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Stochastic interval-based optimal offering model for residential energy management systems by household owners

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ABSTRACT

This paper proposes an optimal bidding strategy for autonomous residential energy management systems. This strategy enables the system to manage its domestic energy production and consumption autonomously, and trade energy with the local market through a novel hybrid interval-stochastic optimization method. This work poses a residential energy management problem which consists of two stages: day-ahead and real-time. The uncertainty in electricity price and PV power generation is modeled by interval-based and stochastic scenarios in the day-ahead and real-time transactions between the smart home and local electricity market. Moreover, the implementation of a battery included to provide energy flexibility in the residential system. In this paper, the smart home acts as a price-taker agent in the local market, and it submits its optimal offering and bidding curves to the local market based on the uncertainties of the system. Finally, the performance of the proposed residential energy management system is evaluated according to the impacts of interval optimistic and flexibility coefficients, optimal bidding strategy, and uncertainty modeling. The evaluation has shown that the proposed optimal offering model is effective in making the home system robust and achieves optimal energy transaction. Thus, the results prove that the proposed optimal offering model for the domestic energy management system is more robust than its non-optimal offering model. Moreover, battery flexibility has a positive effect on the system's total expected profit. With regarding to the bidding strategy, it is not able to impact the smart home's behavior (as a consumer or producer) in the day-ahead local electricity market.

1. Introduction

1.1. Aims and approaches

Customers are going to play a key role in the prospective power systems [1]. This will be possible because power will no longer be generated at centralized facilities, instead different technologies will be used to generate energy locally, this is called distributed generation. The infrastructure of smart grid makes this transition possible [1]. Thus, in power distribution systems' demand-side players – e.g. smart homes – will manage their own electrical energy according to the real and fair price [2]. Besides, current electricity markets are not able to satisfy customers' strategic behavior based on their autonomous decision-makings [3]. Hence, decentralized electricity markets are capable of adapting to the flexible behavior of electrical customers. In this way, smart homes are active agents and play a critical role in the bottom layer of the power systems. Smart homes are prosumers, this means they can be both producers and consumers. Hence, smart homes need energy management systems in order to make optimum decisions related to the management of energy inside the home, such as the choice of the best strategies when trading energy with other players (e.g. aggregators, retailers, local market operator, other consumers) in the distribution power network. In this way, distribution power networks are defined as complex ecosystems consisting of machines, networks, procedures, operators, and players which are organized hierarchically in the bottom layer of power systems in order to deliver electric power to end-users [34]. Different studies have considered distinct aspects of Residential Energy Management Systems (REMSs), e.g. residential electrical appliances [7], the main purposes of residential scheduling [8,15], decision-making under uncertainty [2], the implementation of

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Nomenclatures

Indices

t index of time periods
j index of electrical loads
ω index of real-time scenarios

Objective function variables

EP expected profit ($)

Day-ahead variables

$\lambda^{da}(t)$ day-ahead electricity price at time period t (€/kW h)
$C^{da}(t)$ day-ahead state of the charge of the battery at time period t (kW h)
$EL^{da}(t)$ day-ahead home energy consumption at time period t (kW h)
$k(t)$ day-ahead Dispatched status of PV power system at time period t
$P^{da}_{ch}(t)$ day-ahead battery energy charged at time period t (kW h)
$P^{da}_{dis}(t)$ day-ahead energy discharged from the battery at time period t (kW h)
$P^{da}_{dis,in}(t)$ day-ahead discharged energy of the battery that is injected to the smart home at time period t (kW h)
$P^{da}_{dis,out}(t)$ day-ahead energy discharged from the battery that is injected into the power grid at time period t (kW h)
$P^{da}_{pv,in}(t)$ day-ahead PV energy generation that is injected to the smart home at time period t (kW h)
$P^{da}_{pv,out}(t)$ day-ahead PV energy generation that is injected into the power grid at time period t (€/kW h)
$P^{da}_{pv,p}(t)$ day-ahead PV energy generation at time period t (kW h)
$P^{da}_{net}(t)$ day-ahead energy purchased from the local market at time period t (kW h)
$P^{da}_{sold}(t)$ day-ahead energy sold from home to the local market at at time period t (kW h)
$u^{da}(t)$ day-ahead discharging commitment binary variable for the battery at time period t
$v^{da}(t)$ day-ahead transacted energy status at time period t (kW h)

Real-time variables

$\Delta P^{rt}_{sold}(t,\omega)$ real-time sold energy from home to the local market in scenario ω and at time period t (kW h)
$\Delta P^{rt}_{net}(t,\omega)$ real-time energy purchased from the local market in scenario ω and at time period t (kW h)
$\theta^{in}(t,\omega)$ indoor temperature in scenario ω and at time period t (°C)
$C^{rt}(t,\omega)$ real-time state of charge of the battery in scenario ω and at time period t (kW h)
$EL^{rt}(t,\omega)$ real-time home energy consumption in scenario ω and at time period t (kW h)
$EL^{rt}_{j}(t,\omega)$ real-time energy consumption of load j in scenario ω and at time period t (kW h)
$EL^{rt}_{mrs}(t,\omega)$ real-time energy consumed by the must-run services in scenario ω and at time period t (kW h)
$EL^{rt}_{pp}(t,\omega)$ real-time energy consumed by the pool pump in scenario ω and at time period t (kW h)
$EL^{rt}_{sh}(t,\omega)$ real-time energy consumed by the space heater in scenario ω and at time period t (kW h)
$EL^{rt}_{swh}(t,\omega)$ real-time energy consumed by the storage water heater in scenario ω and at time period t (kW h)
$ES^{rt}(t,\omega)$ load shedding of home in scenario ω and at time period t (kW h)
$ES^{rt}_{j}(t,\omega)$ shedding of load j in scenario ω and at time period t (kW h)
$ES^{rt}_{mrs}(t,\omega)$ load shedding of the must-run services in scenario ω and at time period t (kW h)
$ES^{rt}_{pp}(t,\omega)$ load shedding of the pool pump in scenario ω and at time period t (kW h)
$ES^{rt}_{sh}(t,\omega)$ load shedding of the space heater in scenario ω and at time period t (kW h)
$ES^{rt}_{swh}(t,\omega)$ load shedding of the storage water heater in scenario ω and at time period t (kW h)
$L^{rt}_{mrs}(t,\omega)$ real-time load of the must-run services in scenario ω and at time period t (kW)
$L^{rt}_{pp}(t,\omega)$ real-time load of the pool pump in scenario ω and at time period t (kW)
$L^{rt}_{sh}(t,\omega)$ real-time load of the space heater in scenario ω and at time period t (kW)
$L^{rt}_{swh}(t,\omega)$ real-time load of the storage water heater in scenario ω and at time period t (kW)
$P^{rt}_{ch}(t,\omega)$ real-time battery energy charged in scenario ω and at time period t (kW h)
$P^{rt}_{dis}(t,\omega)$ real-time energy discharged from the battery in scenario ω and at time period t (kW h)
$P^{rt}_{dis,in}(t,\omega)$ real-time energy discharged from the battery that is injected into the smart home in scenario ω and at time period t (kW h)
$P^{rt}_{dis,out}(t,\omega)$ real-time energy discharged from the battery that is injected into the power grid in scenario ω and at time period t (kW h)
$P^{rt}_{pv}(t,\omega)$ real-time PV energy generation in scenario ω and at time period t (kW h)
$P^{rt}_{pv,in}(t,\omega)$ real-time PV energy generation that is injected into the smart home in scenario ω and at time period t (kW h)
$P^{rt}_{pv,out}(t,\omega)$ real-time PV energy generation that is injected into the power grid at scenario ω and at time period t (kW h)
$S^{PV}(t,\omega)$ energy spilled from PV in scenario ω and at time period t (kW h)
$u^{rt}(t,\omega)$ real-time discharging commitment binary variable for the battery in scenario ω and at time period t
$v^{rt}(t,\omega)$ day-ahead transacted energy status at scenario ω and at time period t
$z(t,\omega)$ operation status of the pool pump in scenario ω and at time period t

Parameters

α_{price} optimistic coefficient of price
α_{pv} optimistic coefficient of PV energy generation
$\sigma^{dn}_{price}(t)$ lower bound predicted price error at time period t (€/kW h)
$\sigma^{up}_{price}(t)$ upper bound predicted price error at time period t (€/kW h)
$\sigma^{dn}_{pv}(t)$ lower bound predicted error for PV energy generation at time period t (kW h)
$\sigma^{up}_{pv}(t)$ upper bound predicted error for PV energy generation at time period t (kW h)
$\lambda^{da}(t)$ day-ahead electricity price at time period t (€/kW h)
$\lambda^{pred}(t)$ day-ahead price prediction at time period t (€/kW h)
$\lambda^{rt}_{net}(t,\omega)$ price of the electrical energy purchased from the real-time local market in scenario ω and at time period t (€/kW h)
$\lambda^{rt}_{sold}(t,\omega)$ price of the electrical energy sold to the real-time local market in scenario ω and at time period t (€/kW h)
η_{B2H} discharging efficiency of the battery
η_{H2B} charging efficiency of the battery
γ flexibility coefficient
π_{ω} probability of real-time scenarios in scenario ω
θ^{in}_{des} desired indoor temperature (°C)
θ^{in}_{i} initial indoor temperature (°C)
$\theta^{out,pred}(t,\omega)$ predicted outdoor temperature in scenario ω and at time period t (°C)

C	thermal energy capacity of the building (kW h/°C)
C_i	the initial state of the charge of the battery (kW h)
$EL^{pred}(t)$	day-ahead predicted of home's energy consumption at time period t (kW h)
P_b^{max}	maximum storage level of the battery (kW h)
P_b^{min}	minimum storage level of the battery (kW h)
L_{pp}^{max}	maximum electrical consumption for the pool pump (kW)
L_{sh}^{max}	maximum electrical consumption for the space heater (kW)
L_{swh}^{max}	maximum electrical consumption of the storage water heater (kW)
L_i^{mrs}	initial load consumption of the must-run services (kW)
L_i^{pp}	initial load consumption of the pool pump (kW)
$L_{mrs}^{pred}(t)$	predicted electrical consumption for the must-run services in scenario ω and at time period t (kW)
L_i^{sh}	initial load consumption of the space heater (kW)
L_i^{swh}	initial load consumption of the storage water heater (kW)
$P_{PV}^{pred}(t)$	day-ahead predicted PV energy generation at time period t (kW h)
$P_{pv}^{scen}(t, \omega)$	scenarios of wind energy generation in scenario ω and at time period t (kW h)
R	thermal resistance of the building shell (°C/kW)
S_{max}	maximum capacity of the end-user distributed line (kW h)
T_{ON}	maximum daily-hours that pool pump can be ON (h)
U_{swh}^{max}	daily energy consumption for the storage water heater (kW h)
V_{PV}^{S}	cost of PV Spillage (€/kW h)
$VOLL_j(t)$	value Of Lost Load (VOLL) of load j at time period t (€/kW h)
w^{max}	maximum ramping rate of the battery's state of charge (kW h)
w^{min}	minimum ramping rate of the battery's state of charge (kW h)

the Residential Energy Management Systems (REMSs) [7], and interaction between the REMSs and other systems in their neighborhood or up-stream grid [2].

1.2. Literature review and contributions

Residential energy management systems have been modeled and studied from different points of view. For instance, in [4], a multi agent-based structure is presented to model homes and retailers in the distribution power network. The main purpose of [4] is to optimize residential Demand Response (DR). This way, authors predict electrical loads and implement the optimal load control model for home agents. In [5], authors propose a multi-objective stochastic optimization problem. According to their model, the REMS can control home appliances, and exchange energy and price with the upstream power grid's agents. In [6–8], REMSs are implemented by means of Multi Agent Systems (MASs). In [6], the main goal of authors was to provide optimum DR without negatively impacting the consumers' level of comfort. Demand response was a topic of interest to many authors. Refs. [7,8] propose a MAS-based REMS which enables smart homes to manage their energy autonomously and trade it with the local electricity market based on Time of Use (ToU) tariff. In [9], the home appliances are controlled under uncertainty of outdoor temperature and electricity price based on the DR programs. In [10], a combined DR program based on machine learning and an optimization method is used in the REMS. In [11], the uncertainty of price and residential loads is tackled via chance constrained programming, and the DR is used to optimize the operation of devices. In [12], authors classified domestic appliances into fixed and flexible loads. Moreover, the purpose of their proposed model is to tradeoff between the expected energy cost and the residents' level of comfort. Ref. [13] assumes that the main purpose of REMSs is to improve the energy efficiency of smart homes. In [14], authors presented a MAS-based cooperation smart grid and smart buildings to maximize comfort and energy efficiency, it also proposes a MAS-based approach for energy management in smart homes. According to [14], smart homes can manage their energy independently to decrease expected electricity cost of homes and make a softer load profile.

