Automatica 45 (2009) 2265–2271

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stability analysis for discrete-time switched time-delay systems[☆]

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ARTICLE INFO

Article history:
Received 29 August 2008
Received in revised form
21 January 2009
Accepted 25 May 2009
Available online 25 July 2009

Keywords:
Switched time-delay systems
Discrete-time
Exponential stability
Delay-dependent
Average dwell time

ABSTRACT

The stability analysis problem is studied in this paper for a class of discrete-time switched time-delay systems. By using a newly constructed Lyapunov functional and the average dwell time scheme, a delay-dependent sufficient condition is derived for the considered system to be exponentially stable. The obtained results provide a solution to one of the basic problems in discrete-time switched time-delay systems, that is, to find a switching signal for which the switched time-delay system is exponentially stable. Two illustrative examples are given to demonstrate the effectiveness of the proposed results.

1. Introduction

Switched systems are hybrid dynamical systems composed of subsystems with their own parameterizations subject to a rule orchestrating the switching law between the various subsystems (De la Sen, 2007). They have strong engineering background in various areas and have attracted increasingly more attention since the 1990s (DeCarlo, Branicky, Pettersson, & Lennartson, 2000; Liberzon, 2003; Sun & Ge, 2004). It is commonly agreed that there are three basic problems in stability analysis and the design of switched systems (Liberzon, 1999): (i) to find the conditions for stability under arbitrary switching; (ii) to identify the limited but useful class of stabilizing switching signals; and (iii) to construct a stabilizing switching signal. Many efficient approaches have been presented in the existing literature to deal with these three basic problems. See, for example, the multiple Lyapunov function approach (Branicky, 1998; El Farral, Mhaskar, & Christofides, 2005), the piecewise Lyapunov function approach (Johansson & Rantzer, 1998; Wicks, Peleties, & De Carlo, 1994), the switched Lyapunov function approach (Daafouz, Riedinger, & Iung, 2002; Du, Jiang, Shi, & Zhou, 2007), and the dwell-time or average dwell-time scheme (Hespanha & Morse, 1999; Song, Fan, Fei, & Yang, 2008; Tshii & Francis, 2002; Zhai, Hu, Yasuda, & Michel, 2002; Zhang, Boukas, & Shi, 2008).

On the other hand, time-delay phenomena are very common in practical systems. A switched system with time-delay individual subsystems is called a switched time-delay system; in particular, when the subsystems are linear, it is then called a switched time-delay linear system. Switched time-delay systems have various applications in practical engineering systems, such as power systems and power electronics (Meyer, Schroder, & De Doncker, 2004; Sun & Ge, 2004), time-delay systems with controller or actuator failure (Sun, Liu, Rees, & Wang, 2007), and networked control systems (Kim, Park, & Ko, 2004). The presence of delay makes the analysis and synthesis problems for switched time-delay systems much more complicated. Some of the aforementioned approaches for non-delayed switched systems have been successfully adopted in some existing results to investigate the stability and stabilization of switched time-delay systems. However, most of the results are focused on the basic problem (i) and problem (iii). For example, by considering an arbitrary switching signal, the stability and L_2 gain analysis were studied in Zhai, Sun, Yasuda, and Anthony (2003) for a class of symmetric time-delay systems; the quadratic stability and stabilization problems were considered in Xie and Wang (2004) for discrete-time switched time-delay systems; and H_∞ filters were designed in Du et al. (2007) for a class of discrete-time switched time-delay systems by using switched Lyapunov functions. The problem (iii) was considered, for example, in Kim, Campbell, and Liu (2006), Phat (2005) and Sun, Wang, Liu, and Zhao (2008). In Kim et al. (2006), a switching signal was constructed to guarantee the

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Masayuki Fujita under the direction of Editor Ian R. Petersen.

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doi:10.1016/j.automatica.2009.05.027

asymptotic stability of switched time-delay systems with small constant delay. The results in Kim et al. (2006) were improved in Sun, Wang, et al. (2008) by using a less conservative method and were further extended to the time-varying delay case.

Compared with the results on problem (i) and problem (iii), much fewer results have been reported on the problem (ii) for switched time-delay systems. Problem (ii) is to find stabilizing switching signals on the condition that all the individual subsystems of the switched systems are stable. Basically, we will find that stability is ensured if the switching is sufficiently slow (Liberzon, 1999), and it is known that dwell time and average dwell time are two simple yet useful ways to specify slow switching of the switching signals. By applying the average dwell time scheme, the stability analysis problem is investigated in Sun, Zhao, and Hill (2006) for continuous-time switched time-delay systems. The results presented in Sun et al. (2006) reveal that if all the individual subsystems are exponentially stable and that the average dwell time of the switching signal is not smaller than a certain lower bound, then the exponential stability of the switched time-delay system is preserved. To the best of the authors' knowledge, few results are reported in the existing literature on solving the basic problem (ii) for discrete-time switched time-delay systems via the dwell time or average dwell time scheme. Moreover, the procedures given in Sun et al. (2006) can not be applied to the discrete-time case. This has motivated us to develop some procedures to solve the basic problem (ii) for discrete-time switched time-delay systems via the average dwell time approach.

In this paper, we study the stability analysis problem for a class of discrete-time switched time-delay systems. A class of slow switching signals specified by the average dwell time is identified to guarantee the exponential stability of the considered systems. A sufficient condition, which explicitly characterizes the switching signal, is derived for the switched time-delay system to be exponentially stable by using a properly constructed decay-rate-dependent Lyapunov functional and the state variable transformation technique. The established condition reveals that the exponential stability of the overall switched time-delay system is preserved if all its individual subsystems are exponentially stable and that the average dwell time of the switching signal is not smaller than a lower bound. The effectiveness of the proposed results is finally demonstrated by two illustrative examples.

