO algorithm is applied to solve two important geodetic optimization problems for the first time: the calibration of an EDM instrument using the LS principle and the application of a robust parameter estimator called the LTAV to a leveling network.

## 2 Geodetic Optimization Problems

Optimization has a crucial role in Geodesy. For instance, parameter estimation problems such as the adjustment of geodetic networks using the ordinary LS method or numerous robust estimation methods, calibration of EDM instruments, optimal design of geodetic networks and integer ambiguity resolution are highly important optimization problems. In the present study, we will discuss the calibration of EDM instruments and robust estimation in leveling networks problems.

### 2.1 Calibration of an EDM Device

The EDM devices with their reflectors have to be tested and checked to determine their zero and scale errors. These systematic errors are constant and incremental, respectively. A calibration baseline may be used to accomplish this task. The LS method is generally used to obtain the scaling factor that is linearly proportional to the distance being measured and the zero error value for the instrument-reflector pair. In the LS adjustment computa-

tion, the following observation equation is used:

$$SD_A + C = D_H - D_A + V_{DH} \quad (1)$$

where  $S$  is scaling factor for the EDM device,  $C$  is device-reflector constant,  $D_H$  is the observed horizontal distance (all atmospheric and slope corrections should be applied),  $D_A$  is the published horizontal calibrated distance for the baseline, and  $V_{DH}$  is the residual. This linear system of observation equations can be solved using the ordinary the LS method to obtain mentioned unknowns, i.e.  $S$  and  $C$  (Ghilani, 2010).

As well known, the sum of the squared residuals is the objective function of the LS method. The objective function can be optimized (minimized) using the GWO algorithm in order to compute the zero error and the scaling factor.

### 2.2 Robust LTAV Estimator

Robust estimation methods are very useful outlier diagnosis tools. Furthermore, they might give the least affected results from blunders. In the past, many different robust methods have been used in Geodesy. For example, the Least Absolute Deviations method (Yetkin and Inal 2011), M-estimators (Banaś, 2017), and high-breakdown estimators, i.e., LMS (Least Median of Squares) and LTS (Least Trimmed Squares) methods (Yetkin and Berber, 2014) have been applied successfully.

The parameters such as the elevations of the points of a leveling network can be estimated by optimizing the objective function of the robust LTAV method using the GWO algorithm.

To apply the LTAV estimator in a leveling network, the following observation equation can be written for any height difference observation in the network:

$$h_l - h_k = \Delta h_{kl} + v_{kl} \quad (2)$$

This equation relates the unknown heights of any two points,  $k$  and  $l$ , with the height difference observation  $\Delta h_{kl}$  and its residual  $v_{kl}$ . This equation is basic in performing the LTAV estimator in a leveling network.

The LTAV estimator is a close variation of Rousseeuw's (1984) LTS method that can achieve a breakdown point of approximately 0.5 (Wilcox, 2012).

The LTAV estimator minimizes the sum of the  $m$  smallest absolute residuals. It is based on the following objective function:

$$\sum_{i=1}^m |p_i v_i| \rightarrow \text{minimum} \quad (3)$$

where  $|v_i|$  is the  $i$ th smallest absolute residual and  $m$  is defined as  $m = \lceil n/2 \rceil + 1$ . It should be noted that if  $n$  is an odd number  $\lceil n/2 \rceil$  is rounded to the nearest integer (Wilcox, 2012).  $n$  is the number of height difference observations. The residuals are computed based on the observation equations given in Eq. (2). The weights of the observed height differences ( $p_i$ ) are inversely proportional with the observational variances.

### 3 GWO Algorithm

The implementation of the GWO algorithm is discussed in Mirjalili, et al., (2014). A summary of the process is given below.