In [15,16], different strategies were proposed for the control of energy storage systems (e.g. batteries and EVs). In [15], authors defined a flexibility coefficient to model energy flexibility in the REMS. Moreover, the hybrid interval-stochastic optimization method was used to consider uncertainties in the system. In [16], authors proposed a method to operate renewable generation and minimize cost of lost energy in the home area and bought energy from the power grid. Ref. [17] proposed an REMS which consists of prediction, operation and control units, and utilized stochastic programming in the modeling of the uncertainty of different home appliances. In [18–20], authors predict and estimate the behavior of residential consumers and analyze their impacts on energy consumption. In [21], the bi-level day-ahead REM program was presented. In the first level, electrical customers scheduled their energy autonomously. In the second level, the system operator optimized the centralized multi-objective problem through fuzzy decision-making. In [22], a decomposition approach was adapted in autonomous REMSs. In [23] authors used Markovian processes to model the uncertainty of power generation from renewable energy resources and controllable loads that depend on weather conditions. Ref. [24] presented three methods for reducing the energy costs in the REMS. These methods were based on a partially observable Markov decision process. In [25], authors solved the home energy management problem through the two-point estimate optimization method to model uncertainty of PV power generation and decrease the computational burden of the problem.

The interaction between small consumers and all emerging energy resources is enhanced by the introduction of local energy markets. In Europe for example, interest in energy communities has increased since energy cooperative have already undertaken 2400 initiatives [26]. Energy cooperative initiatives have been driven by the inability of public utilities to provide the kind of services that end-users require, as well as to contest the existing monopoly [26]. In this way the small final users (consumers, producers, and prosumers), are creating local communities of energy. The creation of local electricity markets could be the solution to some of the economic and efficiency challenges that energy cooperatives will face. In [27], it is argued that the market for electricity transactions at the local level was probably constrained due to the focus on the restructuring of electricity markets. In turn, the author affirms that the creation of business models of the local power supply has the potential to give a better way to the production of small community generators, thus supporting the growing sector of local generation. In [28], the author states that the realization of a local commerce contributes significantly to the autonomy of the micro-networks, reducing the demand and dependence of the main network. The author also suggests that it is intriguing to devise a market that allows local commerce of electricity between users who have excess electricity and those who demand it. Marketing at the local level could also be beneficial to the network itself. According to the ENTSO-E harmonized electricity market role model a local electricity market is a geographic area where consumption and production can be metered, there are no transmission capacity restrictions and for which there is one balance responsible party (BRP) and, thus, one price for the imbalance [29]. One of the most promising approaches in this domain is the creation of a local grid controller (LGC) which is responsible for different control tasks, i.e. voltage and frequency control, demand response, DG control

as well as market trading and system monitoring. Therefore, certain participants and actors are clustered and connected to the LGC. Thus, a large number of users (producers, consumers, suppliers, network operators, aggregators, etc.) are connected through a bi-directional flow of energy and information. The interface between the market system operator and the LGC, and consequently the producers, consumers, etc., is given by an aggregator. The aggregator participates in the energy trading and provides further services to the distribution grid [29]. Within local energy markets, different trading mechanism, e.g. bilateral contracts, auctions, supermarkets, etc. are intended.

From this literature review we can see that REMS proposals deal with a wide range of issues as shown in Table 1. Some of these works [1–5,7,8,14–16,21,22] discussed the possible interaction between buildings and distribution power network, retailers and local electricity markets. For instance, authors in [30] proposed a day-ahead bidding approach, which allowed residential consumers to submit their bidding curves to the power system operator. Ref. [31] presented a probabilistic method where the demand aggregator can submit bids for its residential customers to follow the DR programs.

Refs. [32,33] presented pricing methodologies to provide reserve from buildings so as to counteract the uncertainty of renewable energy generation. In [32,33], authors proposed a decentralized approach to obtain distribution locational marginal price. However, a building integration framework has been presented in the distribution grid. Therefore, buildings are not able to participate autonomously in the local market. In other words, aggregators are in charge of managing the buildings' demand flexibility in [32]. Moreover, in [33], authors proposed an extended model of [32] where aggregators handle the uncertainties of flexible loads. Thus, cooperation between the Distribution System Operator (DSO) and the aggregators has been studied in [33]. To the best of our knowledge, however, there is no other study in the literature to present an optimal model for residential energy management systems that would empower buildings and consumers to participate directly as autonomous players in the local electricity market. This is a significant gap in the literature that should be promptly addressed because local energy markets are quickly becoming a reality, and small consumers and prosumers are not prepared to deal with this paradigm change. This may cause significant problems to the successful implementation and execution of local markets, since the consumer is the central player.

This paper presents a probabilistic scenario-based method for the autonomous management of the production and consumption of residential energy and for deriving optimal offering and bidding curves as a price-taker prosumer in a local electricity market. The proposed residential energy management problem consists of two stages: day-ahead and real-time stages. In the day-ahead stage, uncertainty in the electricity price and PV energy generation is modeled by interval-based scenarios. However, uncertainty in the REMS is modeled through scenarios in the real-time stage, to determine optimal transactions between the smart home and the local electricity market. In our proposed REMS, the battery is considered to provide the energy flexibility in the domestic system. According to our proposed model, the REMS can send its optimal offering and bidding curves to the local market based on the uncertainties of the system a price-taker agent in the local market. On the other hand, our proposed REMS without optimal bidding strategy is able to participate in peer-to-peer energy transactions with other small consumers, producers, and prosumers in its neighborhood through its optimum decisions in the management of the smart home.

1.3. Paper organization

The rest of this paper is organized as follows. Section 2 describes our method to model uncertainty in the system and proposes the two-stage probabilistic scenario-based residential energy management problem for which optimal offering and bidding strategies are derived. In Section 3, the effectiveness of our proposed methodology is studied. Finally, Section 4 concludes the paper.

2. Methodology

It is not easy to obtain an accurate market price forecast, due to the main characteristics of market prices. The main features of electricity prices are non-stationary mean and variance, multiple seasonality and the calendar effect. Uncertainty is associated with the forecasted values. Although the electricity market prices are highly volatile, the market agents need to obtain an estimation from the price to make optimal decisions in the market [39]. This section discusses the uncertainty modeling for power generation of the PV solar panels and market prices.

2.1. Uncertainty representation

The modeling of uncertainty is one of the main concerns of the energy management systems. In [35], authors studied energy systems from the perspective of decision making under uncertainty. In this way, in [35], authors classified uncertainty modelling methods into probabilistic, interval, robust, possibilistic, hybrid probabilistic-possibilistic optimization approaches, and information gap decision theory. In [36,37], authors presented a combined forecasting technique using time-varying weights to model uncertainty of distributed energy resources in electric power systems. In this way, uncertainties have been modeled by interval bands and stochastic scenarios to be considered in interval linear programming, mixed-integer linear programming, and chance-constrained programming in a general structure. In addition, in [37], bi-level programming has been presented to control air pollution and plan renewable energy resources in an inexact bi-level optimization model. In [37], authors proposed a multi-level algorithm for decision making problems. According to the proposed model of [38], authors did not concentrate on interval bands of uncertain parameters as inputs of the system. Hence, solutions of the decision-makers have been represented by interval bands, and authors proposed how optimal solutions could be achieved if the solutions desired by the decision-makers

Table 1
Taxonomy of some of the reviewed papers.

References	Model	Purpose	Decision-making	Interaction
[1]	Mathematical-based	Maximizing comfort & energy efficiency	Deterministic	Interaction between smart buildings and smart grid
[4]	Forecasting-based	Optimizing residential DR	Point forecasting	No communication between homes
[5]	Mathematical-based	Optimizing residential energy scheduling	Stochastic programming	Homes transact energy and price signals with transaction energy nodes
[7]	Mathematical-based	Optimizing residential energy based DRP.	Interval optimization	Interaction between the smart home and the local market
[8]	Mathematical-based	optimizing residential energy based DRP	Interval optimization based on moving window algorithm	Interaction between the smart home and the local market
[13]	Real-time metering	Improving energy efficiency	Deterministic	Only interaction between devices inside the home
[14]	Mathematical-based	Minimizing home's electricity bill to flatten total demand curve	Stochastic model	Each home manages energy autonomously, and there is no interaction between homes

are conflicting.

Among the uncertainties that influence the operation of the residential energy management systems, the solar irradiation and the electricity market prices have the highest impact [45]. Hence, the uncertainties associated with these inputs are considered in the proposed model and the scheduling problem is developed as a stochastic scenario-based optimization model [46].

In stochastic models, a set of realizations should be considered, and therefore the foremost problem is to produce a set of scenarios for random variables, which can effectively characterize the probabilistic features of the data [39,47]. The initial set of scenarios is a large data set generated by the Monte Carlo Simulation (MCS) technique for representing power system uncertainties. The MCS parameters are the probability distribution functions of the forecast errors, which are obtained from the historical data [47,49]. An additional term which can be positive or negative is added to the forecasted profile ($x^{forecasted}(t)$) to include the impact of uncertainty.

$$x^s(t) = x^{forecasted}(t) + x^{error,s}(t), \quad \forall\, t, \forall\, s \tag{1}$$

According to Eq. (1), the error term, $x^{error,s}(t)$, is a zero-mean noise with standard deviation σ [47,50]. Scenarios are represented with $x^s(t)$. In this model, the forecast errors are all assumed normally distributed. It is noticeable that electricity prices present very high spikes. However, it depends on the structure of the markets and the behavior of the participants. Some studies, e.g. [55,56], authors in [55] prove that the market price can fit well with the normal distribution function, while [56] adopts normal distribution to model market price uncertainty. Thus, the scenario tree concept can clearly explain how the discrete outcome for each stochastic input can be combined to construct the larger set of scenarios. A scenario tree consists of nodes that represent the states of the random variable at particular time points, branches to show different realizations of the variable and the root which shows the beginning point where the first stage decisions are made [47]. Fig. 1 shows the scenario tree model for the proposed scenario-based stochastic programming model [47]. $x_n^s(t)$ refers to the nth random variable. Variables can be of different nature. In this way, $x_1^s(t)$ may represent PV power generation and $x_2^s(t)$ can denote local market prices. The number of the nodes at the second stage is equal to the total number of scenarios. The occurrence probability of each scenario is equal to the product of the branches' probabilities [47,48].

Using the initial set of generated realizations in the optimization problem will lead to a large-scale optimization model [47]. It is essential to obtain a tradeoff between model accuracy and the computation speed [51,52]. In order to handle the computational tractability of the problem, the standard scenario reduction techniques developed in [53] is implemented. The scenario reduction algorithms exclude the scenarios with low probabilities of occurrence and combines the scenarios that are close to each other in terms of statistic metrics [53]. They determine a scenario subset of the prescribed cardinality and probability which is closest to the initial distribution in terms of a probability metric [49]. The main purpose of scenario reduction is to reduce the dimension of the problem through decreasing the number of variables and equations.

Thus, it would be possible to obtain the solutions more efficiently, without losing the main statistical characteristics of the initial dataset [54]. The drawback of applying these approaches is introducing imprecision in the final solution [52]. The reduction algorithms proposed in [53] incorporate algorithms with different computational performance and accuracy, namely fast backward method, fast backward/forward method and fast backward/backward method. The selection of the algorithms depends on the problem size and the expected solution accuracy [49,53]. For instance, the best computational performance with the worst accuracy can be provided by the fast-backward method for large scenario trees. Furthermore, the forward method provides the best accuracy and the highest computational time. Thus, it is usually used where the size of reduced subset is small [49].