2. Preliminaries and problem formulation

Consider the following discrete-time switched time-delay system:

$$S_{\sigma(k)}: \begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}x(k-d), \\ x(l) = \phi(l), \quad l = k_0 - \bar{d}, k_0 - \bar{d} + 1, \ldots, k_0 \end{cases} \tag{1}$$

where $x(k) \in \Re^n$ is the system state, $\phi(l)$ is a vector-valued initial function, k_0 is the initial time step, d is the bounded constant delay and satisfies $0 \le d \le \bar{d}$, $\sigma(k): Z \to M = \{1, \ldots, m\}$ is the switching signal, m is a finite integer, and Z is the set of positive integers. A_i and B_i, $i = 1, \ldots, m$, are constant matrices.

For any given integer $k \ge 1$ and a piecewise constant switching signal $\sigma(\tau)$, we let $k_1 < \cdots < k_t$, $t \ge 1$, denote the switching instants of $\sigma(\tau)$ for $k_0 \le \tau < k$ and $\{x(k_0); (i_0, k_0), (i_1, k_1), \ldots, (i_t, k_t), (i_{t+1}, k)\}$ denote the switching sequence, which means that the i_jth subsystem is activated when $k_j \le \tau < k_{j+1}$, or equivalently, $\sigma(\tau) = i_j$ when $k_j \le \tau < k_{j+1}$, where $i_j \in M$, $0 \le j \le t$, $k_{t+1} = k$, and $d_{i_j} = k_{j+1} - k_j$ is the dwell time of the switched system on the subsystem i_j, $j = 0, 1, 2, \ldots$. The definition of average dwell time for the discrete-time switched systems is given as follows.

Definition 1 (*Song et al. (2008), Zhai et al. (2002) and Zhang et al. (2008)*). For any $k \ge k_0$ and any switching signal $\sigma(\tau)$, $k_0 \le \tau < k$, let N_σ denote the number of switchings of $\sigma(\tau)$. If $N_\sigma \le N_0 + (k - k_0)/T_a$ holds for $N_0 \ge 0$ and $T_a > 0$, then T_a is called the average dwell time and N_0 the chatter bound.

Remark 1. The concept of average dwell time was originally proposed for continuous-time switched systems in Hespanha and Morse (1999), and it has been modified to fit the discrete-time ones in some existing literature; see, for example, Song et al. (2008), Zhai et al. (2002) and Zhang et al. (2008) and the references therein. The definition of average dwell time in Definition 1 is borrowed from these existing results. For simplicity, but without loss of generality, we choose $N_0 = 0$ in what follows.

The following definition and lemma also will be used in the derivation of the main results.

Definition 2. System (1) is said to be exponentially stable if its solutions satisfy

$$\|x(k)\| \le c\lambda^{-(k-k_0)}\|\phi\|_L, \quad \forall\, k \ge k_0.$$

for any initial conditions $(k_0, \phi) \in \Re^+ \times C^n$, where $\|\phi\|_L = \sup_{k_0 - \bar{d} \le l \le k_0} \|\phi(l)\|$, $c > 0$ is the decay coefficient, and $\lambda > 1$ is the decay rate.

Lemma 1 (*Chen, Yu, & Zhang, 2007*). *Let $\xi(k) \in \Re^n$ be a vector-valued function. Then, the following inequality*

$$\sum_{s=k-d}^{k-1} z^{\mathrm{T}}(s)Rz(s) \le \eta^{\mathrm{T}}(k)\begin{bmatrix} N_1 + N_1^{\mathrm{T}} & -N_1^{\mathrm{T}} + N_2 \\ * & -N_2 - N_2^{\mathrm{T}} \end{bmatrix}\eta(k) + \eta^{\mathrm{T}}(k)\begin{bmatrix} N_1^{\mathrm{T}} \\ N_2^{\mathrm{T}} \end{bmatrix} dR^{-1}[N_1\ N_2]\eta(k)$$

holds for any matrices $R > 0$, N_1, N_2, and a scalar $d \ge 0$, where $z(s) = \xi(s+1) - \xi(s)$ and $\eta(k) = \left[\xi^{\mathrm{T}}(k)\ \xi^{\mathrm{T}}(k-d)\right]^{\mathrm{T}}$.

The objective of this paper is to find a class of switching signals (specified by the average dwell time) which guarantee the exponential stability of the switched time-delay system (1).

3. Main results

Consider the following subsystem of the switched system (1):

$$S_i: \begin{cases} x(k+1) = A_i x(k) + B_i x(k-d), \quad i \in M, \\ x_{k_0}(l) = x(k_0 + l), \quad l = -\bar{d}, -\bar{d}+1, \ldots, 0. \end{cases} \tag{2}$$

Choose the following Lyapunov functional for S_i:

$$V_i(k) = x^{\mathrm{T}}(k)P_i x(k) + \sum_{s=k-d}^{k-1} \lambda^{2(s-k)} x^{\mathrm{T}}(s)Q_i x(s) + \sum_{\theta=-\bar{d}+1}^{0} \sum_{s=k-1+\theta}^{k-1} \lambda^{2(s-k)} y^{\mathrm{T}}(s)R_i y(s) \tag{3}$$

where $P_i > 0$, $Q_i > 0$, $R_i > 0$, and $y(s) = \lambda x(s+1) - x(s)$; $\lambda > 1$ is a given constant.

The decay estimation of the Lyapunov functional $V_i(k)$ along the trajectory of system (2) will first be given in the following proposition, which plays a key role in the derivation of the exponential stability condition for the switched time-delay system (1).

