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Study of the Dynamic Characteristics of BWE Based on Products, Difference With a Two-Stage Supply Chain

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ABSTRACT

In previous studies, many researchers have achieved significant success in reducing the negative effects brought about by the bullwhip effect. In this article, the authors have established a supply chain that consists of one supplier and two retailers and has adopted a Cournot-Bertrand mixed duopoly model that successfully combines a nonlinear complexity dynamic system with the bullwhip effect. In order to partition systems with various degrees of disorder and take appropriate methods to refrain systems from falling into a chaos state, numerical simulation methods are conducted to find the stability range, bifurcation range and the chaos range. This article focuses on three different ranges that the authors analyze about the bullwhip effect and inventory variability. In the end, some practical suggestions on behavioral science management are made.

KEYWORDS

A Two-Stage Supply Chain, Bifurcation, Bullwhip Effect Control, Chaos, Cournot-Bertrand Model, Dynamic Character, Stability

1. INTRODUCTION

In the past decade, researchers have probed into the "Bullwhip effect" in supply chains. However, along with the complication of the structures of supply chains, we find it increasingly important to take bullwhip effect into account when we do researches about supply chains with a view of entropy complexity. Bullwhip effect refers to the phenomenon that the orders are amplified within each level from downstream demand to upstream. Nowadays, this phenomenon becomes especially significant in many industries, such as automotive, commodities, electronic equipment, fast food industries etc. Previous literature has shown that the bullwhip effect can lead to wrong judgments for both retailers and suppliers, especially in the aspect of inventory. And it also can add production cost and cause inactive transportation (Lee, Padmanabhan, & Whang, 1997b). In other words, the bullwhip effect in product orders not only contributes to adding cost and possibilities of inventory oscillations for upstream, but also unavoidably results in larger inventory costs for downstream (Hoberg, Bradley, & Thonemann, 2007).

The bullwhip effect has so many negative influences, accompanying it brings about special concern: how to reduce or eliminate it. In this regard, many scholars have already set foot on these issues.

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"Bullwhip effect" was put forward by Lee et al. (1997a). He demonstrated the five factors causes the bullwhip effect in supply chains, which include demand signal processing, non-zero lead-time, order batching, supply shortages and price fluctuation. To begin with, Kahn quantified the bullwhip effect by using the first-order autoregressive demand process (Kahn, 1987). Graves researched the bullwhip effect in supply chains with an integrated moving average process (Graves, 1999). Chen studied the influence of the methods of exponential smoothing (ES) and MA forecasting on the bullwhip effect in supply chain including one supplier and one retailer (Chen, Drezner, Ryan, Simchi-Levi, 2000a; 2000b). XU developed the ES forecasting method for the lead-time demand in his papers about the bullwhip effect (Xu, Dong, Evers, 2001). Alwan studied the bullwhip effect for the lead-time demand with MMSE-optimal forecasting technique and demand process following a positively or negatively correlated process (Ahmad, 2007). Wang compared the bullwhip effect using correct, MA and EWMA method in a single-stage supply chain (Wang, 2010). Wang published papers about two kinds of fuzzy demand through a revenue-sharing contract (Wang, Li, Du & Wang, 017).

According to preceding studies, many scholars also analyzed the coefficient of the demand forecast on the bullwhip effect subsequently. For example, Zhang studied the impact of bullwhip effect with a first-order autoregressive process and an order up to inventory policy, which indicated that different parameters influence the bullwhip effect to a great extent (Zhang, Cavusgil & Roath, 2003). Luong investigated the effect of autoregressive coefficient and lead time on the bullwhip effect in a supply chain and the demand forecast was performed with the first-order autoregressive model AR(1), he examined the upper limit of the bullwhip effect and demonstrated the upper limit mainly depends on the autoregressive coefficient (Luong, 2007), which is similar to the paper once used by Chen. Duc studied the effects of autoregressive coefficient, the moving average parameter and the lead time on the bullwhip effect with the inventory policy interpreted as first order mixed autoregressive-moving average model (Duc, Luong & Kim, 2008). These results are helpful because they are able to identify whether the bullwhip effect is infinite.

The bullwhip effect is influenced not by the forecasting methods, but by the inventory management. Here some researchers introduced different replenishment strategies to reduce the bullwhip effect. Bowersox studied a proper inventory management to alleviate the related problems of bullwhip effect, such as excessive inventory buildups, lost sales and customer dissatisfaction (Bowersox, 2003). Likewise, Chopra and Meindl (2004) proved that inventory is an important factor which can affect the performance of bullwhip effect in a supply chain significantly (Axsater, 2005). Axsater showed the decision problems in inventory systems with review policy of a two-stage supply chains and provided solutions to solve the problems existed in the inventory management of two stage supply chain (Chopra & Meindl, 2004).

Based on Lyapunov exponent, Ahmad MaKui compared the bullwhip effect in a supply chain with centralized information and decentralized information. Akhtar simulated an optimization model to mitigate the bullwhip effect in a two-echelon supply chain (Sheu, 2005). Qian Cao discussed that "guanxi" in china has a positive impact on business performance and can reduce the bullwhip effect to some extent (Akhtar & Yin-Zhen, 2014). Borut studied a simple three-stage supply chain through seasonal and unseasonal time, through a series of the customers demands data and discuss the influence of different levels on the bullwhip effect (Cao & Baker, 2014).

Efforts have been conducted related to quantifying the bullwhip effect based on the above theoretical. In this paper, we will investigate the impact of bullwhip effect in a nonlinear complexity dynamic system. Specifically, we want to offer some insights that how the bullwhip effect is affected with interacting product demands and what the entropy of the system acts like. Because, retailers are category-sensitive to products, their market sales depend on the products difference. The products of the two retailers are substitutes, so that products sales quantity in each supply chain depends on not only its own products categories, but also the other's products output. Thus, we apply a Cournot-Bertrand mixed duopoly model to investigate the influence of bullwhip effect when the complexity system is in stability, Bifurcation and chaos (Wang & Ma, 2014). As the system evolving from stable to chaos, the entropy increases correspondingly, and it will be growing formidable for the parameters of the system to be forecasted exactly. So, having an insight into the influence of bullwhip effect on systems of various degrees of disorder will be intriguing, which will reveal more details of both bullwhip effect and entropy complexities.

The paper is structured as follows. Section 2 presents a supply chain model that made up of two retailers, which both follow the AR (1) demand process and employ the order-up-to stock policy and use Cournot-Bertrand model to choose output and price as decision variables respectively. Section 3, the retailers ordering process and demand forecasting is obtained. Section 4 examines how the adjusted parameters affect the system in these situations (stable range, bifurcation n range, and chaos) of different degrees of entropy. In Section 5, we use the numerical analysis to study the influence of every parameter on the bullwhip effect and inventory variability, some managerial suggestions are discussed. Section 6, the concluding remarks are discussed.