2.2. Problem formulation

This paper addresses a two-stage probabilistic residential energy problem in which it is necessary to determine optimal offering and bidding curves in the Day-Ahead (DA) and Real-Time (RT) Local Electricity Markets (LEMs). Energy is defined to be the only electrical commodity that is exchanged with the DA and RT local electricity markets. In the DA stage, the uncertainty of the PV energy generation and electricity price is modeled through interval-based scenarios, but

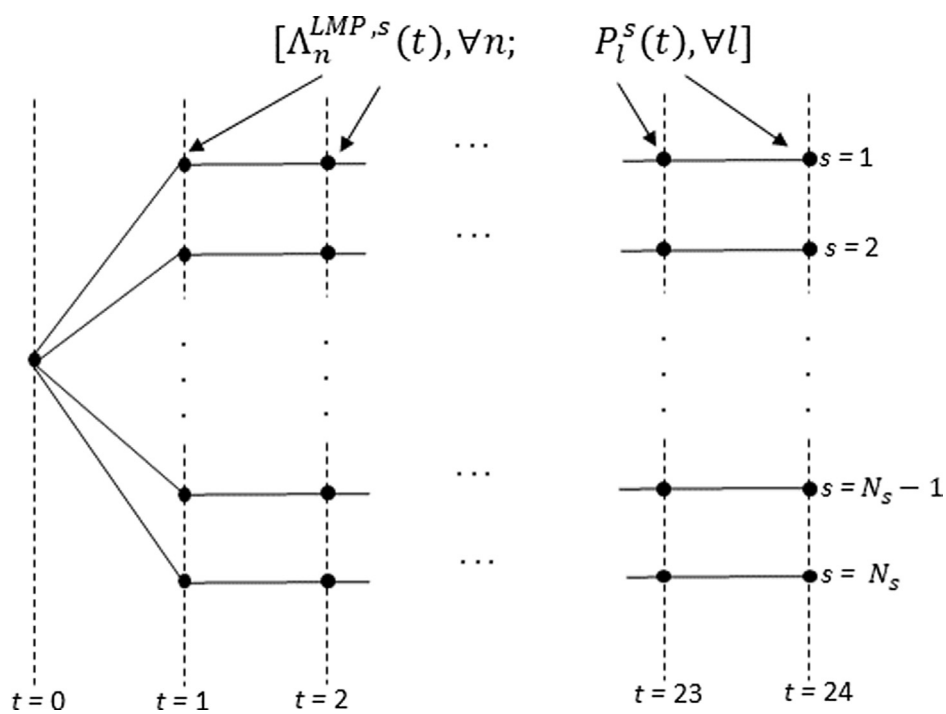

Fig. 1. Scenario tree representation [47].

the scenarios are used to model the corresponding uncertainty of the PV generation and electrical price in the RT stage. In this way, the two-stage interval-stochastic optimization method to solve the residential energy management problem is described. Then, our proposed problem is modeled by a two-stage stochastic programming. The difference between these two methods is to model the DA stage. While the uncertainties in the DA stage are modeled by interval bands in interval-optimization method, the stochastic interval-based scenarios are used to model the DA stage's uncertainty in the two-stage stochastic programming.

2.2.1. Two-stage interval-stochastic model

a. Objective function

In the context of this paper, smart home – as a prosumer – is defined as an active player that can trade energy with the LEM in the DA and RT stages. Fig. 2 shows a schematic of our proposed residential energy management system. Thus, the objective is to maximize the Expected Profit (EP) of the energy served in the home and the energy transacted with the market. In this paper, the PV system is considered as the Distributed Energy Resource (DER) in the domestic energy system. The battery system acts as an Energy Storage System (ESS). Also, Electrical Loads (ELs) consist of Space Heater (SH), Storage Water Heater (SWH), Pool Pump (PP), and Must-Run Services (MRSs).

Max.

$$EP = \overbrace{\sum_t [\lambda^{da}(t)(P^{da}_{sold}(t) - P^{da}_{net}(t))]}^{day-ahead\ profit} + \overbrace{\sum_\omega \pi_\omega \{ \sum_t (\lambda^{rt}_{sold}(t,\omega)\Delta P^{rt}_{sold}(t,\omega) - \lambda^{rt}_{net}(t,\omega)\Delta P^{rt}_{net}(t,\omega) - V^S_{PV} S^{PV}(t,\omega) - \sum_j VOLL_j(t) ES^{rt}_j(t,\omega)) \}}^{real-time\ expected\ profit} \quad (2)$$

As seen in Eq. (2), the EP is represented as an objective function of the two-stage interval-stochastic residential energy management problem. The EP consists of two parts. The first part represents the profit of the day-ahead stage and the second part expresses the real-time expected profit. In the DA part, the revenue of selling the electrical energy to local market is stated as a first term, and the second term states the costs of buying the electrical energy from the market. In the RT part, they are presented in the following order: the revenue of extra energy sold in real-time, the cost of extra energy bought in real-time, PV's spillage cost, and the cost of loads' shedding. The constraints related to the DA and RT stages are represented in the following.

b. Day-ahead stage

As discussed further on, we account that the smart home can transact electrical energy in both day-ahead and real-time local electricity markets. Eqs. (3) and (4) represent the power flow limitation through the distribution line which ends at the home building. In this way, S_{max} expresses the maximum power capacity of the distribution line that links the smart home and the power grid (hereinafter, authors refer to the Smart HomE as "SHE" for short, note that the abbreviation does not intend to make any association with gender). Also, $v^{da}(t)$ is a binary variable which states the transacted energy status. In other words, SHE purchases energy from the local market when $v^{da}(t)$ is equal to 1, and SHE sells energy to the local market when $v^{da}(t)$ equals 0. Eqs. (3) and (4) guarantee that the SHE cannot act as a producer and a consumer, simultaneously. In this model, the smart home provides for its demand first and then SHE sells its extra energy to the local market.

$$0 \le P^{da}_{net}(t) \le S_{max} v^{da}(t), \quad \forall\, t \quad (3)$$

$$0 \le P^{da}_{sold}(t) \le S_{max}(1 - v^{da}(t)), \quad \forall\, t \quad (4)$$

Moreover, Eq. (5) expresses that the energy sold to the local market consists of two terms: the energy produced by the PV system, $P^{da}_{pv,out}(t)$, and the discharged energy, $P^{da}_{dis,out}(t)$, of the battery system; these are injected into the power grid in the day-ahead stage. Besides, the flexibility coefficient, γ, is multiplied by the discharged and charged energy of the battery in the day-ahead stage, obtaining a value between 0 and 1. If γ equals 0 it means that the battery is not considered in the day-ahead residential energy management problem. On the other hand, the battery is considered to have full capacity in the day-ahead stage of the problem when γ equals 1. Also, only the corresponding portion of the battery's capacity will be considered in the day-ahead stage when γ gets

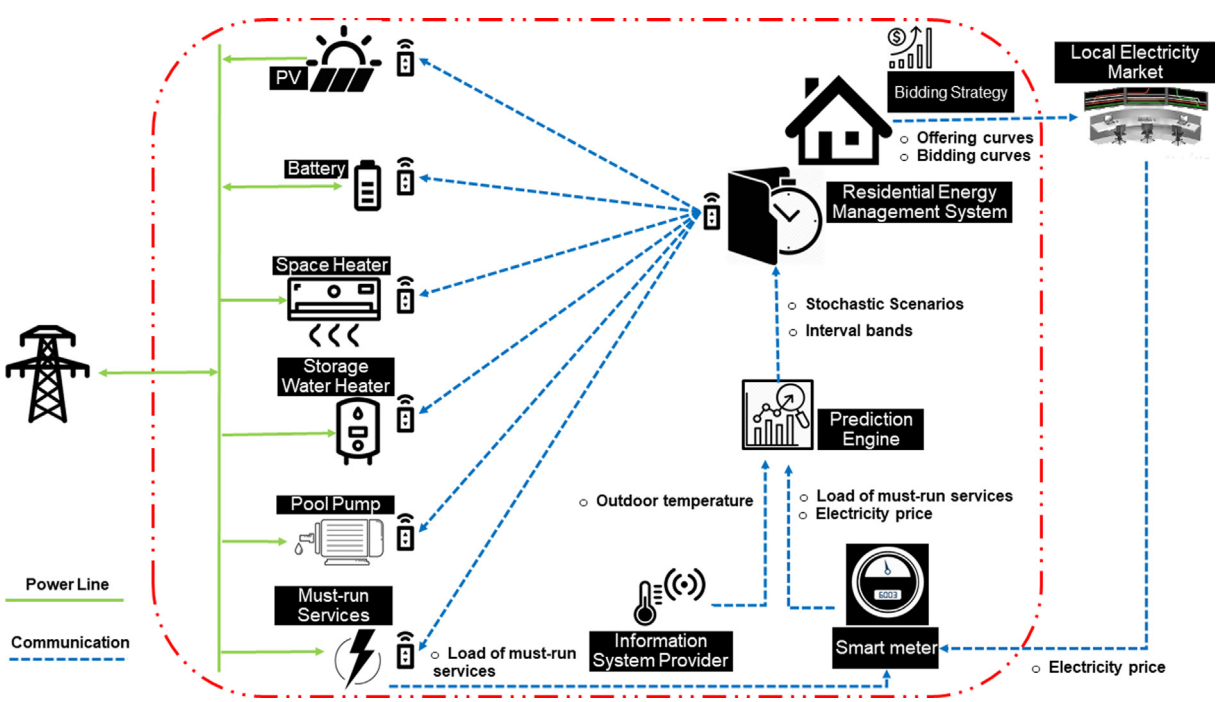

Fig. 2. A generic layout of our residential energy management system.

an amount between 0 and 1.

$$P_{sold}^{da}(t) = P_{pv,out}^{da}(t) + \gamma P_{dis,out}^{da}(t), \quad \forall\, t \tag{5}$$

Eq. (6) establishes the power balance equation due to the energy output of the PV system and the discharged energy of the battery injected into the home ($P_{pv,in}^{da}(t)$ and $P_{dis,in}^{da}(t)$, respectively), the electrical energy bought from the local market, $P_{net}^{da}(t)$, total energy consumption of the domestic loads, $EL^{da}(t)$, and charged energy of the battery system, $P_{t\omega}^{ch}$.

$$P_{net}^{da}(t) + P_{pv,in}^{da}(t) + \gamma P_{dis,in}^{da}(t) = EL^{da}(t) + \gamma P_{ch}^{da}(t), \quad \forall\, t \tag{6}$$

As discussed further in this paper, the DA stage's uncertainty is modeled by interval bands in the two-stage interval-stochastic model. Eq. (7) presents the maximum and minimum bands of the price in the day-ahead local market. Hence, $\lambda^{pred}(t)$ and $\sigma_{price}^{up}(t)/\sigma_{price}^{dn}(t)$ are predicted price and upper/lower predicted price error, respectively. Also, α_{price} is the corresponding Optimistic Coefficient (OC) of the electricity price. The OC is a slack parameter for the decision-maker which can take amounts between 0 and 1. If α_{price} equals 0/1 the uncertainty of price is modeled as conservative/optimistic.

$$\lambda^{pred}(t) - \sigma_{price}^{dn}(t)(1-\alpha_{price}) \le \lambda^{da}(t) \le \lambda^{pred}(t) + \sigma_{price}^{up}(t)\alpha_{price}, \quad \forall\, t \tag{7}$$

Moreover, the following constraints correspond to all devices in the smart home. The total potential energy generated by the PV system in each time period, $P_{pv,p}^{da}(t)$, is the sum of the produced PV's energy that is injected into the home, $P_{pv,in}^{da}(t)$, and the power grid, $P_{pv,out}^{da}(t)$ as represented in Eq. (8). Also, k(t) is a binary variable which states the dispatched status of the PV system.

$$P_{pv,p}^{da}(t)k(t) = P_{pv,in}^{da}(t) + P_{pv,out}^{da}(t), \quad \forall\, t \tag{8}$$

Furthermore, our uncertainty modeling relies on confidence intervals for energy generation of the PV as well as price. Hence, Eq. (9) deals with the possibility of point forecasting error. This way, $P_{PV}^{pred}(t)$ and $\sigma_{pv}^{up}(t)/\sigma_{pv}^{dn}(t)$ are predicted PV energy generation and upper/lower predicted energy error, respectively. Also, α_{pv} is the corresponded Optimistic Coefficient (OC) of the PV energy produced that can be between 0 and 1.

$$P_{PV}^{pred}(t) - \sigma_{pv}^{dn}(t)(1-\alpha_{pv}) \le P_{pv,p}^{da}(t) \le P_{PV}^{pred}(t) + \sigma_{pv}^{up}(t)\alpha_{pv}, \quad \forall\, t \tag{9}$$

The battery is used based on the charging and discharging strategies in the residential energy management problem. Eq. (10) represents the state-of-charge (SOC) balance equation of the battery, where C_i is the initial state of charge in the battery.

$$C^{da}(t) = C^{da}(t-1) + P_{ch}^{da}(t)\eta_{H2B} - \frac{P_{dis}^{da}(t)}{\eta_{B2H}}, \quad \forall\, t \ge 2$$

$$C^{da}(t) = C_i + P_{ch}^{da}(t)\eta_{H2B} - \frac{P_{dis}^{da}(t)}{\eta_{B2H}}, \quad \forall\, t = 1 \tag{10}$$

Eq. (11) presents the maximum and minimum limitations of the battery's SOC.

$$P_b^{min} \le C^{da}(t) \le P_b^{max}, \quad \forall\, t \tag{11}$$

The ramping upper and lower constraints related to the SOC are expressed in Eq. (12).