Proposition 1. *For a given scalar $\lambda > 1$ and any delay d satisfying $0 \le d \le \bar{d}$, if there exist matrices $P_i > 0$, $Q_i > 0$, $R_i > 0$, N_{1i}, N_{2i}, $i \in M$ such that the following matrix inequality*

$$\begin{bmatrix} (1,1) & (1,2) & \bar{A}_i^{\mathrm{T}}P_i & (\bar{A}_i-I)^{\mathrm{T}}R_i & N_{1i}^{\mathrm{T}} \\ * & (2,2) & \bar{B}_i^{\mathrm{T}}P_i & \bar{B}_i^{\mathrm{T}}R_i & N_{2i}^{\mathrm{T}} \\ * & * & -P_i & 0 & 0 \\ * & * & * & -\bar{d}^{-1}R_i & 0 \\ * & * & * & * & -\bar{d}^{-1}R_i \end{bmatrix} < 0, \quad (4)$$

holds, then along any trajectory of system (2), the function $V_i(k)$ in (3) ensures the following decay estimation:

$$V_i(k) \le \lambda^{-2(k-k_0)}V_i(k_0), \quad k \ge k_0, \quad (5)$$

where $\bar{A}_i = \lambda A_i$, $\bar{B}_i = \lambda^{\bar{d}+1}B_i$, and

$$\begin{aligned}(1,1) &= -P_i + Q_i + N_{1i} + N_{1i}^{\mathrm{T}} + (1-\lambda^{-\bar{d}})\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i \\ &\quad + (1-\lambda^{-\bar{d}})\bar{d}(\bar{A}_i-I)^{\mathrm{T}}R_i(\bar{A}_i-I),\end{aligned}$$

$$(1,2) = -N_{1i}^{\mathrm{T}} + N_{2i},$$

$$(2,2) = -Q_i - N_{2i} - N_{2i}^{\mathrm{T}}.$$

Proof. Applying the transformation $x(k) = \lambda^{-(k-k_0)}\xi(k)$ and denoting $\tilde{B}_i = \lambda^{d+1}B_i$, we obtain the following system from (2):

$$\begin{cases} \xi(k+1) = \bar{A}_i\xi(k) + \tilde{B}_i\xi(k-d), \\ \xi_{k_0}(l) = \xi(k_0+l) = \lambda^l x_{k_0}(l), \quad l = -\bar{d}, -\bar{d}+1, \ldots, 0. \end{cases} \quad (6)$$

Choose the following Lyapunov functional for system (6):

$$\begin{aligned} W_i(k) &= \xi^{\mathrm{T}}(k)P_i\xi(k) + \sum_{s=k-d}^{k-1}\xi^{\mathrm{T}}(s)Q_i\xi(s) \\ &\quad + \sum_{\theta=-\bar{d}+1}^{0}\sum_{s=k-1+\theta}^{k-1} z^{\mathrm{T}}(s)R_iz(s), \end{aligned} \quad (7)$$

where $z(s) = \xi(s+1) - \xi(s)$. Then, by Lemma 1 and the fact that $-\sum_{s=k-\bar{d}}^{k-1}z^{\mathrm{T}}(s)R_iz(s) \le -\sum_{s=k-d}^{k-1}z^{\mathrm{T}}(s)R_iz(s)$, the forward difference of the Lyapunov functional $W_i(k)$ along any trajectory of system (6) is given by

$$\begin{aligned} \Delta W_i(k) &= W_i(k+1) - W_i(k) \\ &= \xi^{\mathrm{T}}(k+1)P_i\xi(k+1) - \xi^{\mathrm{T}}(k)P_i\xi(k) \\ &\quad + \xi^{\mathrm{T}}(k)Q_i\xi(k) - \xi^{\mathrm{T}}(k-d)Q_i\xi(k-d) \\ &\quad + z^{\mathrm{T}}(k)\bar{d}R_iz(k) - \sum_{s=k-\bar{d}}^{k-1}z^{\mathrm{T}}(s)R_iz(s) \\ &\le \eta^{\mathrm{T}}(k)\begin{bmatrix}\bar{A}_i^{\mathrm{T}} \\ \tilde{B}_i^{\mathrm{T}}\end{bmatrix}P_i\begin{bmatrix}\bar{A}_i & \tilde{B}_i\end{bmatrix}\eta(k) \\ &\quad + \eta^{\mathrm{T}}(k)\begin{bmatrix}(\bar{A}_i-I)^{\mathrm{T}} \\ \tilde{B}_i^{\mathrm{T}}\end{bmatrix}\bar{d}R_i\begin{bmatrix}\bar{A}_i-I & \tilde{B}_i\end{bmatrix}\eta(k) \\ &\quad + \eta^{\mathrm{T}}(k)\begin{bmatrix}-P_i+Q_i & 0 \\ 0 & -Q_i\end{bmatrix}\eta(k) \\ &\quad + \eta^{\mathrm{T}}(k)\begin{bmatrix}N_{1i}+N_{1i}^{\mathrm{T}} & -N_{1i}^{\mathrm{T}}+N_{2i} \\ -N_{1i}+N_{2i}^{\mathrm{T}} & -N_{2i}-N_{2i}^{\mathrm{T}}\end{bmatrix}\eta(k) \\ &\quad + \eta^{\mathrm{T}}(k)\begin{bmatrix}N_{1i}^{\mathrm{T}} \\ N_{2i}^{\mathrm{T}}\end{bmatrix}\bar{d}R_i^{-1}[N_{1i}\ N_{2i}]\eta(k) \end{aligned} \quad (8)$$

where $\eta(k) = \begin{bmatrix}\xi^{\mathrm{T}}(k)\ \xi^{\mathrm{T}}(k-d)\end{bmatrix}^{\mathrm{T}}$. It follows from the relations $\tilde{B}_i = \lambda^{d-\bar{d}}\bar{B}_i$, $0 \le d \le \bar{d}$, and $\lambda > 1$ that