2. MODEL FRAMEWORK

We consider a supply chain contains one supplier and two retailers. We assume that retailers sell generic products which have some difference but alternatives to customer who have a certain level of income. For example, because the retailers are competitors and incomplete information sharing between them, in order to obtain more profit, each retailer estimates customers demand and makes their product price according to other retailer situation. In this paper, we will discuss that one of retailers applies the Cournot model and takes the products output as decision variable, but the other applies the Bertrand model and takes the products price as decision variable.

We model our supply chain as Figure 1, the prerequisite are as follows:

1. Here retailers have no complete knowledge to the other's price information. We assume that players make decisions at the same time and adopted the quadratic function form as customer's utility function (Wang, Li, Du & Wang, 2017), which can be rewritten as in the following:

$$D\left(d_1, d_2\right) = d_1 + d_2 - \frac{1}{2}\left(d_1^2 + 2bd_1d_2 + d_2^2\right) + \varepsilon \tag{1}$$

2. Retailers have fixed consumer groups, the price and quality of their products can't be too high or low, because in real situation customers have the characteristics of meeting the level of consumption groups. The level of consumption can be interpreted as different incomes levels lead to consumption difference in a certain sequence. Generally, the higher income levels, the higher consumption level, and vice versa. As income level determines consumption level, different consumption levels have more different consumption contents. For example, high earners usually have family car, air conditioner and the ideal housing etc. But it is very hard for the low earning people who often have difficulties to have decent durable consumer goods. Thus, the consumer's budget will be constrained. One equation adopted in previous literatures is given Equation (2):

$$p_1d_1 + p_2d_2 = m \tag{2}$$

Here, P means sales price, d means sales quantity.

In the condition of the utility function's maximization problem, we can obtain the contra variance function of two retailers respectively:

$$\begin{cases} p_{1,t} = \sigma_1 - d_{1,t} - bd_{2,t} + \varepsilon_1 \\ p_{2,t} = \sigma_2 - d_{2,t} - bd_{1,t} + \varepsilon_2 \end{cases} \tag{3}$$

where, $\sigma_i (i = 1,2)$ means the market demand, $\varepsilon_i (i = 1,2)$ is an interference factor and represents the particularity of the product itself.

In the next section, we will describe the two retailers demand function given the condition obtained above.

2.1. Demand Functions

In practice, the structure of our linear demand function that transformed through the Equation (2), which is similar to other researches by previous studiers somewhat. Specifically, retailer 1 chooses the production demand as decision variable to influence the market supply, and retailer 2 chooses the production price as decision variable to interfere the market supply. For the other function adopted in this paper, we also apply linear function, which has been empirically tested by other studies.

For retailer 1:

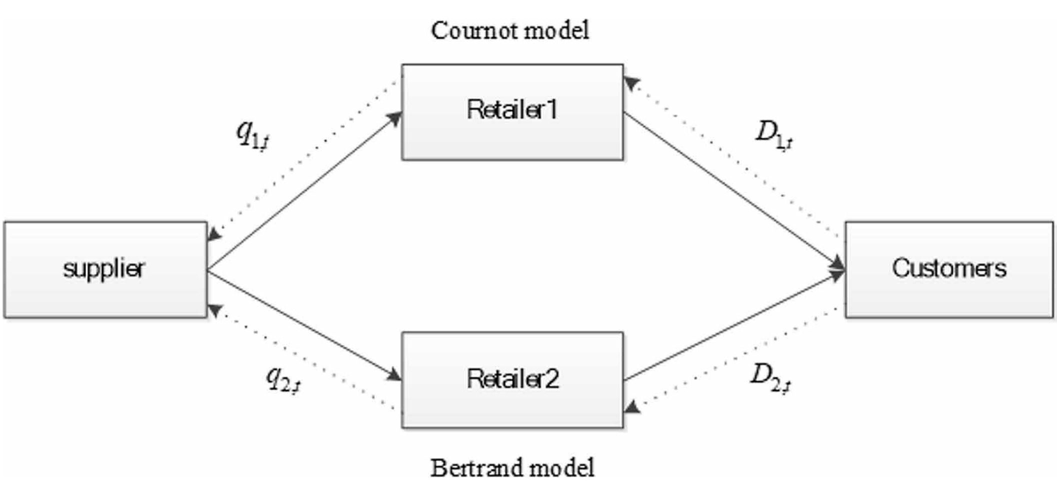
$$p_{1,t} = \sigma_1 - d_{1,t} + bp_{2,t} + b^2 d_{1,t} + \varepsilon_1 \tag{4}$$

For retailer 2:

$$d_{2,t} = \sigma_2 - bd_{1,t} - p_{2,t} + \varepsilon_2 \tag{5}$$

$b(0 < b < 1)$ represents production difference between two retailers, the more b, the smaller production difference between retailers. Which means customers can switch to others production

Figure 1. A two-stage supply chain



according the degree of substitute between retailers. Therefore this assumption can be found in real life. We will prove that in the rest of this paper.

As mentioned above, retailers adopt Cournot and Bertrand model respectively, the retailers' profit can be expressed as:

$$\begin{cases} \pi_{1,t} = d_{1,t}(\sigma_1 - d_{1,t} + bp_{2,t} + b^2 d_{1,t} + \varepsilon_1 - c_1) \\ \pi_{2,t} = (p_{2,t} - c_2)(\sigma_2 - bd_{1,t} - p_{2,t} + \varepsilon_2) \end{cases} \tag{6}$$

Here, $c_i(i = 1,2)$ means the product costs.

By derivation calculation, the retailers' marginal profit function is given in:

$$\begin{cases} \dfrac{\partial \pi_{1,t}}{\partial d_{1,t}} = \sigma_1 - 2d_{1,t} + bp_{2,t} + bd_{1,t} + 2b^2 d_{1,t} + \varepsilon_1 - c_1 \\ \dfrac{\partial \pi_{2,t}}{\partial p_{2,t}} = \sigma_2 + c_2 - bd_{1,t} - 2p_{2,t} + \varepsilon_2 \end{cases} \tag{7}$$

In practice, each retailer has full knowledge about information of itself, but it is unknown to the supplier and other retailer. Although the market information has very high commercial value, it is very hard to gain market strategy and information for downstream players. If they want to obtain, they must pay for higher price and more times. Thus, retailers make their decisions based on incompletely information. So we assume that the two retailers adopt bounded rationality prediction and the Cournot-Bertrand mixed nonlinear complexity dynamical system of retailers expectation can be described different equations:

$$\begin{cases} d_{1,t+1} = d_{1,t} + \alpha d_{1,t} \left(\sigma_1 - 2d_{1,t} + bp_{2,t} + bd_{1,t} + 2b^2 d_{1,t} + \varepsilon_1 - c_1 \right) \\ p_{2,t+1} = p_{2,t} + \beta p_{2,t} \left(\sigma_2 + c_2 - bd_{1,t} - 2p_{2,t} + \varepsilon_2 \right) \end{cases} \tag{8}$$

α, β represent the efficiency parameters of adjust intensity (output, price) for retailers' demand. Equation (8) depicts that the retailers can made the next period product demand and price accordingly according to previous marginal profit. If the previous marginal profit is positive, the following period the players will improve the product demand or price so as to gain more profit. If the previous period marginal profit is negative, the following period the players will reduce product demand and price so as to gain more profit as usual.