$$-w^{min} \le C^{da}(t) - C^{da}(t-1) \le w^{max}, \quad \forall\, t \ge 2$$

$$-w^{min} \le C^{da}(t) - C_i \le w^{max}, \quad \forall\, t = 1 \tag{12}$$

Maximum and minimum limitations of the discharged and charged energy of the battery are stated in Eqs. (13) and (14), respectively.

$$0 \le P_{dis}^{da}(t) \le w^{max} u^{da}(t), \quad \forall\, t \tag{13}$$

$$0 \le P_{ch}^{da}(t) \le w^{min}(1-u^{da}(t)), \quad \forall\, t \tag{14}$$

Eq. (15) represents that the total discharged energy of the battery system, $P_{dis}^{da}(t)$, is the sum of discharged energies that are injected into the home, $P_{dis,in}^{da}(t)$, and the power grid, $P_{dis,out}^{da}(t)$, in the day-ahead stage.

$$P_{dis}^{da}(t) = P_{dis,in}^{da}(t) + P_{dis,out}^{da}(t), \quad \forall\, t \tag{15}$$

In our proposed model, it is considered that the day-ahead electrical loads are equal to the predicted load as seen in Eq. (16), and their corresponding equations are defined only in the real-time stage. Moreover, for the sake of simplicity, the uncertainty of the electrical loads is not considered in this paper.

$$EL^{da}(t) = EL^{pred}(t), \quad \forall\, t \tag{16}$$

c. Real-time stage

In the DA stage the smart home can exchange energy with the LEM. However, in contrast to the DA stage, stochastic programming is used to model the uncertainty of the electricity price and PV energy generation in the RT stage, and the prices of sold and bought electricity can be different in the RT stage. The power balance equation in the RT is expressed in Eq. (17) to represent the mismatch between the DA transacted energy and RT expected exchanged energy. According to Eq. (17), the sum of energy bought in the DA and RT markets, $P_{net}^{da}(t)$ and $\Delta P_{net}^{rt}(t, \omega)$, produced energy of the PV system in the RT, $P_{pv}^{rt}(t, \omega)$, and discharged energy of the battery in the RT, $P_{dis}^{rt}(t, \omega)$, equal total electrical energy consumption in the RT, $EL^{rt}(t, \omega)$, charged energy of the battery in the RT, $P_{ch}^{rt}(t, \omega)$, the energy sold to the local market in the DA and RT, $P_{sold}^{da}(t)$ and $\Delta P_{sold}^{rt}(t, \omega)$, minus total energy loss, $ES^{rt}(t, \omega)$.

$$P_{net}^{da}(t) + P_{pv}^{rt}(t, \omega) + P_{dis}^{rt}(t, \omega) + \Delta P_{net}^{rt}(t, \omega) = EL^{rt}(t, \omega) - ES^{rt}(t, \omega) + P_{ch}^{rt}(t, \omega) + P_{sold}^{da}(t) + \Delta P_{sold}^{rt}(t, \omega), \quad \forall\, t, \forall\, \omega \tag{17}$$

Eq. (18) presents the power flow limitation in a distribution line that ends at the smart home. It is noticeable that both, Eqs. (17) and (18), are coupling constraints that cause the DA and RT problems to be solved simultaneously.

$$-S_{max} \le P_{net}^{da}(t) + \Delta P_{net}^{rt}(t, \omega) - P_{sold}^{da}(t) - \Delta P_{sold}^{rt}(t, \omega) \le S_{max}, \quad \forall\, t, \forall\, \omega \tag{18}$$

In addition, Eqs. (19) and (20) ensure that the smart home cannot be a producer and a consumer in the same scenario in the real-time stage.

$$0 \le \Delta P_{net}^{rt}(t, \omega) \le S_{max} v^{rt}(t, \omega), \quad \forall\, t, \forall\, \omega \tag{19}$$

$$0 \le \Delta P_{sold}^{rt}(t, \omega) \le S_{max}(1 - v^{rt}(t, \omega)), \quad \forall\, t, \forall\, \omega \tag{20}$$

Eq. (21) represents the energy output equation of PV in the real-time stage. According to Eq. (21), $P_{pv}^{scen}(t, \omega)$ presents the stochastic potential PV energy generation, and $S^{PV}(t, \omega)$ is the spilled energy of the PV system.

$$P_{pv}^{rt}(t, \omega) = P_{pv}^{scen}(t, \omega) - S^{PV}(t, \omega), \quad \forall\, t, \forall\, \omega \tag{21}$$

As well as Eq. (8), Eq. (22) presents the total energy generated by the PV system in the RT stage that is the sum of energy produced by the PV which is injected into the home, $P_{pv,in}^{rt}(t, \omega)$, and the energy grid, $P_{pv,out}^{rt}(t, \omega)$.

$$P_{pv}^{rt}(t, \omega) = P_{pv,in}^{rt}(t, \omega) + P_{pv,out}^{rt}(t, \omega), \quad \forall\, , \forall\, \omega \tag{22}$$

The maximum and minimum bands of spilled PV energy produced are represented in Eq. (23).

$$0 \le S^{PV}(t, \omega) \le P_{pv}^{scen}(t, \omega), \quad \forall\, t, \forall\, \omega \tag{23}$$

In the following, the battery system's constraints in the RT stage are stated in Eqs. (24)–(29).

$$C^{rt}(t,\omega) = C^{rt}(t-1,\omega) + P_{ch}^{rt}(t,\omega)\eta_{H2B} - \frac{P_{dis}^{rt}(t,\omega)}{\eta_{B2H}}, \quad \forall\, t \geq 2, \forall\, \omega$$
$$C^{rt}(t,\omega) = C_i, \quad \forall\, t = 1, \forall\, \omega \tag{24}$$

$$P_b^{min} \leq C^{rt}(t,\omega) \leq P_b^{max}, \quad \forall\, t, \forall\, \omega \tag{25}$$

$$-w^{min} \leq C^{rt}(t,\omega) - C^{rt}(t-1,\omega) \leq w^{max}, \quad \forall\, t \geq 2, \forall\, \omega$$
$$0 \leq P_{dis}^{rt}(t,\omega) \leq w^{max} u^{rt}(t,\omega), \quad \forall\, t = 1, \forall\, \omega, \tag{26}$$

$$0 \leq P_{dis}^{rt}(t,\omega) \leq w^{max} u^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{27}$$

$$0 \leq P_{ch}^{rt}(t,\omega) \leq w^{min}(1-u^{rt}(t,\omega)), \quad \forall\, t, \forall\, \omega \tag{28}$$

$$P_{dis}^{rt}(t,\omega) = P_{dis,in}^{rt}(t,\omega) + P_{dis,out}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{29}$$

In the context of this paper, electrical loads consist of loads that can be controllable and/or shiftable, or not. In this paper, Space Heater (SH) is a controllable load, Storage Water Heater (SWH) and Pool Pump (PP) are modeled as shiftable loads, and Must-Run Services (MRSs) are defined as a class of loads that are non-controllable and non-shiftable. Eqs. (30) and (32) represent the total electrical energy consumption and energy shedding in the RT stage.

$$EL^{rt}(t,\omega) = \sum_j EL_j^{rt}(t,\omega) = EL_{sh}^{rt}(t,\omega) + EL_{swh}^{rt}(t,\omega) + EL_{pp}^{rt}(t,\omega) + EL_{mrs}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{30}$$

$$ES^{rt}(t,\omega) = \sum_j ES_j^{rt}(t,\omega) = ES_{sh}^{rt}(t,\omega) + ES_{swh}^{rt}(t,\omega) + ES_{pp}^{rt}(t,\omega) + ES_{mrs}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{31}$$

Space heater controls the indoor temperature at the desired temperature band. Eq. (32) states the linear equation between the indoor and outdoor temperature and the electrical consumption of the space heater. Here, θ_i^{in} is the initial indoor temperature which, in this model is assumed to equal the desired temperature, θ_{des}^{in}.

$$\theta^{in}(t+1,\omega) = e^{\frac{-1}{RC}}\theta^{in}(t,\omega) + R\left(1-e^{\frac{-1}{RC}}\right)L_{sh}^{rt}(t,\omega) + \left(1-e^{\frac{-1}{RC}}\right)\theta^{out,pred}(t,\omega), \quad \forall\, t \geq 2, \forall\, \omega$$
$$\theta^{in}(t,\omega) = \theta_i^{in} = \theta_{des}^{in}, \quad \forall\, t = 1, \forall\, \omega \tag{32}$$

Eq. (33) represents that the indoor temperature is limited to 1 degree above and below the desired temperature.

$$-1 \leq \theta^{in}(t,\omega) - \theta_{des}^{in} \leq 1 \tag{33}$$

Besides, the corresponding maximum and minimum bands of the space heater's load consumption in Eq. (34).

$$0 \leq L_{sh}^{rt}(t,\omega) \leq L_{sh}^{max}, \quad \forall\, t, \forall\, \omega \tag{34}$$

Eq. (35) presents how energy consumption of the SH is determined based on its power consumption.

$$EL_{sh}^{rt}(t,\omega) = \frac{(L_{sh}^{rt}(t,\omega) - L_{sh}^{rt}(t-1,\omega))}{2}, \quad \forall\, t \geq 2, \forall\, \omega$$
$$EL_{sh}^{rt}(t,\omega) = \frac{(L_{sh}^{rt}(t,\omega) - L_i^{sh})}{2}, \quad \forall\, t = 1, \forall\, \omega \tag{35}$$

Energy shedding constraint of the SH is expressed in Eq. (36).

$$0 \leq ES_{sh}^{rt}(t,\omega) \leq EL_{sh}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{36}$$

SWH is in charge of storing the heat in the water tank. The maximum and minimum constraints of the storage water heater's load consumption are stated in Eq. (37).

$$0 \leq L_{swh}^{rt}(t,\omega) \leq L_{swh}^{max}, \quad \forall\, t, \forall\, \omega \tag{37}$$

Besides, Eq. (38) represents that the total energy consumption of the SWH should be equal to its maximum energy capacity, and it guarantees that the SWH is only a shiftable load, not a shavable load.

$$\sum_{t=1}^{N_T} L_{swh}^{rt}(t,\omega) = U_{swh}^{max}, \quad \forall\, t, \forall\, \omega \tag{38}$$

Also, Eq. (39) represents that relation between energy and load consumption of the SWH.

$$EL_{swh}^{rt}(t,\omega) = \frac{(L_{swh}^{rt}(t,\omega) - L_{swh}^{rt}(t-1,\omega))}{2}, \quad \forall\, t \geq 2, \forall\, \omega$$
$$EL_{swh}^{rt}(t,\omega) = \frac{(L_{swh}^{rt}(t,\omega) - L_i^{swh})}{2}, \quad \forall\, t = 1, \forall\, \omega \tag{39}$$

Eq. (40) states the energy shedding constraint related to the SWH.

$$0 \leq ES_{swh}^{rt}(t,\omega) \leq EL_{swh}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{40}$$

PP should not run more than T_{ON} hours in a day as represented in Eq. (41).

$$L_{pp}^{rt}(t,\omega) = L_{pp}^{max} z(t,\omega) \tag{41}$$

Besides, Eq. (42) represents when the PP is "ON" it consumes its maximum load capacity. In Eqs. (41) and (42), z(t, ω) is a binary variable that represents the "ON"/"OFF" status of the PP. This way, z(t, ω) is equal to 1 when the PP is "ON", and z(t, ω) equals 0 when the PP is "OFF".

$$\sum_{t=1}^{N_T} z(t,\omega) \leq T_{ON} \tag{42}$$

The relation between the energy and power consumption of the PP is stated in Eq. (43).

$$EL_{pp}^{rt}(t,\omega) = \frac{(L_{pp}^{rt}(t,\omega) - L_{pp}^{rt}(t-1,\omega))}{2}, \quad \forall\, t \geq 2, \forall\, \omega$$
$$EL_{pp}^{rt}(t,\omega) = \frac{(L_{pp}^{rt}(t,\omega) - L_i^{pp})}{2}, \quad \forall\, t = 1, \forall\, \omega \tag{43}$$

Also, the limitations regarding the shedded energy of the PP is expressed in Eq. (44).

$$0 \leq ES_{pp}^{rt}(t,\omega) \leq EL_{pp}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{44}$$

MRSs include the loads that should be provided quickly – e.g. lighting, entertainment, etc. Hence, MRS are not dispatchable, and the quantity of them are determined based on the prediction as seen in Eq. (45).

$$L_{mrs}^{rt}(t,\omega) = L_{mrs}^{pred}(t) \tag{45}$$

The relation between the energy and power consumption and energy shedding of the MRSs are obtained the same as SH, SWH and PP as represented in Eqs. (46) and (47), respectively.