$$\begin{aligned} &\eta^{\mathrm{T}}(k)\begin{bmatrix}\bar{A}_i^{\mathrm{T}} \\ \tilde{B}_i^{\mathrm{T}}\end{bmatrix}P_i\begin{bmatrix}\bar{A}_i & \tilde{B}_i\end{bmatrix}\eta(k) \\ &= \xi^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i\xi(k) + 2\xi^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\tilde{B}_i\xi(k-d) \\ &\quad + \xi^{\mathrm{T}}(k-d)\tilde{B}_i^{\mathrm{T}}P_i\tilde{B}_i\xi(k-d) \\ &= \lambda^{d-\bar{d}}[\xi^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i\xi(k) + 2\xi^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\bar{B}_i\xi(k-d) \\ &\quad + \xi^{\mathrm{T}}(k-d)\bar{B}_i^{\mathrm{T}}P_i\bar{B}_i\xi(k-d)] \\ &\quad + (1-\lambda^{d-\bar{d}})\xi^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i\xi(k) \\ &\quad + (\lambda^{2(d-\bar{d})} - \lambda^{d-\bar{d}})\xi^{\mathrm{T}}(k-d)\bar{B}_i^{\mathrm{T}}P_i\bar{B}_i\xi(k-d) \\ &\le \eta^{\mathrm{T}}(k)\begin{bmatrix}\bar{A}_i^{\mathrm{T}} \\ \bar{B}_i^{\mathrm{T}}\end{bmatrix}P_i\begin{bmatrix}\bar{A}_i & \bar{B}_i\end{bmatrix}\eta(k) \\ &\quad + (1-\lambda^{-\bar{d}})\xi^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i\xi(k). \end{aligned} \quad (9)$$

Similar to the result in (9), we have

$$\begin{aligned} &\eta^{\mathrm{T}}(k)\begin{bmatrix}(\bar{A}_i-I)^{\mathrm{T}} \\ \tilde{B}_i^{\mathrm{T}}\end{bmatrix}\bar{d}R_i\begin{bmatrix}\bar{A}_i-I & \tilde{B}_i\end{bmatrix}\eta(k) \\ &\le \eta^{\mathrm{T}}(k)\begin{bmatrix}(\bar{A}_i-I)^{\mathrm{T}} \\ \bar{B}_i^{\mathrm{T}}\end{bmatrix}\bar{d}R_i\begin{bmatrix}\bar{A}_i-I & \bar{B}_i\end{bmatrix}\eta(k) \\ &\quad + (1-\lambda^{-\bar{d}})\xi^{\mathrm{T}}(k)(\bar{A}_i-I)^{\mathrm{T}}\bar{d}R_i(\bar{A}_i-I)\xi(k). \end{aligned} \quad (10)$$

Substituting (9) and (10) into (8) yields

$$\Delta W_i(k) \le \eta^{\mathrm{T}}(k)\Omega_i\eta(k), \quad (11)$$

where

$$\begin{aligned} \Omega_i &= \begin{bmatrix}\bar{A}_i^{\mathrm{T}} \\ \bar{B}_i^{\mathrm{T}}\end{bmatrix}P_i\begin{bmatrix}\bar{A}_i & \bar{B}_i\end{bmatrix} + \begin{bmatrix}(\bar{A}_i-I)^{\mathrm{T}} \\ \bar{B}_i^{\mathrm{T}}\end{bmatrix}\bar{d}R_i\begin{bmatrix}\bar{A}_i-I & \bar{B}_i\end{bmatrix} \\ &\quad + \begin{bmatrix}(1-\lambda^{-\bar{d}})\left(\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i + (\bar{A}_i-I)^{\mathrm{T}}\bar{d}R_i(\bar{A}_i-I)\right) & 0 \\ 0 & 0\end{bmatrix} \\ &\quad + \begin{bmatrix}-P_i+Q_i & 0 \\ 0 & -Q_i\end{bmatrix} + \begin{bmatrix}N_{1i}+N_{1i}^{\mathrm{T}} & -N_{1i}^{\mathrm{T}}+N_{2i} \\ -N_{1i}+N_{2i}^{\mathrm{T}} & -N_{2i}-N_{2i}^{\mathrm{T}}\end{bmatrix} \\ &\quad + \begin{bmatrix}N_{1i}^{\mathrm{T}} \\ N_{2i}^{\mathrm{T}}\end{bmatrix}\bar{d}R_i^{-1}[N_{1i}\ N_{2i}]. \end{aligned}$$

By Schur complement, inequality (4) is equivalent to $\Omega_i < 0$. Therefore, we have by (11) that $\Delta W_i(k) \le 0$, which implies that $W_i(k) \le W_i(k_0)$ for any $k \ge k_0$. Furthermore, by the relation

$$\begin{aligned} V_i(k) &= \lambda^{-2(k-k_0)}\xi^{\mathrm{T}}(k)P_i\xi(k) \\ &\quad + \sum_{s=k-d}^{k-1}\lambda^{2(s-k)}\lambda^{-2(s-k_0)}\xi^{\mathrm{T}}(s)Q_i\xi(s) \\ &\quad + \sum_{\theta=-\bar{d}+1}^{0}\sum_{s=k-1+\theta}^{k-1}\lambda^{2(s-k)}[\lambda\cdot\lambda^{-(s+1-k_0)}\xi(s+1) \\ &\quad - \lambda^{-(s-k_0)}\xi(s)]^{\mathrm{T}}\cdot R_i\cdot[\lambda\cdot\lambda^{-(s+1-k_0)}\xi(s+1) \\ &\quad - \lambda^{-(s-k_0)}\xi(s)] \\ &= \lambda^{-2(k-k_0)}\xi^{\mathrm{T}}(k)P_i\xi(k) + \lambda^{-2(k-k_0)}\sum_{s=k-d}^{k-1}\xi^{\mathrm{T}}(s)Q_i\xi(s) \\ &\quad + \lambda^{-2(k-k_0)}\sum_{\theta=-\bar{d}+1}^{0}\sum_{s=k-1+\theta}^{k-1}z^{\mathrm{T}}(s)R_iz(s) \\ &= \lambda^{-2(k-k_0)}W_i(k), \end{aligned}$$

and the fact that $W_i(k_0) = V_i(k_0)$, we have $\lambda^{2(k-k_0)}V_i(k) = W_i(k) \le W_i(k_0) = V_i(k_0)$, i.e., $V_i(k) \le \lambda^{-2(k-k_0)}V_i(k_0)$. The proof is completed. □