3. ORDERING PROCESS AND DEMAND FORECASTING

3.1. Ordering Process

Without loss of generality, the order-up-to inventory policy is one of the most commonly used inventory strategies in supply chain. It based on this hypothesis: the demand of downstream players in supply chain are stationary and the supply are infinite, purchase cost of products is constant and not considering the order cost, holding cost, and shortage cost. It also holds for real life cases, based on target inventory level $S_{i,t}$, under the current period t, the downstream players send orders to upstream players, when upstream players receive order, after a fixed period of time, will feedback to downstream players demand orders. Thus, we describe the inventory policy to express the net inventory level of the players as the following:

$$q_{i,t} = S_{i,t} - S_{i,t-1} + d_{i,t-1} \tag{9}$$

Here, $S_{i,t}$ represents the order-up-to level. It is a safety stock and must meet the lead-time demand so as to deal with the suddenly unexpected demand. Therefore, the order-up-to inventory level of each supply chain often update as follows:

$$S_{i,t} = \widehat{D_{i,t}} + = \widehat{\sigma_{i,t}} \tag{10}$$

Here, $\widehat{D_{i,t}}$ is the forecast order quantities during the lead-time and it depends on the forecasting method, $\widehat{\sigma_{i,t}}$ is the variance of lead-time demand forecast error, z is the normal z-score that can be determined by the service level of inventory policy. We can prove that $\widehat{\sigma_{i,t}}$ does not depend on t and has no influence on the bullwhip effect, which is proved by Luong (2008). So we can also rewrite Equation (9) as follows:

$$q_{i,t} = \widehat{D_{1,t-}}\widehat{D_{1,t-1}} + d_{1,t-1}. \tag{11}$$

3.2. Lead-Time Demand Forecasting

In the traditional statistics, the forecasting method of lead-time demand often use the linear times-series forecasting, including Moving-averages, Exponential Smoothing, Minimum mean square error and Autoregressive-moving average process. In fact, in real situation the upstream players want to choose forecasting technique to control the bullwhip effect on downstream orders. They usually adopt this forecast of the lead-time demand forecasting associated with the actual value D_{t-1} and estimated value $\widehat{D_{i,t-1}}$ of prior period demand. Thus, in this paper we assumes that the upstream player use the ES technique to forecast the lead-time demand of two retailers, and the prediction is influenced by single exponential smoothing parameter.

The Exponential Smoothing Forecasting is defined as follows:

$$\widehat{d_{1,t}} = \lambda d_{1,t-1} + \left(1-\lambda\right)\widehat{d_{1,t-1}} \tag{12}$$

where, $\lambda \in \left(0,1\right)$.

At the end of lead-time, retailers observe the customer demands. According to previous demand data, they make products plan and placed an order to its upstream supplier. This order has been placed at the end of period t and received in period $t + L$, where L is the lead time that is a fixed number. We make hypothesis the demand forecasts $d_{1,t}\, d_{1,t+1} \ldots\ldots d_{1,t+L-1}$ are equal at the beginning of period t. Thus, we can get:

$$\widehat{d_{1,t}} = \widehat{d_{1,t+1}} = \ldots = \widehat{d_{1,t+1}} = \overline{d_{1,t+L-1} = \lambda d_{1,t-1} + \left(1-\lambda\right)}\widehat{d_{1,t-1}} \tag{13}$$

After iterations, we have:

$$\widehat{D_{1,t}} = \sum_{i=0}^{L-1}\widehat{d_{1,t+1}} = L\widehat{d_{1,t}} = L\left(\lambda d_{1,t} + \left(1-\lambda\right)\widehat{d_{1,t}}\right) \tag{14}$$

Depending on the ES forecasting method, we can rewrite Equation (11) for retailer 1:

$$q_{1,t} = (\widehat{d_{1,t-}}\ \overline{d_{1,t-1})+ = \left(\widehat{s_{1,t-}}\ \widehat{s_{1,t-1}}\right) + d_{1,t-1}}$$

$$= L\left[\left(\lambda d_{1,t-1} + \left(1-\lambda\right)\widehat{d_{1,t-1}}\right) - \widehat{d_{t-1}}\right] + = \left(\widehat{s_{1,t-}}\ \widehat{s_{1,t-1}}\right) + d_{1,t-1}$$

$$= \left(1+\lambda L\right)d_{1,t-1} - \lambda L\widehat{d_{1,t-1}} + = \left(\widehat{s_{1,t-}}\ \widehat{s_{1,t-1}}\right) \tag{15}$$

For simplicity of Equation (15), we assume the safety factor ==0. Accordingly we have the following equation:

$$q_{1,t} = \left(1+\lambda L\right)d_{1,t} - \lambda L d_{1,t} \tag{16}$$

Therefore, we also have the lead-time forecasting quantity of retailer 2:

$$q_{2,t} = \left(1+\lambda L\right)d_{2,t} - \lambda L d_{2,t} \tag{17}$$

Furthermore, we have the following total quantity regarding the two retailers:

$$\begin{gathered} q_{2,t} = q_{1,t} + q_{2,t} \\ = \left(1+\lambda L\right)d_{1,t} - \lambda L\widehat{d_{1,t}} + \left(1+\lambda L\right)d_{2,t} - \lambda L\widehat{d_{2,t}} \end{gathered} \tag{18}$$

4. EQUILIBRIUM POINT AND LOCAL STABILITY

4.1. Equilibrium Analysis

In system Equation (8), we let:

$$d_{1,t+1} = d_{1,t}, p_{2,t+1} = p_{2,t}$$

$$\alpha d_{1,t}(\sigma_1 - 2d_{1,t} + bp_{2,t} + bd_{1,t} + 2b^2 d_{1,t} + \varepsilon_1 - c_1) = 0$$

$$\beta p_{2,t}(\sigma_2 + c_2 - bd_{1,t} - 2p_{2,t} + \varepsilon_2) = 0$$

We apply the method of Jacobian matrix to analysis the system (8):

$$J(E) = \begin{pmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{pmatrix}$$

where:

$$j_{11} = 1 + a(\sigma_1 - 4d_1 + 2bd_1 + 6b^2 d_1 + bp_2 + \varepsilon_1 - c_1)$$
$$j_{22} = 1 + \beta(\sigma_2 + c_2 - bd_1 - 4p_2 + \varepsilon_2)$$
$$j_{12} = abd_1$$
$$j_{21} = -\beta bp_2$$

We can get four fixed points. The fixed points $E_i (i = 1,2,3)$ are all boundary equilibrium point, and E_4 is the Nash Equilibrium:

$$E_1\left(0,0\right)\ E_2\left(0, \frac{\sigma_2 + c_2 + \varepsilon_2}{2}\right)\ E_3\left(\frac{\sigma_1 + \varepsilon_1 - c_1}{2 - b - 2b^2}, 0\right)\ E_4\left(d_1^*, p_2^*\right) \tag{19}$$

where:

$$d_1^* = \frac{2\sigma_1 - \sigma_2 b + c_2 b + \varepsilon_2 b + \varepsilon_1 - c_1}{3 - b^2}$$

$$p_2^* = \frac{3\sigma_2 + 3c_2 + 3\varepsilon_2 - 2c_2 b - 2\varepsilon_2 b^2 - 2\sigma_1 b - b\varepsilon_1 + bc_1}{6 - 2b^2} \tag{20}$$

In reality, even if the two retailers have competition, they must have their own sales price and fixed quantity production. Thus, in order to make the complexity system Equation (8) more meaningful we assume $\sigma_\iota > c_i$.