$$EL_{mrs}^{rt}(t,\omega) = \frac{(L_{mrs}^{rt}(t,\omega) - L_{mrs}^{rt}(t-1,\omega))}{2}, \quad \forall\, t \geq 2, \forall\, \omega$$
$$EL_{mrs}^{rt}(t,\omega) = \frac{(L_{mrs}^{rt}(t,\omega) - L_i^{mrs})}{2}, \quad \forall\, t = 1, \forall\, \omega \tag{46}$$

$$0 \leq ES_{mrs}^{rt}(t,\omega) \leq EL_{mrs}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega \tag{47}$$

All equations – which are represented above – described physical home system's objective and constraints, and our proposed model for optimal bidding strategy has not been represented up to now. In the following, we present an optimal bidding strategy for our proposed residential energy management system.

d. Optimal bidding strategy

The equations presented in this section derive optimal offering (when SHE is a producer) and biding (when SHE is a consumer) curves of the smart home for each decision-making time period in the DA and RT local electricity markets. In the context of this work, the offering curves should be ascending. However, the bidding curves should be descending. Eqs. (48) and (49) represent the offering model of the smart

home in the RT stage.

$$\Delta P_{net}^{rt}(t,\omega) \geq \Delta P_{net}^{rt}(t,\omega'), \quad \forall\, \omega > \omega' \& \lambda_{net}^{rt}(t,\omega) < \lambda_{net}^{rt}(t,\omega'), \quad \forall\, t \tag{48}$$

$$\Delta P_{sold}^{rt}(t,\omega) \geq \Delta P_{sold}^{rt}(t,\omega'), \quad \forall\, \omega > \omega' \& \lambda_{sold}^{rt}(t,\omega) > \lambda_{sold}^{rt}(t,\omega'), \forall\, t \tag{49}$$

As seen in the above constraints, Eq. (48) makes the descending bidding curves. On the other hand, Eq. (49) guarantees that the offering curves should be ascending. However, the above equations are not practical in an offering model of the smart home in the day-ahead stage because the uncertainty of PV energy generation and day-ahead electricity price is modeled through interval bands. In this situation, one solution is to use an iterative algorithm according to [40] to derive offering and bidding curves for the smart home in the day-ahead stage. However, the PV energy generation/electricity price will get its maximum/minimum amount in each iteration interval. Hence, using the iterative algorithm is not an appropriate solution for an offering model in the DA stage. This way, a new method for bidding strategy via interval-based scenarios is presented in this paper as described in next subsection.

2.2.2. Two-stage stochastic model

According to our proposed method, the scenarios for the day-ahead stage are come from the interval bands. This way, interval bands of the day-ahead PV energy generation and electricity price are divided into two scenarios that consist of: minimum and maximum bands (however, these scenarios can be extended). In this case, total day-ahead scenarios, N_φ, equals $N_{ib}{}^{N_p}$. In this way, N_{ib} and N_p represent number of scenarios in each interval band in each time period, and number of uncertain parameters. Therefore, in this paper, N_φ equals 4, N_{ib} equals 2 as mentioned above, and N_p is equal to 2 because only the uncertainty of the PV energy generation and electricity price is considered in this paper. Also, the corresponding probability, π_φ, for all scenarios equal $1/N_\varphi$ which are equal to 0.25 (=1/4) in this paper.

Therefore, new scenarios are added to the variables of the DA stage instead of interval bands of the PV energy generation and electricity price. The scenarios in the DA stage will be represented by φ. In this way, the expected profit based on the two-stage stochastic model of the REMS is represented in Eq. (50).

$$EP = \overbrace{\sum_{\varphi} \pi_\varphi \left\{ \sum_t \left[\lambda^{da}(t,\varphi)(P_{sold}^{da}(t,\varphi) - P_{net}^{da}(t,\varphi)) \right] \right\}}^{day-ahead\ expected\ profit} + \overbrace{\sum_{\omega} \pi_\omega \{ \sum_t (\lambda_{sold}^{rt}(t,\omega)\Delta P_{sold}^{rt}(t,\omega) - \lambda_{net}^{rt}(t,\omega)\Delta P_{net}^{rt}(t,\omega) - V_{PV}^{S} S^{PV}(t,\omega) - \sum_j VOLL_j(t) ES_j^{rt}(t,\omega)) \}}^{real-time\ expected\ profit} \tag{50}$$

As seen in Eq. (50), only variables and parameters of the day-ahead stage depend on φ in comparison to Eq. (2). In the following, Eqs. (3)–(18) will be redefined in Eqs. (51)–(66), respectively. In this way, Eqs. (51) and (52) express the power flow limitation for the distribution line which ends at the building.

$$0 \leq P_{net}^{da}(t,\varphi) \leq S_{max} v^{da}(t,\varphi), \quad \forall\, t, \forall\, \varphi \tag{51}$$

$$0 \leq P_{sold}^{da}(t,\varphi) \leq S_{max}(1 - v^{da}(t,\varphi)), \quad \forall\, t, \forall\, \varphi \tag{52}$$

Eq. (53) represents that the energy sold to the local market consists of energy produced by the PV system discharged energy of the battery system.

$$P_{sold}^{da}(t,\varphi) = P_{pv,out}^{da}(t,\varphi) + \gamma P_{dis,out}^{da}(t,\varphi), \quad \forall\, t, \forall\, \varphi \tag{53}$$

Eq. (54) states the power balance equation in the building.

$$P_{net}^{da}(t,\varphi) + P_{pv,in}^{da}(t,\varphi) + \gamma P_{dis,in}^{da}(t,\varphi) = EL^{da}(t,\varphi) + \gamma P_{ch}^{da}(t,\varphi), \quad \forall\, t, \forall\, \varphi \tag{54}$$

Eq. (55) presents the scenarios for the day-ahead electricity price which are come from its interval bands.

$$\begin{aligned} \lambda^{da}(t,\varphi_1) = \lambda^{da}(t,\varphi_2) = \lambda^{pred}(t) - \sigma_{price}^{dn}(t)(1-\alpha_{price}), \quad \forall\, t \\ \lambda^{da}(t,\varphi_3) = \lambda^{da}(t,\varphi_4) = \lambda^{pred}(t) + \sigma_{price}^{up}(t)\alpha_{price}, \quad \forall\, t \end{aligned} \tag{55}$$

Eq. (56) represents the potential energy generated by the PV system.

$$P_{pv,p}^{da}(t,\varphi)k(t,\varphi) = P_{pv,in}^{da}(t,\varphi) + P_{pv,out}^{da}(t,\varphi), \quad \forall\, t, \forall\, \varphi \tag{56}$$

The scenarios for the day-ahead PV energy generation based on its interval bands are represented in Eq. (57).

$$\begin{aligned} P_{pv,p}^{da}(t,\varphi_1) = P_{pv,p}^{da}(t,\varphi_3) = P_{PV}^{pred}(t) - \sigma_{pv}^{dn}(t)(1-\alpha_{pv}), \quad \forall\, t \\ P_{pv,p}^{da}(t,\varphi_2) = P_{pv,p}^{da}(t,\varphi_4) = P_{PV}^{pred}(t) + \sigma_{pv}^{up}(t)\alpha_{pv}, \quad \forall\, t \end{aligned} \tag{57}$$

Eq. (58) expresses the state-of-charge (SOC) equation of the battery in the day-ahead stage.

Table 2
Data of the domestic system.

Battery			Space heater		
η_{H2B}	Charging efficiency	0.90	L_i^{sh}	Initial load consumption	1.00 kW
η_{B2H}	Discharging efficiency	0.90	θ_i^{in}	Initial indoor temperature	23 °C
C_i	Initial state of the charge	0.48 kW h	C	Thermal energy capacity	0.525 kW h/°C
P_b^{max}	Maximum storage level	2.40 kW h	R	Thermal resistance of the building	18 °C/kW
P_b^{min}	Minimum storage level	0.48 kW h	L_{sh}^{max}	Maximum electrical consumption	5.525 kW
w^{max}	Maximum ramping rate	0.40 kW	Storage water heater		
w^{min}	Minimum ramping rate	0.40 kW	L_{swh}^{max}	Maximum electrical consumption	3.00 kW
Pool pump			L_i^{swh}	Initial load consumption	0.00 kW
L_{pp}^{max}	Maximum electrical consumption	1.10 kW	U_{swh}^{max}	Daily energy consumption	10.46 kW h
L_i^{pp}	Initial load consumption	0.00 kW	End-user distributed line		
T_{ON}	Maximum daily-hours	1.00 h	S_{max}	Maximum capacity	10.00 kW
Must-run services			PV system		
L_i^{mrs}	Initial load consumption	0.00 kW	V_{PV}^{S}	Cost of PV Spillage	1 $/kW h

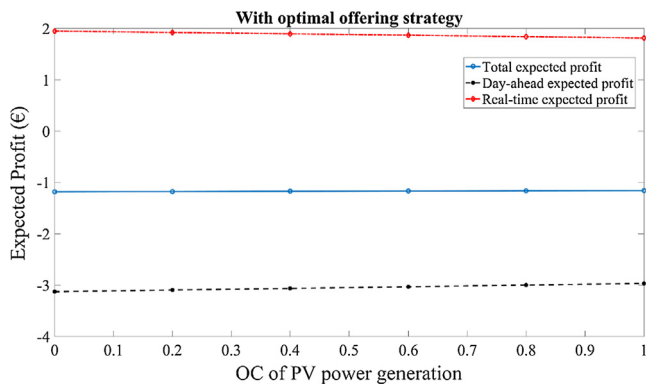

Fig. 3. Impact of α_{pv} on total, day-ahead and real-time expected profits of the residential energy management problem considering α_{price} and γ equal 1.

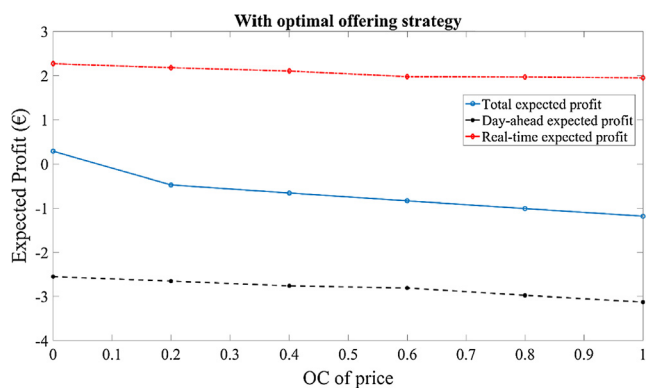

Fig. 4. Impact of α_{price} on total, day-ahead and real-time expected profits of the residential energy management problem considering α_{pv} equals 0 and γ equals 1.

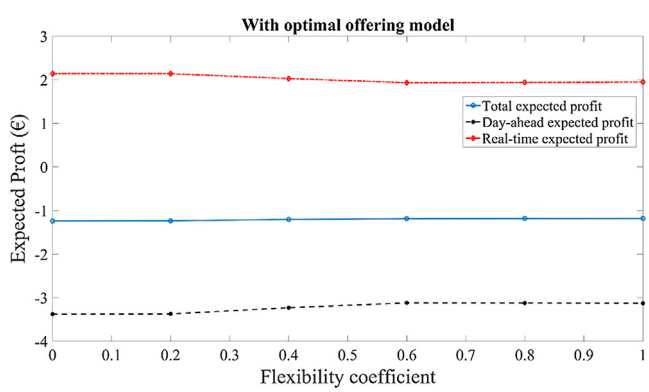

Fig. 5. Impact of γ on total, day-ahead and real-time expected profits of the residential energy management problem considering α_{pv} equals 0 and α_{price} equals 1.

$$C^{da}(t,\varphi) = C^{da}(t-1,\varphi) + P_{ch}^{da}(t,\varphi)\eta_{H2B} - \frac{P_{dis}^{da}(t,\varphi)}{\eta_{B2H}}, \quad \forall\, t \geq 2, \forall\, \varphi$$
$$C^{da}(t,\varphi) = C_i, \quad \forall\, t = 1, \forall\, \varphi \tag{58}$$

Eq. (59) represents the maximum and minimum bands of the battery's SOC.

$$P_b^{min} \leq C^{da}(t,\varphi) \leq P_b^{max}, \quad \forall\, t, \forall\, \varphi \tag{59}$$

The ramping upper and lower limitations related to the SOC are stated in Eq. (60).

$$-w^{min} \leq C^{da}(t,\varphi) - C^{da}(t-1,\varphi) \leq w^{max}, \quad \forall\, t \geq 2, \forall\, \varphi$$
$$-w^{min} \leq C^{da}(t,\varphi) - C_i \leq w^{max}, \quad \forall\, t = 1, \forall\, \varphi \tag{60}$$

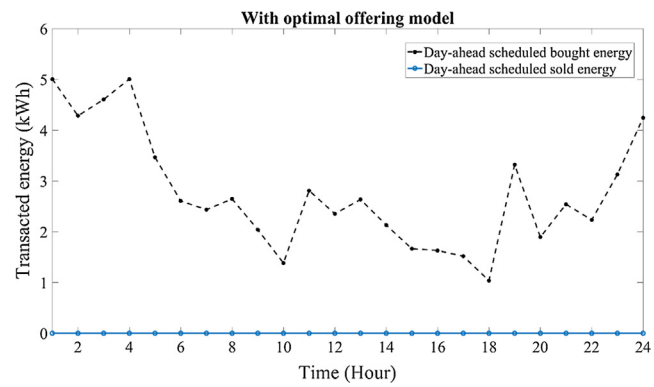

Fig. 6. The optimal scheduled transacted energy for the smart home in the day-ahead stage.