Remark 2. The condition (4), under which the function $V_i(k)$ satisfies the decay estimation (5), is dependent on both the delay bound $\bar{d}$ and the decay rate λ. Such a condition is derived by virtue of the simultaneous usage of the properly constructed decay-rate-dependent Lyapunov functional (3) and the state variable transformation technique.

Remark 3. The finite sum inequality, which is the discrete-time version of the integral inequality originally presented in Zhang, Wu, She, and He (2005), is applied in deriving the delay-dependent condition (4). Recently, various delay-dependent approaches, such as the so-called free-matrix method (Wu, He, & She, 2004), have been presented in the literature to reduce the conservatism of the analysis or synthesis results for time-delay systems. Delay-dependent conditions based on these recently developed approaches can also be established by following similar procedures to those presented in this section.

In what follows, the average dwell time method incorporated with the decay estimation of the function $V_i(k)$ will be used to derive the delay-dependent exponential stability condition for the switched time-delay system (1), and the results are presented in the following theorem.

Theorem 1. *For given scalars $\lambda > 1$, $\mu \geq 1$, and any delay d satisfying $0 \leq d \leq \bar{d}$, if there exist matrices $P_i > 0$, $Q_i > 0$, $R_i > 0$, N_{1i}, N_{2i}, $i = 1, \ldots, m$, such that the inequalities (4) and*

$$P_\alpha \leq \mu P_\beta, \qquad Q_\alpha \leq \mu Q_\beta, \qquad R_\alpha \leq \mu R_\beta, \quad \forall\, \alpha,\ \beta \in M, \tag{12}$$

$$T_a > T_a^* = \frac{\ln \mu}{2 \ln \lambda}, \tag{13}$$

hold, then the switched time-delay system (1) is exponentially stable and ensures a decay rate λ^ρ, where $\rho = -\frac{\ln \mu}{2T_a \ln \lambda} + 1$ and T_a is the average dwell time of system (1).

Proof. Choose the following Lyapunov functional for system (1):

$$V_{\sigma(k)}(k) = x^{\mathrm{T}}(k) P_{\sigma(k)} x(k) + \sum_{s=k-d}^{k-1} \lambda^{2(s-k)} x^{\mathrm{T}}(s) Q_{\sigma(k)} x(s) + \sum_{\theta=-\bar{d}+1}^{0} \sum_{s=k-1+\theta}^{k-1} \lambda^{2(s-k)} y^{\mathrm{T}}(s) R_{\sigma(k)} y(s), \tag{14}$$

where $P_i > 0$, $Q_i > 0$, $R_i > 0$, $i = 1, \ldots, m$ are the solutions of (4), (12) and (13). We have by (12) and (14) that

$$\begin{aligned} V_{\sigma(k_t)}(k_t) &= x^{\mathrm{T}}(k_t) P_{\sigma(k_t)} x(k_t) + \sum_{s=k_t-d}^{k_t-1} \lambda^{2(s-k_t)} x^{\mathrm{T}}(s) Q_{\sigma(k_t)} x(s) \\ &\quad + \sum_{\theta=-\bar{d}+1}^{0} \sum_{s=k_t-1+\theta}^{k_t-1} \lambda^{2(s-k_t)} y^{\mathrm{T}}(s) R_{\sigma(k_t)} y(s) \\ &\leq x^{\mathrm{T}}(k_t) \mu P_{\sigma(k_t-1)} x(k_t) \\ &\quad + \sum_{s=k_t-d}^{k_t-1} \lambda^{2(s-k_t)} x^{\mathrm{T}}(s) \mu Q_{\sigma(k_t-1)} x(s) \\ &\quad + \sum_{\theta=-\bar{d}+1}^{0} \sum_{s=k_t-1+\theta}^{k_t-1} \lambda^{2(s-k_t)} y^{\mathrm{T}}(s) \mu R_{\sigma(k_t-1)} y(s) \\ &= \mu V_{\sigma(k_t-1)}(k_t). \end{aligned} \tag{15}$$

On the other hand, the function $V_i(k)$ ensures the decay estimation (5) under the condition (4). Therefore, we have by the fact that $\sigma(k_t - 1) = \sigma(k_{t-1})$ that

$$\begin{aligned} V_{\sigma(k)}(k) &\leq \lambda^{-2(k-k_t)} V_{\sigma(k_t)}(k_t) \\ &\leq \lambda^{-2(k-k_t)} \mu V_{\sigma(k_t-1)}(k_t) \\ &\leq \mu \lambda^{-2(k-k_t)} \lambda^{-2(k_t-k_{t-1})} V_{\sigma(k_{t-1})}(k_{t-1}) \\ &\ \ \vdots \\ &\leq \mu^{N_\sigma} \lambda^{-2(k-k_t)} \lambda^{-2(k_t-k_{t-1})} \cdots \lambda^{-2(k_1-k_0)} V_{\sigma(k_0)}(k_0) \\ &= \mu^{N_\sigma} \lambda^{-2(k-k_0)} V_{\sigma(k_0)}(k_0) \\ &= \lambda^{N_\sigma \frac{\ln \mu}{\ln \lambda}} \lambda^{-2(k-k_0)} V_{\sigma(k_0)}(k_0) \\ &= \lambda^{-2(k-k_0) \frac{N_\sigma}{-2(k-k_0)} \frac{\ln \mu}{\ln \lambda}} \lambda^{-2(k-k_0)} V_{\sigma(k_0)}(k_0) \\ &\leq \lambda^{-2(k-k_0)(-\frac{\ln \mu}{2T_a \ln \lambda}+1)} V_{\sigma(k_0)}(k_0) \\ &= (\lambda^\rho)^{-2(k-k_0)} V_{\sigma(k_0)}(k_0). \end{aligned} \tag{16}$$