To solve any optimization problem using the GWO algorithm, initially a random timber wolf population that is scattered in the search space is generated. The population includes candidate solutions (grey wolves). The objective function values of the initial solutions are calculated. The first three best solutions (solutions with the smallest objective function values or largest fitness values) are determined and they are named as alpha ( $\alpha$ ), beta ( $\beta$ ), and delta ( $\delta$ ), respectively. The remaining solutions are omega ( $\omega$ ) wolves. As well known, the fitness value of any solution is inversely proportional with its objective function value.

During optimization, the  $\omega$  wolves try to improve their positions with respect to  $\alpha$ ,  $\beta$ , or  $\delta$  as follows:

$$D_\alpha = |C_1 * X_\alpha - X| \quad (4)$$

$$D_\beta = |C_2 * X_\beta - X| \quad (5)$$

$$D_\delta = |C_3 * X_\delta - X| \quad (6)$$

$X_\alpha$ ,  $X_\beta$ , and  $X_\delta$  are the position vectors of the alpha, beta and delta, respectively.  $C_1$ ,  $C_2$ , and  $C_3$  are random vectors and  $X$  shows the position of the current solution.  $C_{1,2,3} = 2 * r_2$  The random vector  $r_2$  takes the values between 0 and 1.

Equations (4), (5), and (6) shows the approximate distance between the current solution ( $\omega$ ) and  $\alpha$ ,  $\beta$ , and  $\delta$ , respectively. Then, the new position of a  $\omega$  wolf is computed by

$$X_1 = X_\alpha - A_1 * (D_\alpha) \quad (7)$$

$$X_2 = X_\beta - A_2 * (D_\beta) \quad (8)$$

$$X_3 = X_\delta - A_3 * (D_\delta) \quad (9)$$

$$X_{new} = \frac{(X_1 + X_2 + X_3)}{3} \quad (10)$$

$A_1$ ,  $A_2$  and  $A_3$  random vectors.  $A_{1,2,3} = 2a * r_1 - a$ . The vector  $r_1$  includes random numbers that are extracted in the interval  $[0, 1]$ . The components of the vector  $a$  are linearly decreased from 2 to 0 as the iterations progress.

Balancing exploration and exploitation has a key role in the success of any metaheuristic search method. Exploration means the generation of diverse solutions in order to globally scout the search space. Nevertheless, exploitation focuses on a current good solution that is found in a local region. The importance of exploration and exploitation in the success of any metaheuristic algorithm was emphasized by Yang (2010). In the GWO algorithm, linearly decreasing  $A$  during iterative optimization process provides exploitation. On the other hand, the randomization in the generation of  $C$  is important for exploration and exploitation at any stage. Thus, the solutions being trapped at local optima are avoided (Mirjalili, 2015).

The GWO algorithm is iteratively applied. The  $\alpha$ ,  $\beta$ ,  $\delta$ , and  $\omega$  wolves are updated in each iteration step. It should be noted that, we keep the best solutions untouched, i.e., only the positions of  $\omega$  wolves are updated.

### 4 Numerical Results

In this section the GWO algorithm is applied to solve two different geodetic optimization problems for the verification of the above formulation.

#### Example I: Calibration of an EDM device

Calibration baseline observations and their published values are listed in Table 1 (Ghilani, 2010).

The aim is to determine the instrument-reflector pair constant ( $C$ ) and the scaling factor ( $S$ ). Using the ordinary LS method, Ghilani (2010) gives the solution as  $S = -0.7$  ppm and  $C = 20.3$  mm.

To make a comparison, the problem is subsequently solved using the GWO algorithm. The objective function to be minimized is the sum of the squared residuals. The number of the wolves in the population and the iteration number are 20 and 100, respectively. The search space is defined by using the surveying expert knowledge. The GWO algorithm was run 10 times and the average estimate solution is obtained as  $-0.69$  ppm and 20.3 mm. The results are almost identical to the values obtained by the LS method. These results verify the efficiency of the GWO algorithm in the solution of the geodetic optimization problem.