Proposition 1: E_1, E_2, E_3 are the boundary equilibrium point, which is instable.
Proof: Represented by the method of substitution, at fixed point E_1, we can get the Jacobi matrix of system as following:

$$J_1\left(d_1, p_2\right) = \begin{pmatrix} 1 + a\left(\sigma_1 + \varepsilon_1 - c_1\right) & 0 \\ 0 & 1 + \beta\left(\sigma_2 + c_2 + \varepsilon_2\right) \end{pmatrix} \tag{21}$$

and the Eigen value of matrix is:

$$\lambda_1 = 1 + a\left(\sigma_1 + \varepsilon_1 - c_1\right), \lambda_2 = 1 + \beta\left(\sigma_2 + c_2 + \varepsilon_2\right) \tag{22}$$

Here, we have $\lambda_1 > 0, \lambda_2 > 0$, so E_1 is unstable and not belong to equilibrium point.

Similarly, we also can get the Eigen value of Jacobi matrix at fixed point E_2. By substituting obtain the Jacobi matrix:

$$J_2\left(d_1, p_2\right) = \begin{pmatrix} 1 + a\left(\sigma_1 + \dfrac{b\left(\sigma_2 + c_2 + \varepsilon_2\right)}{2} + \varepsilon_1 - c_1\right) & 0 \\ -\beta b\dfrac{\sigma_2 + c_2 + \varepsilon_2}{2} & 1 - \beta\left(\sigma_2 + c_2 + \varepsilon\right) \end{pmatrix} \tag{23}$$

Also, the Eigenvalue of matrix as:

$$\lambda_1 = 1 + a\left(\sigma_1 + \frac{b\left(\sigma_2 + c_2 + \varepsilon_2\right)}{2} + \varepsilon_1 - c_1\right)$$

$$\lambda_2 = 1 - \beta(\sigma_2 + c_2 + \varepsilon) \tag{24}$$

We can get $\lambda_1 > 0$. So E_1 is also not an equilibrium point.

Also it can be proved is also unstable. By substituting obtain the Jacobi matrix we can get:

$$J_3(d_1, p_2) = \begin{pmatrix} 1 + a\left(\sigma_1 + \varepsilon_1\text{-}c_1 + \dfrac{b(\sigma_1 + \varepsilon_1\text{-}c_1)}{2\text{-}b\text{-}2b^2}\left(2b\text{-}6b^2\text{-}4\right)\right) & ab\dfrac{\sigma_1 + \varepsilon_1\text{-}c_1}{2\text{-}b\text{-}2b^2} \\ 0 & 1 + \beta\left(\sigma_2 + c_2 + \varepsilon_2 - b\dfrac{\sigma_1 + \varepsilon_1\text{-}c_1}{2\text{-}b\text{-}2b^2}\right) \end{pmatrix} \tag{25}$$

$$\lambda_1 = 1 + a\left(\sigma_1 + \varepsilon_1\text{-}c_1 + \frac{b(\sigma_1 + \varepsilon_1\text{-}c_1)}{2\text{-}b\text{-}2b^2}\left(2b\text{-}6b^2\text{-}4\right)\right)$$

$$\lambda_2 = 1 + \beta(\sigma_2 + c_2 + \varepsilon_2 - b\frac{\sigma_1 + \varepsilon_1\text{-}c_1}{2\text{-}b\text{-}2b^2}) \tag{26}$$

Because $b(0 < b < 1)$ We take the maximum and minimum values, can calculate $\lambda_1 > 0$ or $\lambda_2 > 0$ also an equilibrium point.

Proposition 1 suggests that if the market only has one retailer, there will be no competition, it's very likely to lead to monopoly, therefore, the customers will be have to buy this production. But this result will not continue too long under the condition: if this business can obtain higher profits, there will be other businessman to enter this industry, thus the competition is coming. In order to keep market competition steady, there must have Nash equilibrium range. Therefore we infer the below corollary regarding the competition.

So, in the condition of jury, if E_1 is the Nash equilibrium point, the following full and necessary conditions must be met:

$$\begin{cases} 1 + Tr(J(E^*)) + Det(J(E^*)) > 0 \\ 1 - Tr(J(E^*)) + Det(J(E^*)) > 0 \\ 1 - Det(J(E^*)) > 0 \end{cases} \tag{27}$$

If only satisfy any one of the Equation (22). The complexity system Equation (8) can lead to different bifurcation for two retailers simultaneously.

The managerial insight is that there always exists a Nash equilibrium point, which can make each retailers' sales price and demands spontaneously but no cooperation. But the premise is that both retailers' sale prices are retained in the customers' reservation price level. Specifically, in the level of consumption, if sale prices exceed the reservation price, customers will not buy, and these prices do not become tightly bound. In other words, it also has confirmed the Bertrand view that only two enterprises can also form the competition is right.

In what follows, we will discuss the stability, bifurcation, and chaos regional based on the products difference in nonlinear dynamic models.

When the system reaches the Nash equilibrium point, the two players will neither be able to get a higher profit, so the entropy of the system is relatively low when it's at a stable Nash equilibrium point.

4.2. Stability Analysis

We set the parameters for $\alpha = 0.3$, $\beta = 0.4$, $\sigma_1 = 0.5$, $\sigma_2 = 0.6$, $c_1 = 0.2$ $c_2 = 0.1$, $d = 0.3$, $d = 0.9$, $b = 0.8$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.08$. As depicted in Figure 2, it is very easy to see that the system's stable range (blue) of the Nash equilibrium are made up of coordinate axis and three section of smooth curves, which is not symmetrical so as to make the output parameters $\left(\alpha\right)$ adjustment range is wider than range of price parameters $\left(\beta\right)$. It can also be explained that the market is more sensitive to price adjustment and the entropy of the system will change relatively more rapidly when changing the price. Furthermore, when the products' difference (b) takes different values, the situation of retailers will change as Figure 3.

Towards this, we can have the following proposition:

Proposition 2: The greater of products difference, the fiercer of competition between retailers will be.

Obviously, if the products difference becomes larger, the stable range shape will change, which means obtaining a different pattern for the system to remain low entropy. The managerial insight is that the disadvantaged enterprise will face a situation that their products will be out of demand, because of sale price are same for the customers who are in the same consumption level and they will pay more attention to products difference. Therefore, the conditions for the system having a low degree of disorder will be different and the stable range will be changed. The more products difference changes, the more the stable range will change. So, both retailers continue games until the system reaches another new stable region.