The definition of the average dwell time T_a is applied in deriving the last inequality in (16). We further obtain from (16) that

$$\begin{aligned} \beta_1 \|x(k)\|^2 &\leq V_{\sigma(k)}(k) \leq (\lambda^\rho)^{-2(k-k_0)} V_{\sigma(k_0)}(k_0) \\ &\leq (\lambda^\rho)^{-2(k-k_0)} \beta_2 \|\phi\|_L^2, \end{aligned}$$

which yields $\|x(k)\| \leq \sqrt{\frac{\beta_2}{\beta_1}} (\lambda^\rho)^{-(k-k_0)} \|\phi\|_L$, where

$$\beta_1 = \min_{i \in M} \lambda_{\min}(P_i),$$

$$\begin{aligned} \beta_2 &= \max_{i \in M} \lambda_{\max}(P_i) + \bar{d} \max_{i \in M} \lambda_{\max}(Q_i) \\ &\quad + 2\bar{d}(\bar{d}+1)(1+\lambda)^2 \max_{i \in M} \lambda_{\max}(R_i), \end{aligned}$$

and $\lambda_{\min}(\star)$ and $\lambda_{\max}(\star)$ denote the minimum and the maximum eigenvalues of the matrix $\star$, respectively. Besides, condition (13) and $\lambda > 1$ guarantee that $\lambda^\rho > 1$. Therefore, it is concluded from Definition 2 that the switched time-delay system (1) is exponentially stable and ensures the decay rate λ^ρ. The proof is completed. □

Remark 4. Denote $\|x\|_L = \sup_{k_0-\bar{d} \leq l \leq k_0} \|x(l)\|$, $\beta_{1i} = \lambda_{\min}(P_i)$, and $\beta_{2i} = \lambda_{\max}(P_i) + \bar{d}\lambda_{\max}(Q_i) + 2\bar{d}(\bar{d}+1)(1+\lambda)^2 \lambda_{\max}(R_i)$. Then, it can be obtained from (5) that the inequality $\|x(k)\| \leq \sqrt{\frac{\beta_{2i}}{\beta_{1i}}} \lambda^{-(k-k_0)} \|x\|_L$ holds for the subsystem (2), which indicates that the subsystem (2) is exponentially stable under the condition (4). So, Theorem 1 implies that the exponential stability of the switched time-delay system (1) is preserved if all its subsystems are exponentially stable and the subsystems switch slowly enough (note that condition (13) implies that the average dwell time should have a lower bound to guarantee exponential stability). Since $\lambda^\rho = \varepsilon\lambda$, where $\varepsilon = \lambda^{-\frac{\ln \mu}{2T_a \ln \lambda}} < 1$, the decay rate of the switched time-delay system (1) is smaller than that of the subsystems. Moreover, λ^ρ is a monotonic increasing function on T_a, which indicates that fast switching degrades the system stability performance (characterized by the decay rate). In particular, when there is no switching, we have by Definition 1 that $T_a \to \infty$. In this case, the decay rate λ^ρ is reduced to λ, which is just the decay rate of the subsystems.

Remark 5. In Theorem 1, μ is a given parameter, and the feasibility of matrix inequalities (4) and (12) depends on the selection of μ. We first choose a large initial value of μ such that (4) and (12) may have a feasible solution, then decrease μ step by step with a certain step length to look for a smaller μ so as to relax condition (13) without destroying the feasibility of inequalities (4) and (12).

Remark 6. For given scalars λ and μ, the delay bound $\bar{d}$ is coupled with several matrix variables. Therefore, it is not a convex problem to include $\bar{d}$ as a parameter to be maximized. As commonly applied in the existing results on time-delay systems (such as Chen et al. (2007), Kim et al. (2004), Sun, Liu, Rees, and Wang (2008), Sun et al.

(2006), Zhang et al. (2005), and Wu et al. (2004)), one may simply use a one-dimensional search algorithm to obtain the delay bound $\bar{d}$ that guarantees the exponential stability of the switched system (1). Note that the delay bound $\bar{d}$ is related to the decay rate of the subsystems λ, and a smaller λ allows for a larger delay bound $\bar{d}$, and vice versa. Moreover, the decay rate λ^{ρ} is a monotonic increasing function on the parameter λ. So, there is a tradeoff between the stability performance of the switched time-delay system (1) and the allowable delay upper bound, and one may sacrifice the decay rate to gain a larger delay bound $\bar{d}$. These will be further illustrated in the examples.