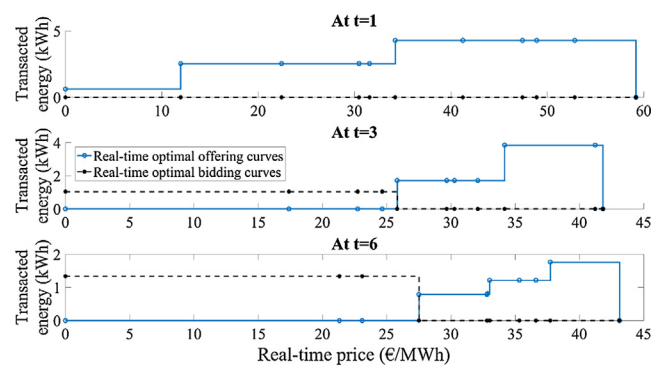

Fig. 7. The optimal bidding and offering curves for the smart home in t equals 1, 3, and 6 in the real-time stage.

Eqs. (61) and (62) represent maximum and minimum constraints of the discharged and charged energy of the battery, respectively.

$$0 \leq P_{dis}^{da}(t,\varphi) \leq w^{max} u^{da}(t,\varphi), \quad \forall\, t, \forall\, \varphi \tag{61}$$

$$0 \leq P_{ch}^{da}(t,\varphi) \leq w^{min}(1 - u^{da}(t,\varphi)), \quad \forall\, t, \forall\, \varphi \tag{62}$$

Eq. (63) presents that the total discharged energy of the battery system.

$$P_{dis}^{da}(t,\varphi) = P_{dis,in}^{da}(t,\varphi) + P_{dis,out}^{da}(t,\varphi), \quad \forall\, t, \forall\, \varphi \tag{63}$$

As highlighted before, in this paper, the uncertainty of the electrical loads is not seen in the day-ahead stage, and the day-ahead electrical loads are considered to be equal to their point forecasting as seen in Eq. (64).

$$EL^{da}(t,\varphi) = EL^{pred}(t), \quad \forall\, t, \forall\, \varphi \tag{64}$$

Eq. (65) represents the power balance equation in the real-time stage.

$$P_{net}^{da}(t,\varphi) + P_{pv}^{rt}(t,\omega) + P_{dis}^{rt}(t,\omega) + \Delta P_{net}^{rt}(t,\omega) = EL^{rt}(t,\omega) - ES^{rt}(t,\omega) + P_{ch}^{rt}(t,\omega) + P_{sold}^{da}(t,\varphi) + \Delta P_{sold}^{rt}(t,\omega), \quad \forall\, t, \forall\, \omega, \forall\, \varphi \tag{65}$$

Eq. (66) states the power flow constraints a distribution line which end at the building.

$$-S_{max} \leq P_{net}^{da}(t,\varphi) + \Delta P_{net}^{rt}(t,\omega) - P_{sold}^{da}(t,\varphi) - \Delta P_{sold}^{rt}(t,\omega) \leq S_{max}, \quad \forall\, t, \forall\, \omega, \forall\, \varphi \tag{66}$$

Hence, the offering model for deriving the offering and bidding curves of the smart home presented according to Eqs. (67) and (68):

$$P_{net}^{da}(t,\varphi) \geq P_{net}^{da}(t,\varphi'), \quad \forall\, \varphi > \varphi' \& \lambda^{da}(t,\varphi) < \lambda^{da}(t,\varphi'), \forall\, t \tag{67}$$

Table 3
Status of energy transaction between the smart home and local market in the real-time stage.

Time (hour)	$v^{rt}(t,\omega)$									
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
1										
2			■						■	
3		■	■			■			■	
4	■									
5	■				■		■			
6		■		■			■			
7										
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14										
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21										
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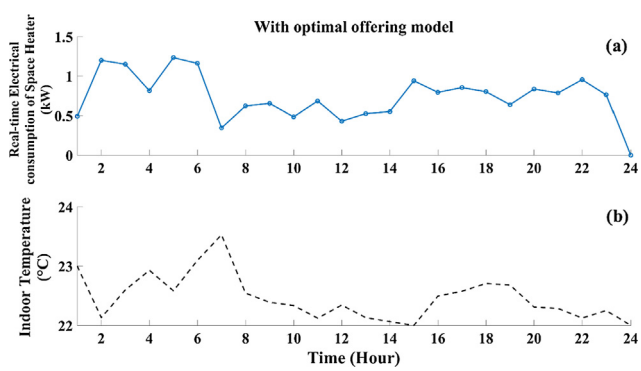

Fig. 8. Real-time expected electrical consumption of the space heater (a), real-time expected indoor temperature (b) in the optimal offering model of the REMS.

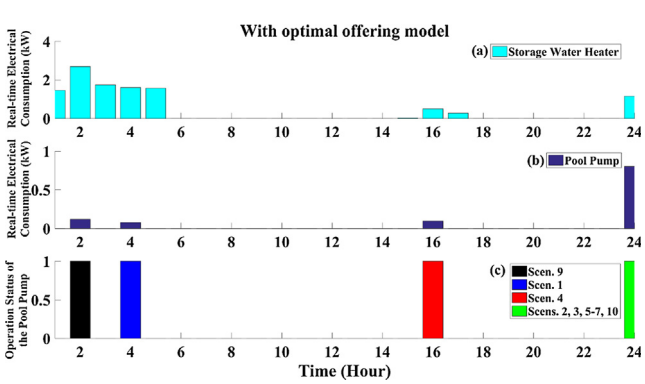

Fig. 9. Real-time expected electrical consumption of the storage water heater (a), real-time expected electrical consumption of the pool pump (b), real-time operation status of the pool pump (c) in optimal offering model of the REMS.

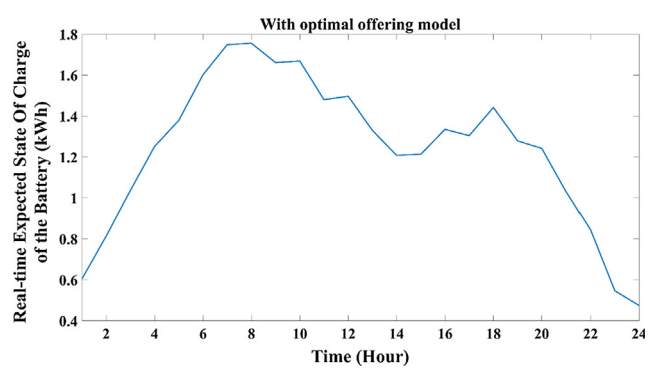

Fig. 10. Real-time expected state of charge of the battery in the optimal offering model of the REMS.

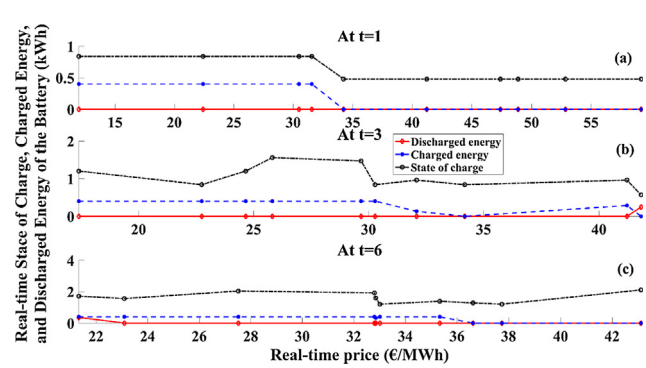

Fig. 11. Real-time state of charge, charged energy, and discharged energy of the battery at t = 1 (a), t = 3 (b), t = 6 (c).

$$P^{da}_{sold}(t,\varphi) \geq P^{da}_{sold}(t,\varphi'), \quad \forall\, \varphi > \varphi' \& \lambda^{da}(t,\varphi) > \lambda^{da}(t,\varphi'), \quad \forall\, t \tag{68}$$

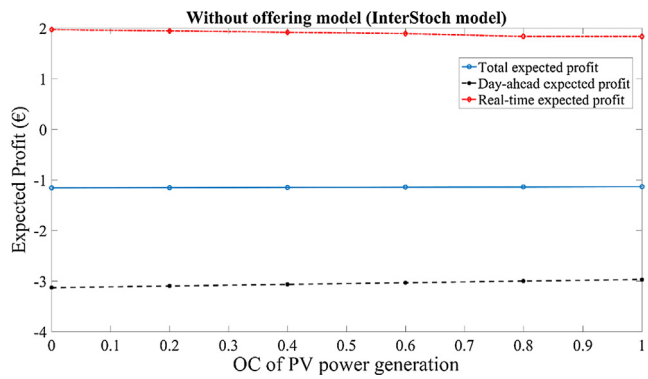

Fig. 12. Without offering model (InterStoch model): impact of α_{pv} on total, day-ahead and real-time expected profit of the residential energy management problem considering α_{price} and γ equal 1.

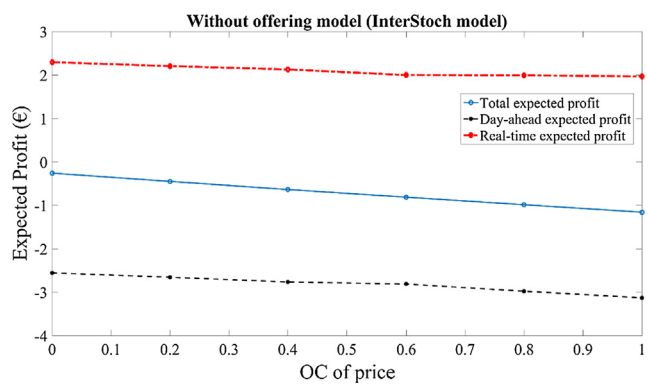

Fig. 13. Without offering model (InterStoch model): impact of α_{pice} on total, day-ahead and real-time expected profit of the residential energy management problem considering α_{pv} equals 0 and γ equals 1.

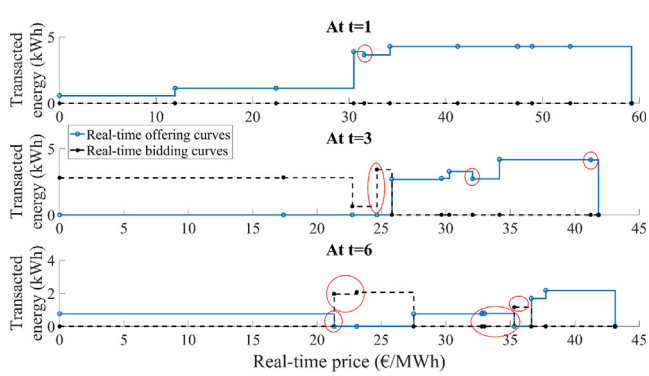

Fig. 14. Without offering model (InterStoch model): the bidding and offering curves for the smart home in t equals 1, 3, and 6 in the real-time stage.

This way, according to the reformulated equations, the decision-making problem is represented below:

Max.

Eq. (50)

Subject to

Eqs. (51)–(68) and (19)–(49).

The above model expressed our proposed optimal bidding strategy for the REMS via two-stage stochastic programming.

3. Case studies

3.1. Cases

The residential system that has been used in [7,8,41–43] is utilized as a test system in this paper. However, electric vehicle is not considered in this paper. The proposed Mixed Integer Linear Programming (MILP) is solved in GAMS 24.2.3 [44]. Also, Table 2 presents data of the proposed domestic system. Prediction, interval bands, and scenarios data are presented in Tables 7–11 in Appendix Section.