Hence the extremely competitive will leads to each retailer changing his products difference constantly. Slowly, both retailers competing will move from the stable region into the bifurcation range, which means the system obtains a higher entropy.

Figure 2. The system's stable range of two retailers (d=0.3)

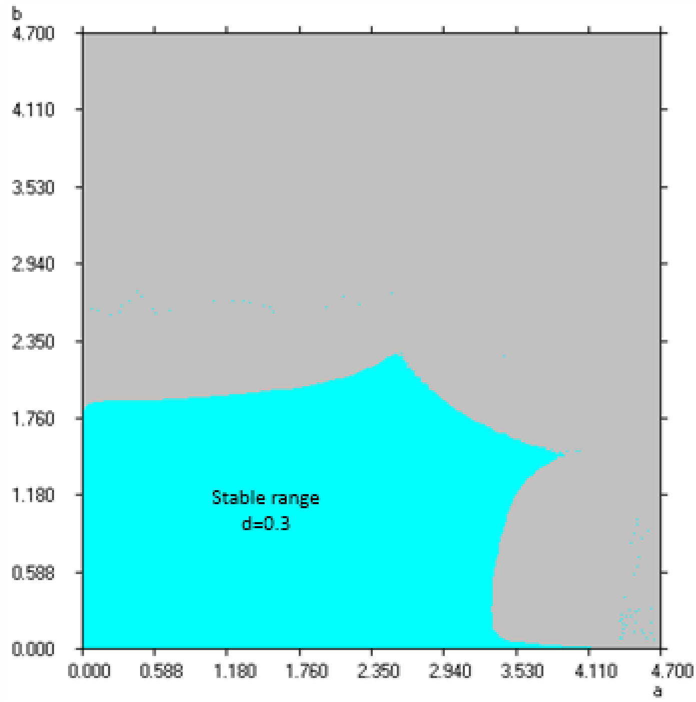
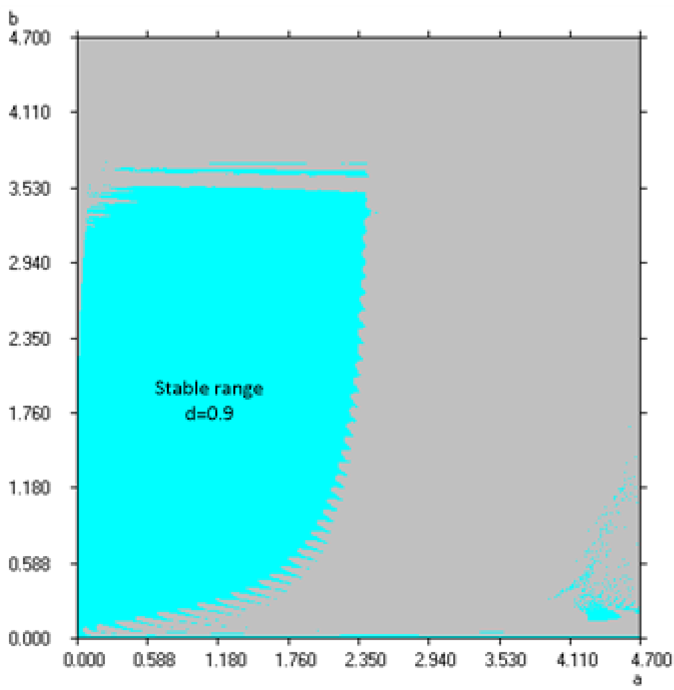


Figure 3. The system's stable range of two retailers (d=0.9)



4.3. Bifurcation Analysis

As Figure 4 when the parameter goes from the light green range passes to the blue, purple and yellow area to the grass green area, the system changes into chaos by flip. In this process, the district grows larger, which means the system does not operate orderly and has a growing trend to be out of control.

From the managers point of views, if the development of competition arrives in a certain extent, two possible conditions may occur: (1) Because of vicious competition between players, they tend to add production costs such as having unique advertising, packing, quality, and trademark, thus leading to a quantity or price beyond the ability of customer consumption, which will fix a level of customer reservation price. Gradually, the customers will no longer buy, and normally this situation will lead to one player evacuating the market, after a rise in the system occurs. (2) The manufacturers want to maintain price stability, but also hope to have unique advertising, packing, quality, and trademark, under this situation they have to reduce the production cost. In term of customers, with the same price but cannot buy the same quality, so the customers will turn to other products. In this situation, this would also lead to players leaving the market.

4.4. Largest Lyapunov Exponent

In previous studies, it has been proved that Irregular dynamic behavior of non- linear systems can be shown by Largest Lyapunov exponent (Wen-lin, Zhi-bin, Liu, Wang, 2014), and the Equation (8) also in compliance with the equations of Lyapunov exponent form:

$$x_{n+1} = f(x_n)$$

Thus, the Figure 5 and Figure 6 depict the parameter's change of Lyapunov exponent, however one side of this picture reveals that time plot of state of the system shows its curves being approximately stable region cyclical. With the system diverging and its entropy increasing, the system enters the period cyclical, bifurcation, until reach chaos regions. On the other hand, the largest Lyapunov exponent can be understood as normally phenomenon that an error is amplified within system. Also, it can be interpreted as the "bullwhip effect" in the supply chain.

5. BULLWHIP EFFECT ANALYSIS

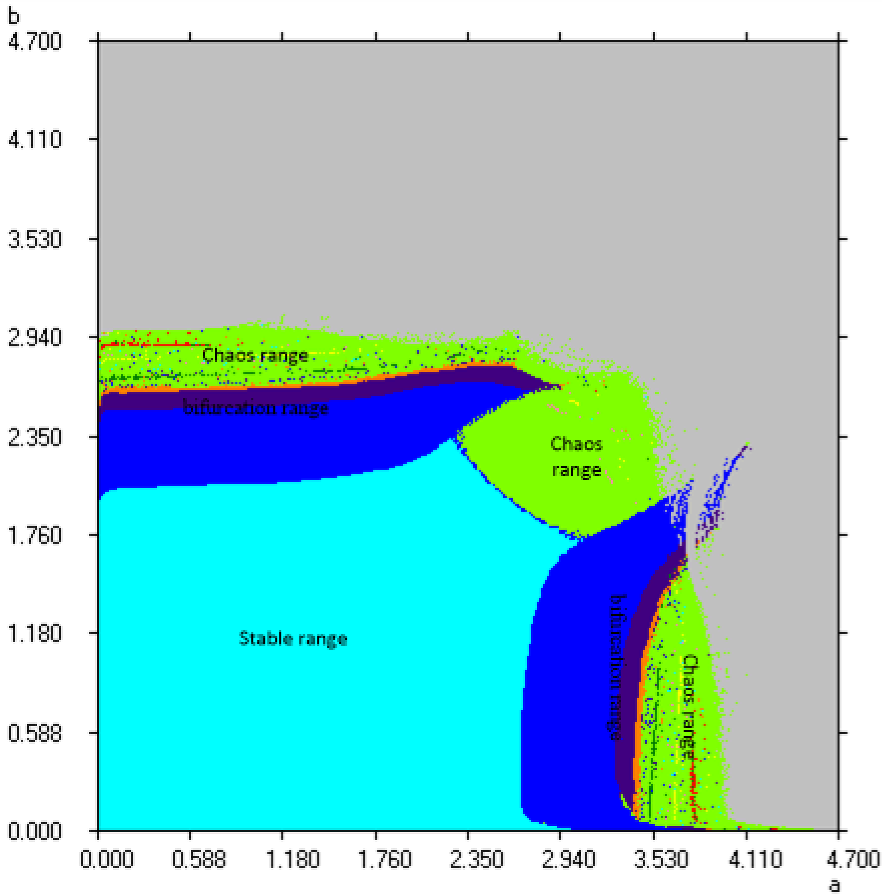
As we mentioned before, with the adjustment of parameters (α, β), we get the change scopes of the complexity system model. In this section, we would execute inventory and orders variance corresponding with nonlinear system to explain the bullwhip effect and inventory oscillation, which are also results of proposed model.