Remark 7. The obtained results can be extended to discrete-time switched time-delay systems with time-varying delays. Suppose that the time-varying delay $d(k)$ is lower-bounded and upper-bounded, and satisfies $d_m \leq d(k) \leq d_M$; then exponential stability conditions for the switched time-delay systems can be obtained by following similar procedures presented in this section and choosing the following Lyapunov functional

$$\begin{aligned} V_{\sigma(k)}(k) &= x^{\mathrm{T}}(k)P_{\sigma(k)}x(k) + \sum_{s=k-d(k)}^{k-1} \lambda^{2(s-k)}x^{\mathrm{T}}(s)Q_{\sigma(k)}x(s) \\ &\quad + \sum_{\theta=-d_M+1}^{0} \sum_{s=k-1+\theta}^{k-1} \lambda^{2(s-k)}y^{\mathrm{T}}(s)R_{\sigma(k)}y(s) \\ &\quad + \sum_{\theta=-d_M+2}^{-d_m+1} \sum_{s=k-1+\theta}^{k-1} \lambda^{2(s-k)}x^{\mathrm{T}}(s)Q_{\sigma(k)}x(s). \end{aligned}$$

4. Illustrative examples

Example 1. Consider system (1) with

$$A_1 = A_2 = \begin{bmatrix} 0 & 0.3 \\ -0.2 & 0.1 \end{bmatrix}, \qquad A_3 = A_4 = \begin{bmatrix} 0 & 0.3 \\ -0.2 & -0.1 \end{bmatrix},$$
$$B_1 = B_3 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \qquad B_2 = B_4 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}. \tag{17}$$

System (17) is a switched system with four subsystems denoted by S_i, $i = 1, 2, 3, 4$, respectively. Suppose that the average dwell time is $T_a = 2$. Choosing $\lambda = 1.5$ and $\mu = 1.1$ yields $T_a^* = \frac{\ln 1.1}{2 \ln 1.5} = 0.1175$. Thus, the condition (13) is satisfied. By applying the one-dimensional search algorithm, the delay upper bound that guarantees the feasibility of the linear matrix inequalities (LMIs) (4) and (12) is found to be $\bar{d} = 2$. Therefore, for $T_a = 2$, $\lambda = 1.5$ and $\mu = 1.1$, it is concluded from Theorem 1 and the calculations that system (17) is exponentially stable with decay rate $\lambda^{\rho} = 1.5^{-\frac{\ln 1.1}{2\times 2\times \ln 1.5}+1} = 1.4647$ if the delay is not larger than the bound $\bar{d} = 2$. The one-dimensional search algorithm used in finding the delay bound is described as follows. **Step 1:** Setting the initial value of $\bar{d}$ as $\bar{d} = 1$ and solving the LMIs (4) and (12), it is found that the LMIs are feasible for all $\lambda \leq 1.83$. So, the selection of $\lambda = 1.5$ is reasonable. **Step 2:** For $\lambda = 1.5$, increase $\bar{d}$ by a step length of $\Delta\bar{d} = 1$ and solve the LMIs (4) and (12). **Step 3:** Repeat Step 2 until the LMIs (4) and (12) are no longer feasible. Then, the largest $\bar{d}$ that guarantees the feasibility of the LMIs (4) and (12) is the desired delay bound for $\lambda = 1.5$. The calculated values of the delay bound $\bar{d}$ and the decay rate λ^{ρ} for different values of λ and $T_a = 2$ are given in Table 1. It can be seen from Table 1 that the delay upper bound is related to the decay rate, and a smaller decay rate λ^{ρ} allows for a larger delay bound $\bar{d}$. Therefore, one may sacrifice the decay rate to obtain a larger delay bound $\bar{d}$ that guarantees the exponential stability of the switched time-delay system.

Table 1
Delay bound $\bar{d}$ and decay rate λ^{ρ} (Example 1).

λ	1.8	1.4	1.3	1.2	1.1
λ^{ρ}	1.7576	1.3670	1.2694	1.1717	1.0741
$\bar{d}$	1	2	4	6	13

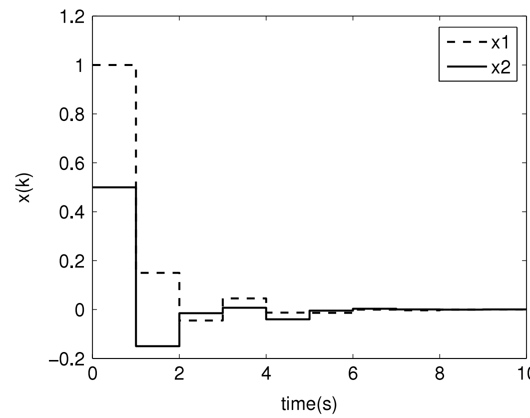

Fig. 1. State trajectories (Example 1).

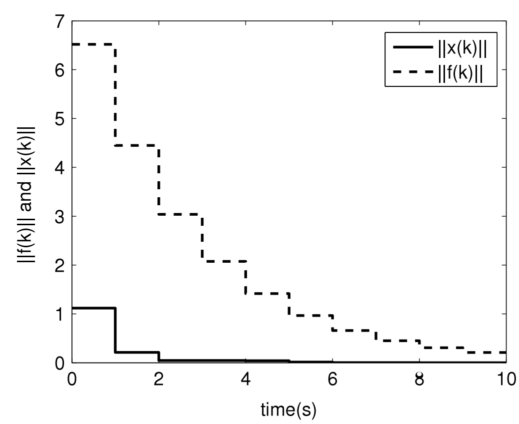

Fig. 2. Trajectories of $\|f(k)\|$ and $\|x(k)\|$ (Example 1).

For $\lambda = 1.5$, the state of system (17) ensures the following exponential decay estimation:

$\|x(k)\| \leq \|f(k)\| = 5.3002 \times 1.4647^{-(k-k_0)}\|\phi\|_L$.