**Table 1.** Calibration Data

Distance	$D_A$ (m)	$D_H$ (m)	Distance	$D_A$ (m)	$D_H$ (m)
0-150	149.9975	150.0175	150-0	149.9975	150.0174
0-430	430.0101	430.0302	430-0	430.0101	430.0304
0-1400	1400.0030	1400.0223	1400-0	1400.0030	1400.0221
150-430	280.0126	280.0327	430-150	280.0126	280.0331
150-1400	1250.0055	1250.0248	1400-150	1250.0055	1250.0257
430-1400	969.9929	970.0119	430-1400	969.9929	970.0125

### Example II: Robust LTAV estimator for a leveling network

A leveling network is used to make a comparison between the ordinary LS method and the robust LTAV method. Table 2 shows the observed height differences and their standard deviations (Ghilani, 2010).

The elevation of  $A$  is 437.596 m, i.e., datum of the network is defined by minimum constraints. Three schemes of adjustment of the leveling network are performed:

Scheme 1: The LS method without any outlier;

Scheme 2: The LS method with an outlier (an error of +1.0 m was added to  $\Delta h_{BC}$ );

Scheme 3: The LTAV method with an outlier (an error of +1.0 m was added to  $\Delta h_{BC}$ ).

The LTAV method was performed via the GWO algorithm. The number of the wolves in the population and the iteration number are 100 and 25000, respectively. The search space may be defined by using the method proposed by Baselga (2007); the LS solution is taken as the center of the search space and the boundaries of the search space are defined by applying positive and negative signed increments to every element of the LS solution vector.

**Table 2.** Leveling data

From	To	$\Delta h$ (m)	$\sigma$ (m)
$A$	$B$	10.509	0.006
$B$	$C$	5.360	0.004
$C$	$D$	-8.523	0.005
$D$	$A$	-7.348	0.003
$B$	$D$	-3.167	0.004
$A$	$C$	15.881	0.012

The adjusted elevations are listed in the Table 3.

The elevations obtained from Scheme 1 are used as reference values because the LS method gives the best results if the observations are free from outliers. Therefore, if a method gives closer results to reference values it can be considered more successful against outlying observations. Therefore, the sums of the error square are computed for Scheme 2 and Scheme 3 (see the last row of Table 3). In other words, the values of the last row of Table 3 are the

**Table 3.** The Elevations of the Leveling Network for the three schemes (m)

Station	Scheme1	Scheme2	Scheme3
$B$	448.1087	447.8973	448.1107
$C$	453.4685	453.9277	453.4761
$D$	444.9436	444.9677	444.9439
$\sum$ (cm <sup>2</sup> )	0	2561.35	0.62

sum of the squares of the differences between the heights obtained with two different adjustment schemes and the heights obtained with Scheme 1. According to this, the robust method that is implemented by the GWO algorithm has produced better results than the ordinary LS method in the case of outlying observation. Furthermore, for Scheme 2 and Scheme 3, the maximum differences among adjusted elevations with respect to Scheme 1 are 45.92 cm and 0.76 cm, respectively. This also confirms that the proposed approach's results are better than the ordinary LS method in the case of outlying observation.

## 5 Conclusion

In the present work, a novel metaheuristic algorithm called the GWO algorithm has been applied to solve two geodetic optimization problems. In the first example, the method has been used to compute the calibration parameters of an EDM instrument. The results have been compared with the results of the ordinary LS method. Both methods have produced nearly the identical values for the calibration parameters. Thus, the first example shows that the GWO algorithm can be efficiently used to solve LS based problems.

In the second example, the GWO algorithm has been used to implement a robust method called the LTAV estimator in a leveling network. The ordinary LS method has stuck in a bad local optimum solution because of the outlying observation. On the other hand, the results of proposed robust estimation approach are very close to the results of the ordinary LS method without any outlier. Thus, the second example shows that the robust LTAV method may be

efficiently applied in a geodetic network using the GWO algorithm.

We applied the LTAV method along with the GWO algorithm to a leveling network. As a future work, the method proposed in the present paper may be compared with iterative procedures for excluding outliers such as Baarda's data snooping method.

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