5.1. Bullwhip Effect Definition

Chen proposed the most common bullwhip-related measure metric. Disney and Towil proposed the metric measuring net stock instability respectively (Zhang & Wang, 2017). Thus, order variance ratio can be explained as the ratio of the orders rate variance meaning order value and the market demand variance meaning demand value. Similarly inventory variance ratio can be quantified as the fluctuations between the inventory variance to inventory value and demand variance to mean demand value. Specifically, the calculation formula of the investigated variables as follows:

$$OrderVarianceRation = (\sigma_q^2 / \mu_p) / (\sigma_d^2 / \mu_d) \tag{28}$$

Figure 4. The system's two-dimensional bifurcation diagram



$$InventoryVarianceRation = (\sigma_I^2 / \mu_I) / (\sigma_d^2 / \mu_d) \tag{29}$$

According to what we have analyzed before, the complex system diverges into the stability, bifurcation period and chaos as the parameters varies. So, we provide parameters (Table 1) three values to classify three typical behavioral patterns (stable range, bifurcation range and chaos range), which can characterize the distortion of demand under bullwhip effect and inventory oscillation. (L: the order lead time; z: safety factor; T: periodic length of the experimental data).

5.2. To Investigate the Impact of Fixed Parameters on the Bullwhip

When the complex system conforms to three typical behavioral patterns respectively, we calculate inventory oscillation and bullwhip effect of every period, concluding the comparative analysis.

As depicted in Figure 7. Firstly, when the system becomes stable, bullwhip effect suddenly sharply increases and achieves maximum value. But with the time going, the bullwhip effect falls gradually. In bifurcation period the bullwhip effect also shows a suddenly peak value, then fall down slowly. At last, when system is in chaos, the curves change the same as the above situation.

Figure 5. The maximum value of system's largest Lyapunov exponent (β = 1.1, α = 0~4.7)

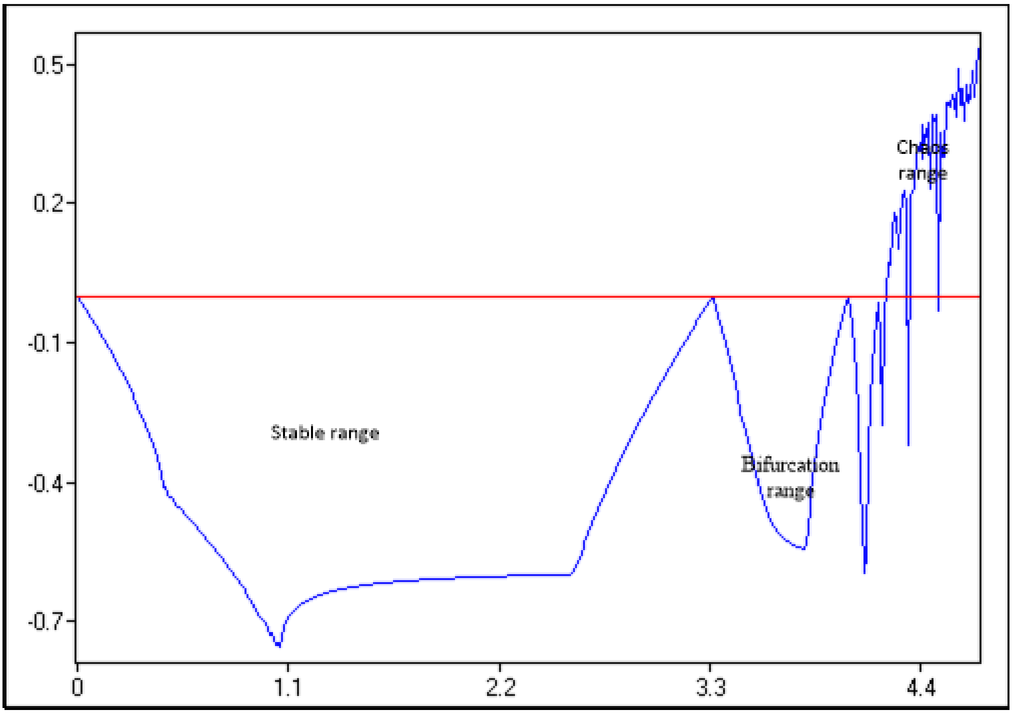


Figure 6. The maximum value of system's largest Lyapunov exponent (α = 1.5, β = 0~2.7)

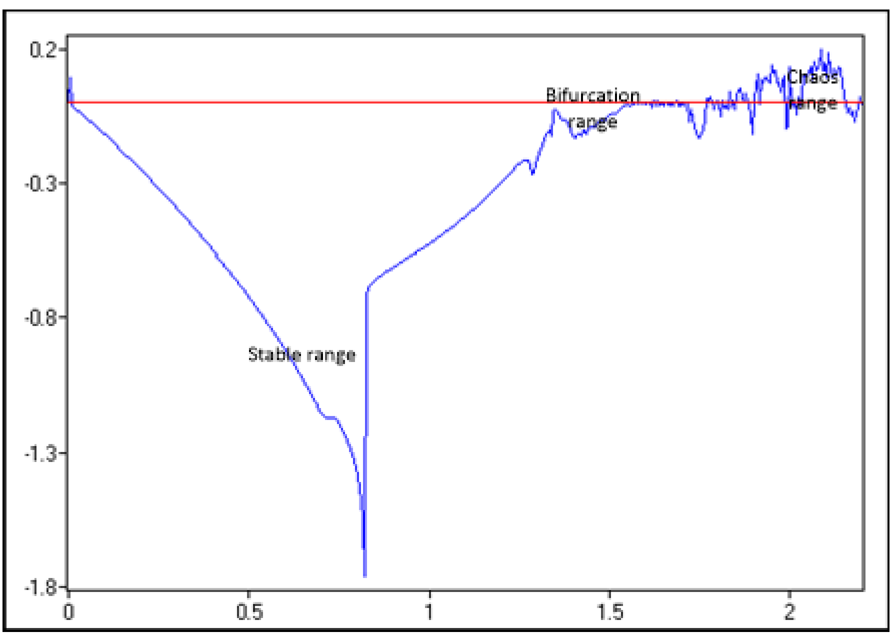


Table 1. Parameters set

Parameters	α	β	L	Z	λ	T
stable range	$\alpha = 2.8$	$\beta = 1.8$	$L = 3$	$Z = 0$	$\lambda = 0.8$	T = 55
bifurcation range	$\alpha = 3.9$	$\beta = 1.8$	$L = 3$	$Z = 0$	$\lambda = 0.8$	T = 55
Chaos range	$\alpha = 4.8$	$\beta = 1.8$	$L = 3$	$Z = 0$	$\lambda = 0.8$	T = 55

Comparing the three kinds of curves under three periods, obviously we can see that the bullwhip effect be in stable state is lighter than that in Bifurcation period and Chaos situation. Besides, the bullwhip effect in Bifurcation period is lighter than that in Chaos situation, under which it is very fierce.