In the simulation setup, let $k_0 = 0$ and suppose that the subsystems of system (17) are activated in the following sequence:

$$S_1S_1S_2S_2S_3S_3S_4S_4S_1S_1S_2S_2S_3S_3S_4S_4\cdots. \tag{18}$$

It can be seen from the switching sequence that $T_a = 2$. Choose $\phi(l) = [1; 0.5]$ for all $l = -2, -1, 0$; then we obtain $\|\phi\|_L = 1.1180$. The simulations are shown in Figs. 1 and 2, where Fig. 1 depicts the trajectories of the system states, while Fig. 2 depicts the trajectories of the functions $\|f(k)\|$ and $\|x(k)\|$. The simulations demonstrate that the switched time-delay system (17) with switching sequence (18) is exponentially stable.

Example 2. Consider the following time-delay system:

$$\begin{cases} x(k+1) = Ax(k) + Bx(k-d) + Du(k), \\ x(l) = \phi(l), \quad l = k_0 - \bar{d}, k_0 - \bar{d} + 1, \ldots, k_0, \end{cases} \tag{19}$$

where $d \leq \bar{d}$, $\bar{d}$ is the delay bound to be determined, and

$$A = \begin{bmatrix} 0.7 & 0 \\ 0.05 & 0.8 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.1 & 0 \\ -0.3 & -0.1 \end{bmatrix}, \qquad D = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}.$$

Table 2
Delay bound $\bar{d}$ and decay rate λ^{ρ} (Example 2).

λ	1.25	1.20	1.15	1.12
λ^{ρ}	1.1295	1.0843	1.0391	1.0120
$\bar{d}$	1	2	3	4

We consider the actuator failure case; that is, the two actuators may fail during the running time. It is supposed that at least one actuator does not fail at any time k, and the failed actuator will be repaired within a certain time interval. The corresponding column of matrix D is zeroed out when an actuator fails. So, we have under these assumptions that

$$D_1 = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \qquad D_3 = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}.$$

The state feedback controller is $u(k) = Kx(k)$ with $K = [0.1510 - 0.2176]$. By the above analysis, it can be seen that system (19) with actuator failure can be written as a switched time-delay system in the form of (1) with $M = \{1, 2, 3\}$, $A_i = A + D_iK$, and $B_i = B$.

Setting $\lambda = 1.2$ and $\mu = 1.5$, we have $T_a^* = \frac{\ln 1.5}{2 \ln 1.2} = 1.1120$. By applying the one-dimensional search algorithm as in Example 1, it is found that the delay upper bound is $\bar{d} = 2$. Therefore, if in practice the delay is smaller than 2, and the frequency of actuator failure is not too high, for example, on average not more than once every four sampling periods, then the average dwell time T_a will be large enough to guarantee the exponential stability of system (19). Specifically, if the average dwell time T_a is larger than 1.1120 and the delay is not larger than the bound $\bar{d} = 2$, then it can be said by Theorem 1 that the time-delay system (19) with actuator failure is exponentially stable.

Denote S_1, S_2, and S_3 the subsystems with D_1, D_2, and D_3, respectively. Suppose that the three subsystems are activated in the following sequence:

$$S_3S_3S_3S_1S_3S_3S_3S_2S_3S_3S_3S_1S_3S_3S_3S_2 \cdots.$$

It can be seen from the switching sequence that $T_a = 2 > T_a^*$, and we have by simple calculation that $\lambda^{\rho} = 1.0843$. Therefore, system (19) with actuator failures is exponentially stable and ensures the following state decay estimation:

$$\|x(k)\| \leq 14.6530 \times 1.0843^{-(k-k_0)} \|\phi\|_L.$$

By choosing $k_0 = 0$ and $\phi(l) = [0.5; -0.5]$ for all $l = -2, -1, 0$, the state trajectories are depicted in Fig. 3, which shows the stability of system (19).

For $T_a = 2$, the delay bound $\bar{d}$ and the decay rate λ^{ρ} for different values of λ are given in Table 2. Again, it can be seen from Table 2 that the delay bound is related to the exponential decay rate, and the smaller the decay rate the larger the delay bound. So, there is a tradeoff between the stability performance of the switched delay system and the allowable delay bound.

5. Conclusions

In this paper, a sufficient delay-dependent condition has been derived for the exponential stability of a class of discrete-time switched time-delay systems. A class of switching signals has been identified for the considered switched time-delay systems to be exponentially stable under the average dwell time scheme. Note that, in deriving the stability condition, we considered the case that all the individual subsystems ensure the same decay rate, which may be restrictive in practice. A general case is that the decay rates of the subsystems may be different from each other. In this case, how to derive the delay-dependent exponential stability condition is a problem that needs further investigation. The techniques presented in Sun, Liu, et al. (2008), which studies the stability of discrete-time systems with large delay sequence, may be applicable to solve this problem, and more research work needs to be done on this issue.

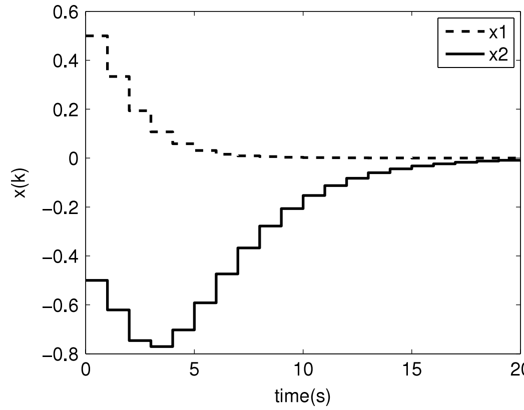

Fig. 3. State trajectories (Example 2).

Acknowledgements

The authors appreciate the anonymous reviewers' valuable suggestions for improving the quality of the paper. This work was supported by the National Natural Science Funds for Distinguished Young Scholar of China under Grant no. 60525304 and the National Natural Science Funds of China under Grant no. 60834003.

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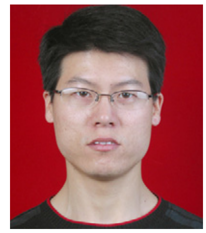

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