As mentioned before, the predicted day-ahead home's energy consumption and load of must-run services in the real-time do not depend on the scenarios in our proposed model, only their point forecasting is modeled in this paper. Characteristics of the residential system are described in the following:

- Battery can store between 0.48 kW h and 2.4 kW h, and its maximum charging and discharging rates are 400 W. Charging and discharging rates represent maximum amount of power of the battery that can be charged or discharged in each decision-making time step. Also, the charging and discharging efficiencies of the battery are 90% [41–43].
- Maximum load capacity of the space heater in each time period is equal to 5.525 kW.
- Daily energy capacity of the storage water heater is 10.46 kW h (180 l). Also, it has a 3 kW heating element.
- The desired temperature of the building is assumed to equal 23 °C. Furthermore, the thermal resistance of the building shell and C are equal to 18 °C/kW and 0.525 kW h/°C, respectively.

The assessment of the performance of the proposed residential energy management problem is done in two cases that are described as follows:

- Case 1: The residential energy management problem is solved by Mixed-Integer Linear Programming (MILP) through a two-stage stochastic optimal bidding strategy which. In this way, scenarios of the first stage come from interval bands, while stochastic scenarios are used in the second stage. In this case, influences of the optimistic and flexibility coefficients are assessed in the performance of the proposed residential energy management system based on the optimal bidding strategy.

In this way, the stochastic optimal bidding strategy for the REMS will be:

Max.

Eq. (50),

Subject to

Eqs. (51)–(68) and (19)–(49).

- Case 2: The residential energy management problem is solved without considering the bidding strategy. In this case, the uncertainty of price and PV energy output in the day-ahead stage is modeled by both methods: Interval-based scenarios and interval bands. In this way, the performance of the system is evaluated according to the impacts of optimistic coefficients on both methods.

In the two-stage stochastic scenario-based method (hereinafter, this method is called InterStoch), the proposed residential energy management problem without the optimal bidding strategy will be:

Max.

Eq. (50),

Table 4
Without offering model (InterStoch model): status of energy transaction between the smart home and local market in the real-time stage.

Time (hour)	$v^{rt}(t,\omega)$									
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
1										
2										
3										
4										
5										
6										
7										
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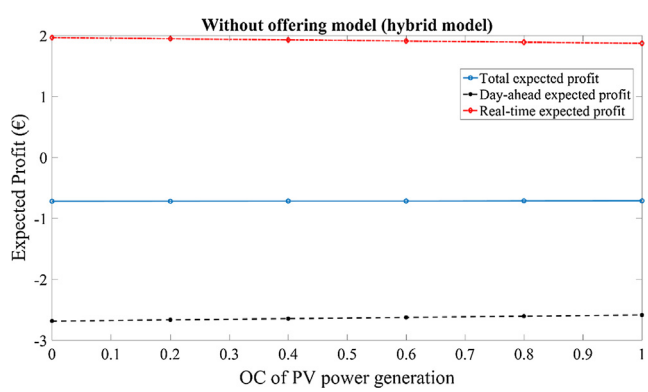

Fig. 15. Without offering model (Hybrid model): impact of α_{pv} on total, day-ahead and real-time expected profit of the residential energy management problem considering α_{price} and γ equal 1.

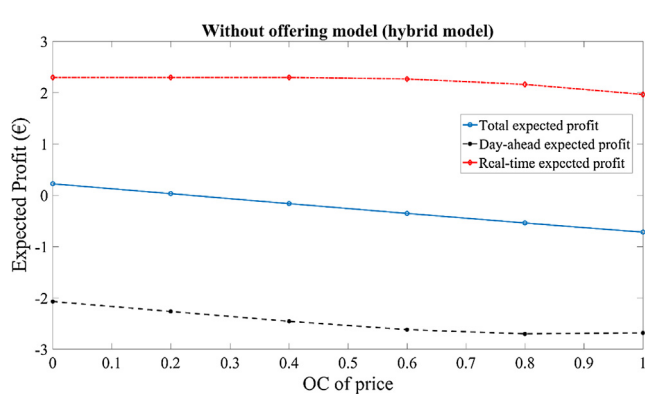

Fig. 16. Without offering model (Hybrid model): impact of α_{pice} on total, day-ahead and real-time expected profit of the residential energy management problem considering α_{pv} equals 0 and γ equals 1.

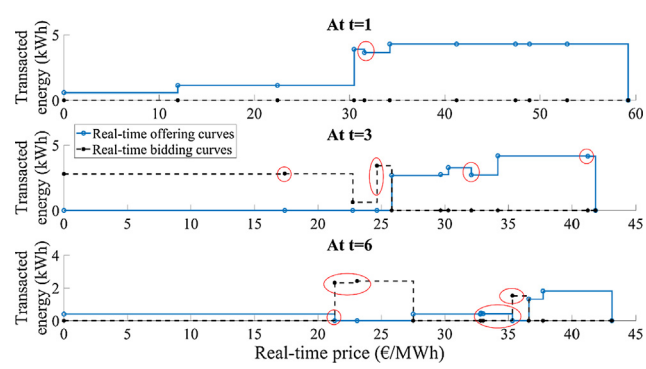

Fig. 17. Without offering model (Hybrid model): the bidding and offering curves for the smart home in t equals 1, 3, and 6 in the real-time stage.

Table 5
Total expected profit of the residential energy management problem considering optimal and non-optimal strategies in the worst scenario (α_{pv} equals 0 and α_{pice} equals 1).

	Non-optimal offering models		Optimal offering model
	Hybrid method	InterStoch method	
Day-ahead EP (€)	−2.684	−3.130	−3.130
Real-time EP (€)	1.965	1.971	1.945
Total EP (€)	−0.719	−1.159	−1.185

Subject to

Eqs. (51)–(66) and (19)–(47).

However, for the two-stage interval-stochastic optimization method

Table 6
Without offering model (Hybrid model): status of energy transaction between the smart home and local market in the real-time stage.

Time (hour)	$v^{rt}(t,\omega)$									
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
1										
2										
3										
4										
5										
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Table 7
Day-ahead predicted energy consumption of the home and predicted load of the must-run services in real-time.

Time (h)	$EL^{pred}(t)$ (kW h)	$L^{pred}_{mrs}(t)$ (kW)
1	4.605	0.005
2	4.605	0.005
3	4.605	0.005
4	4.605	0.005
5	3.065	0.005
6	2.605	0.005
7	2.435	0.005
8	2.245	0.005
9	2.055	0.005
10	1.865	0.005
11	1.675	0.005
12	1.675	0.005
13	1.675	0.005
14	1.675	0.005
15	1.675	0.005
16	1.675	0.005
17	1.85	0.005
18	1.935	0.005
19	2.278	1.218
20	2.452	0.262
21	2.582	0.262
22	2.59	0.14
23	2.727	0.127
24	2.605	0.005

(hereinafter, this method is called Hybrid), the residential energy management problem without the optimal bidding strategy is represented in the following:

Max.

Eq. (2)

Subject to

Eqs. (3)–(47).

Although InterStoch method optimizes the residential energy management problem by MILP, uncertainty modeling based on Hybrid method in our proposed energy management problem is solved by Mixed-Integer Non-Linear Programming (MINLP).

Table 8
Central forecasting and interval forecasting error of the market price and the PV energy output in the day-ahead stage.

Time (h)	λ^{da}(t) (€/MW h)		$P^{da}_{pv,p}$(t)(kW h)	
	Central forecasting	Forecasting error	Central forecasting	Forecasting error
1	39.13	13.11	0	0
2	35.51	12.77	0	0
3	33.13	12.59	0	0
4	31.91	12.37	0	0
5	31.62	12.32	0	0
6	33.25	12.34	0	0
7	38.04	13.03	0.042	0.042
8	43.30	13.81	11.78	11.78
9	45.95	13.58	91.47	75.02
10	46.61	12.75	271.1	147.7
11	46.31	12.82	494.1	215.7
12	45.39	12.83	698.7	275.8
13	44.88	12.84	853.2	312.8
14	44.73	13.00	973.7	328.2
15	43.52	13.31	1066.1	312.7
16	42.42	13.74	1071.8	285.7
17	42.40	14.11	972.6	285.0
18	43.73	14.47	800.8	259.4
19	45.19	14.86	589.6	230.5
20	46.75	14.13	370.1	169.7
21	47.44	13.42	146.3	105.3
22	47.18	12.12	25.06	25.06
23	44.43	11.63	0.680	0.680
24	40.84	11.86	0	0

Table 9
Scenarios of the market price in the real-time stage.

Time (hour)	$\lambda^{rt}(t,\omega)$ (€/MWh)									
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
1	11.96	22.42	30.48	31.56	34.23	41.23	47.42	48.90	52.85	59.20
2	29.05	34.35	19.32	31.38	27.70	33.83	32.40	50.91	21.01	34.40
3	41.22	17.40	25.82	29.68	41.83	22.75	32.09	30.28	24.66	34.18
4	24.20	30.15	28.56	39.21	29.80	31.45	34.78	40.37	39.17	29.35
5	13.49	33.30	39.75	37.96	26.81	30.57	20.85	38.17	41.58	53.09
6	36.62	23.10	43.12	21.33	32.85	37.74	27.52	33.01	32.80	35.33
7	40.18	46.20	37.62	41.97	32.70	43.00	35.64	47.69	32.14	30.76
8	46.71	38.62	47.66	43.39	49.46	33.96	49.51	46.68	47.04	48.18
9	49.29	42.51	47.84	36.27	43.07	41.45	58.31	44.12	41.82	48.57
10	40.14	61.71	64.09	36.72	39.46	52.21	43.75	36.02	35.13	40.04
11	37.05	39.08	29.24	43.01	55.78	47.82	47.79	53.98	54.90	55.20
12	34.53	39.80	55.38	32.61	37.76	64.35	44.50	54.37	34.39	42.58
13	37.50	48.56	43.54	39.54	50.76	45.38	67.95	23.15	46.28	45.44
14	43.32	42.59	52.83	33.82	39.99	40.04	49.73	52.87	34.58	50.54
15	42.47	42.89	32.35	47.86	51.53	41.00	47.19	27.01	35.75	43.31
16	26.11	30.76	49.69	23.35	46.66	36.85	27.31	57.41	32.81	45.03
17	45.90	30.30	47.90	16.84	39.27	24.37	72.74	34.35	41.71	67.24
18	28.00	49.67	35.27	31.16	29.82	40.23	44.97	40.25	31.91	38.66
19	53.04	40.93	47.06	49.15	40.53	61.46	54.31	53.95	54.42	57.43
20	28.17	58.00	27.05	49.46	58.08	28.05	48.24	40.36	55.23	48.96
21	41.61	51.30	51.10	47.98	60.90	42.25	45.62	51.61	39.05	45.47
22	27.68	53.03	41.27	51.70	37.96	47.51	31.93	48.34	45.07	53.13
23	56.34	48.24	49.41	46.56	51.08	43.00	38.23	52.57	47.93	36.63
24	46.38	29.20	50.56	22.86	33.41	33.68	27.80	43.71	50.39	38.75

3.2. Results

3.2.1. Case 1: with optimal offering model

In this section, the performance of the proposed two-stage stochastic residential energy management problem is assessed taking into account optimal bidding strategy. In this way the performance of the proposed problem is evaluated based on the impacts of the optimistic coefficients of the PV energy output and electricity price, and flexibility coefficient on the expected profit of the system and transacted energy between the smart home and the local market.

a. Impact of α_{pv}, α_{price}, and γ

In this section, impacts of α_{pv} and α_{price} on total, day-ahead and real-time expected profits of the smart home are studied. Moreover, their influences on the exchanged energy through smart home and the local

Table 10
Scenarios of the PV energy output in the real-time stage.

Time (h)	$P_{t\omega}^{pv,scen}$ (W)									
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	7.22	26.69	0	0.37	20.80	22.48	17.71	6.50	9.91	13.30
9	80.52	140.6	125.7	99.96	34.55	69.55	107.6	84.43	97.65	63.73
10	203.5	206.9	287.6	195.2	307.8	278.8	320.7	210.3	275.2	160.0
11	531.6	607.5	526.4	452.9	585.5	530.4	476.0	507.5	554.5	513.9
12	895.5	666.1	654.8	864.2	747.0	832.3	586.4	676.5	725.1	610.2
13	745.8	792.3	405.0	994.4	1007.4	1015	805.3	791.6	788.1	1082
14	1165	637.2	899.8	994	1336.9	1138.9	825.7	810.4	1106.2	1049.4
15	916.0	1267.8	1024.5	1282.8	1003.9	1211.2	1074.9	1292.2	923.7	874.7
16	870.9	1306.8	1068.9	988.1	1077.7	1120	1246.6	861.8	903.7	1092.7
17	1152.4	938.9	1061.8	882.2	1072.9	1065.6	1083.8	1058.5	895	925.4
18	773.8	777.5	795.2	738.9	881.5	814.09	950.9	725	868.1	714.9
19	434.8	540.2	582.4	523.9	654.2	493.3	443.9	612.2	615.3	561.4
20	314.3	313.2	305.5	415.3	282.9	379.4	332.7	378.09	360.7	346
21	160.8	150.9	148.1	261.9	98.10	120.1	149.7	87.45	106.8	163
22	24	35.40	19.70	13.29	8.640	48.21	17.18	22.75	0	21.75
23	0.465	0	1.783	0.435	0.851	0	0.694	0.330	0.553	0.054
24	0	0	0	0	0	0	0	0	0	0

Table 11
Scenario Probabilities in the real-time stage.