The managerial insight of above analysis is that due to the lack of the market information and accumulated experience, high bullwhip effect easily occurs in the initial period. So, in the process of enterprise management they should accumulate more experience and study more, understanding the market demand clearly to make the bullwhip effect weakened gradually. When competition turns fervor between retailers, they are advised to reduce intensity of adjustment of output and price under bifurcation period and chaos situation (As shown in Figure 8, reduce the parameters $\alpha = 1.5, \beta = 0.8$). So, to ensure the bullwhip effect of the whole market can be controlled in stable range is of vital importance, else the system may easily obtain larger bullwhip effect as the competition turns fervor and the entropy increases.

As depicted in Figure 9. Firstly, when the system is stable, the inventory variance ration declines to the minimum value, gradually the inventory variance ration increases with time adding. When be in bifurcation period, inventory variance ration also declines to the minimum value, but after turning out, the inventory variance ration suddenly increases. So, this curve showed a rising trend at a certain speed. Thirdly when the system is in chaos, we can clearly see that the value of inventory variance ration is apparently higher than the value of system under stable period and chaos.

The managerial insight of above analysis is also that at the beginning of the retailing, the supplier lacks information about retailers, thus can easily have high bullwhip effect. But with the more information they acquire, the bullwhip effect weakens gradually. When competition turns whit-hot between retailers, supplier should also suggest them to reduce intensity of adjustment of output and price under bifurcation period and chaos (As Figure 10. To reduce the parameters $\alpha = 1.5, \beta = 0.8$). So, ensure the inventory variance ration of the whole market can be controlled in stable period.

5.3. To Investigate the Impact of Adjustment Parameters on Bullwhip Effect

We have calculated the sum of inventory oscillation and orders volatility of both retailers according to the 55 period data. Furthermore, we will examine that with the change trend of adjustment degree of the output, the change of order variance ration and inventory variance ration.

As depicted in Figure 11, when the system is in stable period, with the rising of output adjustment of retailer 1, the bullwhip effect of supplier first suddenly increases, then decreases and finally keeps in a certain numerical value. With the increase of parameter (α), the system falls into the bifurcation period, the bullwhip effect becomes larger with high speed during intervals of changes in small parameters change. Until the system arrive at chaos, the bullwhip effect just happens to vibrate violently and achieves maximum value.

Through comparison of three section curve we can find that when the complexity system has increasing entropy and evolves from stable state into bifurcation period and finally in chaos, on the whole the bullwhip effect changes greatly and keep in growing. From the management point of view, if take the difference of products to compare, for one of the retailers, in stable range there have an optimal value of output products parameter to make the bullwhip effect minimum in supply chain.

Figure 7. The order variance ration timing diagram (α=2.8, β=1.8)

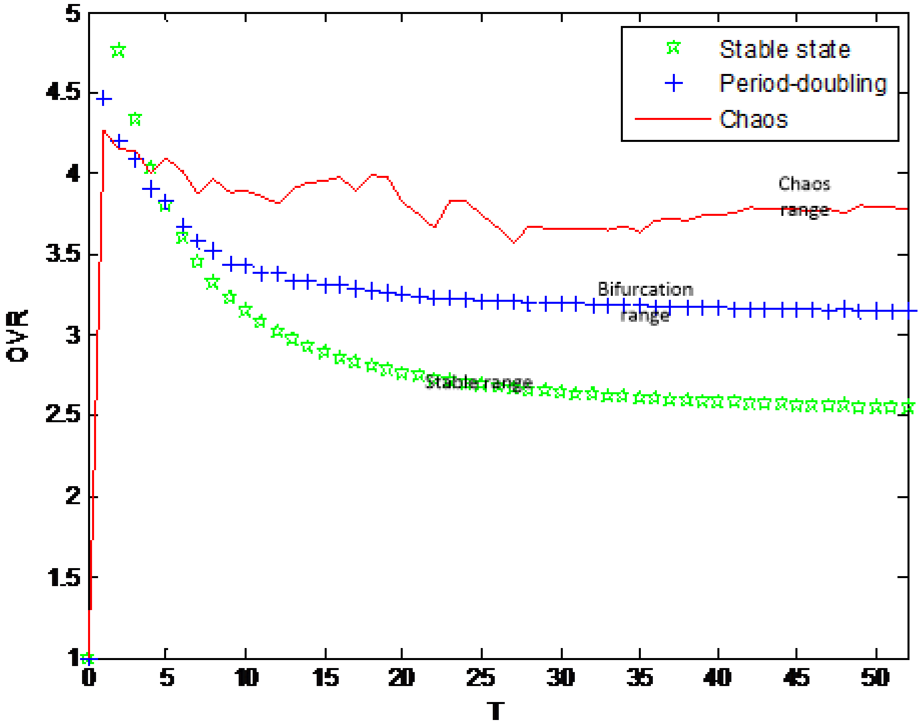


Figure 8. The order variance ration timing diagram (α=1.5, β=0.8)