	Real-time scenarios									
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
π_ω	0.07	0.10	0.10	0.09	0.08	0.11	0.12	0.08	0.11	0.13

market is evaluated. In Fig. 3, impact of the α_{pv} on the expected profits of the system is studied considering α_{price} and γ equal 1. As seen in Fig. 4, increment of α_{pv} increases total expected profit, and the maximum amount of the total expected profit is where α_{pv} is equal to 1. However, the worst case is where α_{pv}equals 0, and the total expected profit of the system gets its minimum amount. Thus, modeling a residential energy management system considering α_{pv} equals 0 increases the robustness of the system. On the hand, the increment of α_{price} has a negative effect on the total expected profit of the system where α_{pv} and γ equal 0 and 1, respectively. This way, worst and robust case of the system is when α_{pv} equals 0 and α_{price} equals 1. Fig. 5 demonstrates the impact of the flexibility coefficient on the expected costs in the worst case of the system when α_{pv} and α_{price} equal 0 and 1, respectively. As shown in Fig. 5, increment of the flexibility coefficient increases the total expected profit of the system. Hence, the maximum amount of the expected profit is where γequals 1. In this case, the best case is more interested to model energy flexibility of the smart home since the best case to manage energy flexibility in the domestic energy management problem is where γequals 1.

b. Optimal offering and bidding curves

In this section, optimal offering and bidding curves of the residential energy management problem through the two-stage stochastic model are represented. As in the day-ahead stage, the home energy management system only offers and bids one quantity for all price scenarios, since the optimal bought/sold energy curve of the smart home from/to the local market is shown in Fig. 6. As seen in Fig. 6, the offering setpoints of the home in all scenarios and time steps in the day-ahead stage equal 0. It means that the proposed home is eager to participate only as a consumer in the day-ahead local market. However, Fig. 7 and Table 3 represent that the smart home acts as a prosumer, and SHE submit both its optimal and bidding curves to the real-time local market in all time steps. Table 3 expresses the state of $v^{rt}(t, \omega)$ in green and orange cells. In this way, green cells represent states in which $v^{rt}(t, \omega)$ is equal to 1. In this way, in t equals 1, 7 to 15, 18, 19, and 21 to 24, the smart home only acts as a producer and there is no green cell. However, in t equal 6, SHE plays as a consumer in scenarios 2, 4 and 7. In Fig. 7, optimal offering and bidding curves are demonstrated at t = 1, t = 3, and t = 6. Although it is expected that SHE only participates as a consumer in the day-ahead local market in all time steps, according to Table 3 and Figs. 6 and 7, SHE plays as both a consumer and a producer and submits both optimal offering and bidding curves at t = 2, t = 3, t = 4, t = 5, t = 6, t = 16, t = 17, and t = 20 in the real-time local market.

As it has been explained in Section 2, three types of electrical loads – controllable, shiftable and non-dispatchable – are defined in this paper. In this way, the space heater is modeled as controllable load based on Eqs. (32)–(34). Fig. 8 shows real-time expected electrical consumption of the space and indoor temperature. In the case study, it is considered that the desired indoor temperature of the home equals 23 °C. Hence, the real-time expected indoor temperature is constrained to 22 °C and 24 °C according to Eq. (33) as it is shown in Fig. 8. On the other hand, the Storage Water Heater (SWH) and Pool Pump (PP) are defined as shiftable loads in this system. Hence, shiftable loads are switched off in the time periods of higher electricity price. As highlighted in Table 9, electricity price is the highest amount in the time period from t = 6 to t = 15. Hence, both SWH and PP are not committed by the REMS from t = 6 to t = 15 as shown in Fig. 9Although the maximum daily operational time period of the PP has been assumed to be 1 h (T_{ON} = 1), Fig. 9(b) shows the amount of real-time expected electrical

consumption is nonzero in four time steps. For this reason, Fig. 9(b) presents expected electrical consumption of the PP in each time period of the residential energy management problem. In this way, real-time operation status on the PP ($z(t, \omega)$) is shown in Fig. 9(c). As it is seen in Fig. 9(c), $z(t, \omega)$ is only committed to one time period of each scenario. However, $z(t, \omega)$ is committed to six scenarios (w2, w3, w5, w6, w7, w10) in t = 24, so real-time expected electrical consumption of the PP is the highest at t = 24.

Fig. 10 shows the real-time expected state of charge of the battery. In this paper, it is considered the battery' SOC is in the minimum storage level in the initial state (C_i = 0.48 kW h). As it is shown in Fig. 10, the SOC of battery is at its minimum level of charge at t = 24. Fig. 11 shows real-time SOC, charged energy, and energy discharged from the battery at t = 1 (a), t = 3 (b), and t = 6 (c). By comparing Figs. 10 and 11, it can be deduced that there is not fixed incremental or decreasing relationship between the SOC of the battery and electricity price. Thus, the use of a battery as an energy storage system can provide energy flexibility to make optimal offering and bidding curves for the REMS.

3.2.2. Case 2: without optimal offering model

In this section, performance of the proposed residential energy management problem is studied while constraints related to the optimal bidding strategy are not seen in the problem, and both proposed methods are used to model uncertainty of the PV energy generation and electricity price. In the following, the results of the system based on InterStoch and Hybrid methods are demonstrated and compared.

a. Results of the InterStoch method

In this section, the uncertainty of the system is modeled by the InterStoch method. Hence, effectiveness of the optimal bidding strategy that consists of constraints (48), (49), (67), and (68) is evaluated in this section.

As seen in Figs. 12 and 13, increment of the optimal coefficients of the PV energy generation and electricity price has positive and negative influence on the expected profit of the system. In other words, the worst case of the system is to consider that α_{pv} and α_{price} equal 0 and 1, respectively. In this way, in Fig. 14 and Table 4, the real-time offering and bidding curves of the domestic energy management system are assessed in the worst case without the optimal bidding strategy. Fig. 10 demonstrates the real-time bidding and offering curves in t = 1, t = 3, and t = 6. As mentioned before, optimal offering and bidding curves must be ascending and descending, respectively. In Fig. 14, red circles indicate offering and bidding transacted energy steps that are descending and ascending, respectively, and they cause the offering and bidding curves to not be optimal. Moreover, Table 4 presents the state of $v^{rt}(t, \omega)$. In this case, dark green cells represent states in which $v^{rt}(t, \omega)$ equals 1 in optimal and non-optimal strategies, and dark orange cells express that $v^{rt}(t, \omega)$ equals 0 in both strategies. Also, light green cells are related to the states in which $v^{rt}(t, \omega)$ equals 1 only in the non-optimal strategy, and light orange cells show states that $v^{rt}(t, \omega)$ equals 1 only in the optimal strategy. Eventually, as it is seen in Table 4, the smart home is committed more as a consumer in the non-optimal bidding strategy. Hence, in non-optimal offering model, SHE is not able to submit its offering and bidding curves to the local market so as to maximize its expected profit. This is because offering and bidding curves are not optimal in this Case. Hence, an appropriate strategy for SHE is to transact energy with other local market players – e.g. small consumers, producers, and prosumers – according to its optimum decisions in home energy management.

participate in the local market to maximize its profit because its bidding and offering curves are not optimal. However, their setpoints decisions regarding transacted energies are optimal. In this way, the REMS can exchange energy with other local market players in its neighborhood according to the results of this section.

b. Results of the Hybrid method

In this section, the Hybrid method is used to model uncertainty of the PV's energy generation and electricity price in the residential energy management problem. In this case, interval bands are defined to consider uncertainty in the day-ahead stage of our proposed energy management problem. Moreover, as it has been highlighted before, the optimization problem will be MINLP.

Fig. 15 demonstrates that increment of the optimistic coefficient of the PV energy generation increases the total expected profit of the system. However, increasing the α_{price} decreases the total expected profit as it is shown in Fig. 16. Hence, these facts state that impact patterns of the optimistic coefficients on the expected profit of the system are the same in both methods. Moreover, Fig. 17 proves that bidding and offering curves are not optimal in this case. By comparing between Tables 4 and 6, it can be observed that the smart home is eager to act as a consumer in the model based on the hybrid method as opposed to the model based on the InterStoch method. The results of this eagerness can be seen in Table 5.

In this way, although total and day-ahead expected profits of the system in the hybrid method is less than the InterStoch method, the real-time expected profit of the system in the InterStoch method is higher. For this reason, SHE prefers to play as a consumer in more scenarios in the hybrid method in comparison to the InterStoch one. Besides, Table 5 compares expected profits of the REMD in non-optimal and optimal offering models in the worst scenario where α_{PV} equals 0 and α_{price} equals 1. As seen in Table 5, total EP of the REMS is the highest when the non-optimal model is solved by hybrid method. Moreover, total EP of the system is lowest in optimal offering model of the REMS. In other words, Table 5 shows that the InterStoch optimization method is more robust than the hybrid method because it provides a lower total expected profit of the system in this case study. Also, the optimal offering model is more robust than the non-optimal offering one.

4. Conclusions and discussions

In this paper, a probabilistic scenario-based method was presented for the management of residential energy and energy trading with the local electricity market based on an optimal bidding strategy. Our residential energy management problem includes two stages: day-ahead and real-time. In the day-ahead stage, two methods have been proposed to model the uncertainty of electricity price and PV energy generation. Their uncertainty is modeled by interval bands and interval-based scenarios. In the real-time stage, stochastic scenarios have been used to consider the uncertainty affecting the system. In addition, energy flexibility is provided by a battery system. Our proposed optimal offering model for the REMS is assessed in two different cases. Case 1 assesses the impacts of optimistic and flexibility coefficients on the REMS considering the optimal bidding strategy. However, in case 2, the performance of the two different optimization methods – called InterStoch and Hybrid – on the REMS are evaluated without considering the optimal bidding strategy. According to the simulation results in our case study:

- The robustness of our proposed residential energy management system is increased where α_{pv} and α_{price} – the optimistic coefficients of PV power generation and electricity price – equal 0 and 1, respectively. In other words, increment of α_{pv} is in line with increment of the expected profit of the system. However, increment of α_{price} has a negative impact on the REMS' expected profit. In this way, worst and robust case of the system is where α_{pv} equals 0 and α_{price} equals 1.
- Optimistic coefficients have the same pattern of impact on the system's expected profit in both InterStoch and Hybrid methods.
- Robustness of the InterStoch optimization method is higher than the Hybrid method because the total expected profit of the system is

lower in the case study that is solved by the InterStoch optimization method. Besides, the Hybrid optimization method obtains sub-optimal results because it is solved by MINLP, and it is not as efficient as the InterStoch optimization method.

- Our proposed optimal offering model for the residential energy management system is more robust than its non-optimal offering model because the optimal offering model brings lower expected profit to the system in the worst scenario where α_{pv} equals 0 and α_{price} equals 1.
- Increment of the flexibility coefficient is in line with the total expected profit of the system. Therefore, the best case of the system is where flexibility coefficient equals 1.
- Our proposed residential energy management system only offers and bids one quantity for all price scenarios in the day-ahead stage. In other words, modeling the domestic system with or without bidding strategy represents that it cannot impact on the smart home's behavior (as a consumer or producer) in the day-ahead local electricity market.

In our future works, we will model different energy management strategies in the distribution power systems based on a community of smart buildings in order to look at how smart buildings can impact local energy trading as price-maker agents.

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Appendix A

The loads prediction data is stated in Table 7. Table 8 presents the predicted day-ahead central forecasting and interval errors of price and PV energy generation. As it can be seen in Table 8, upper and lower forecasting errors are considered to be equal in this paper. Moreover, the real-time electricity price and PV energy generation scenarios are reduced to ten scenarios for each time period as presented in Tables 9 and 10, respectively. The corresponding probabilities of the real-time scenarios are stated in Table 11. It should be highlighted that the sold and bought electricity price in the real-time are considered to be equal in this case study. Hence, $\lambda^{rt}(t, \omega)$ is defined in Table 5 instead of $\lambda^{rt}_{net}(t, \omega)$ and $\lambda^{rt}_{sold}(t, \omega)$.

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