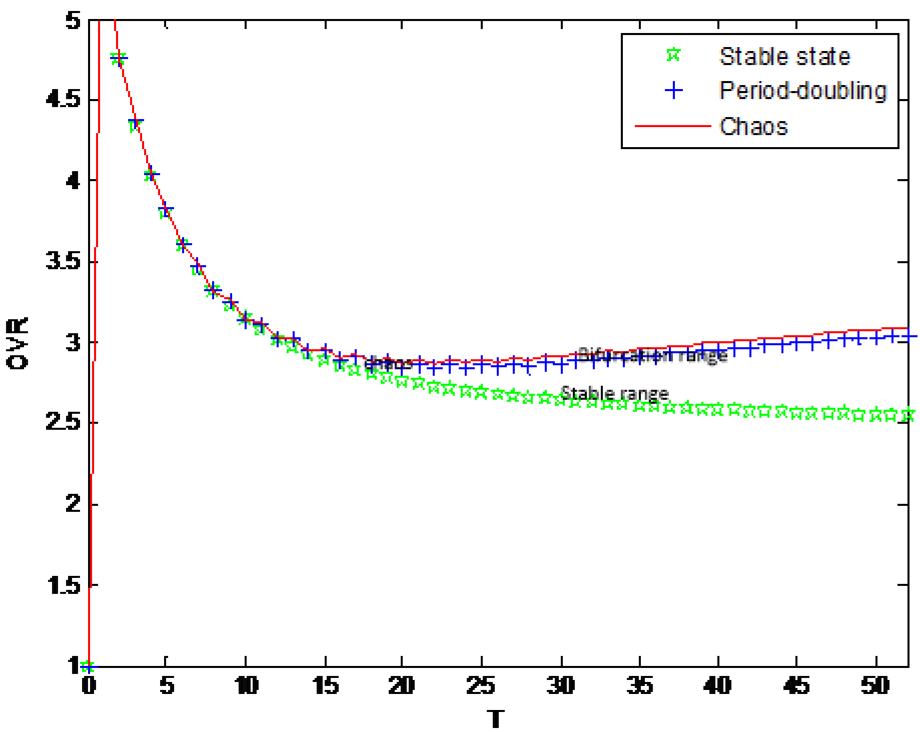
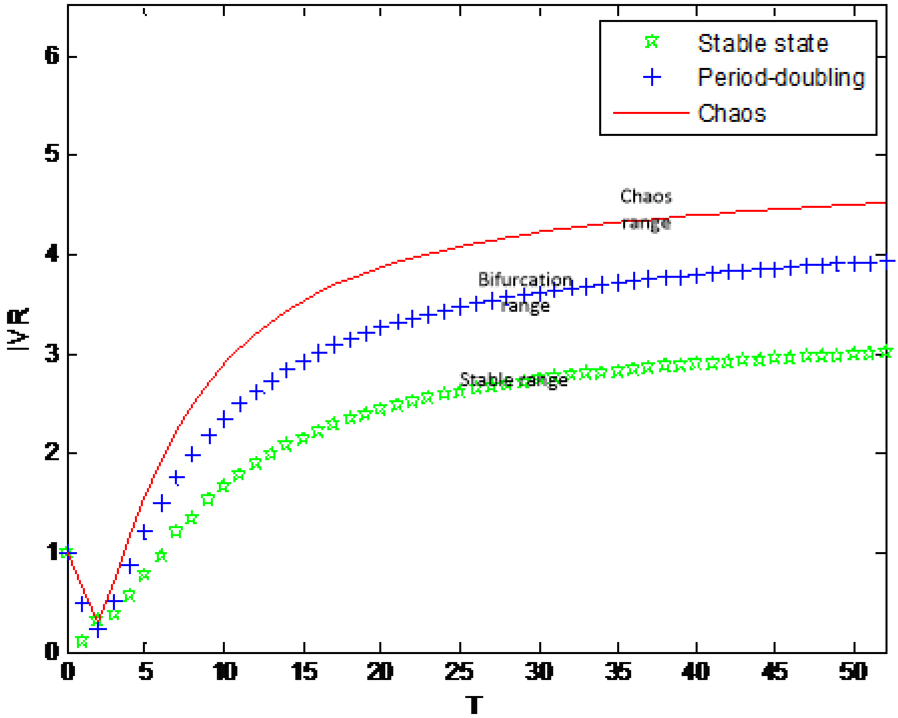


Figure 9. The inventory variance ration timing diagram (α=2.8, β=1.8)



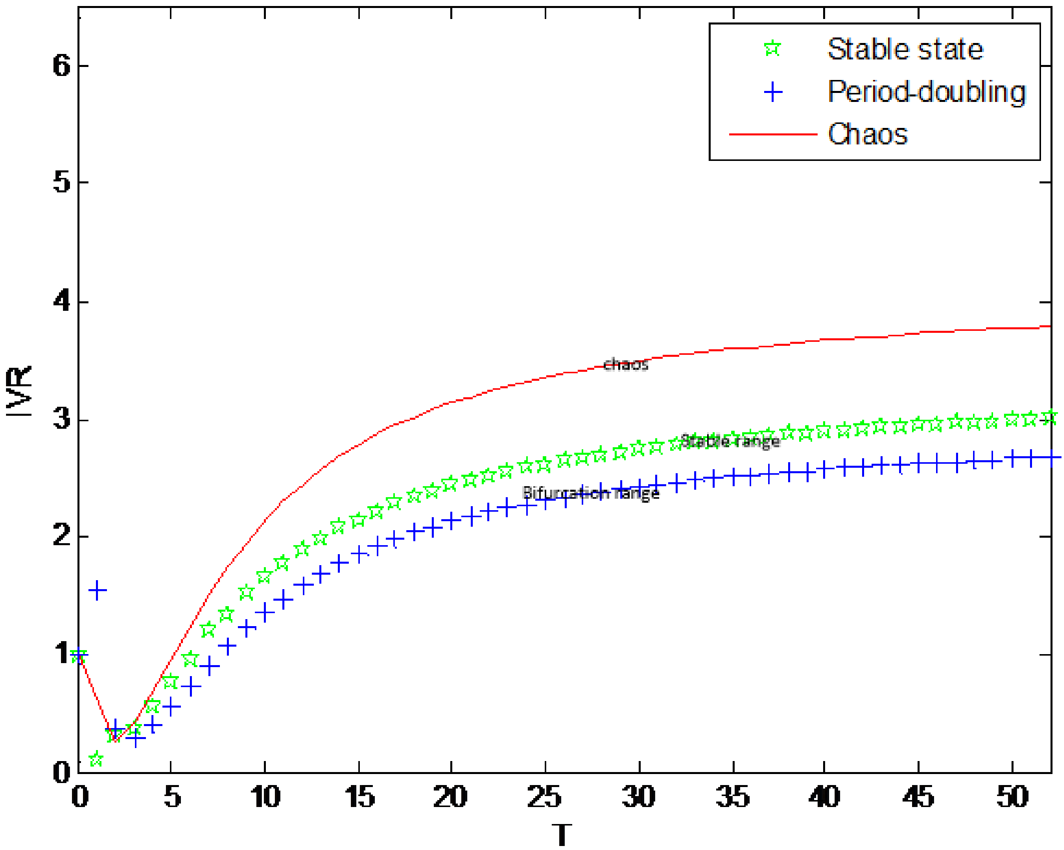
As depicted in Figure 12 and associated with Figure 11, initially the high bullwhip effect value corresponds to the high inventory variance ration. When the bullwhip effect keeps stable value, the inventory variance ration is also decreasing accordingly. As the system turning more disordered and decline into bifurcation period and chaos, the value of inventory variance ration will not increase but decrease instead, which is not the same as the bullwhip effect chart. Thus, for managers, if want to keep good inventory in products difference competition market, it is a good choice by doing adjustment of the degree of output products parameter.

6. CONCLUSION

This paper has presented a supply chain that consists of a supplier and two retailers. In which we adopt a Cournot-Bertrand mixed duopoly model that successfully combines a nonlinear complexity dynamic system with bullwhip effect. Both retailers apply Cournot-Bertrand model and choose output and price as decision variables. Considering two retailer competitors, we assume that they are bounded rational to decide output and price.

Since the bullwhip effect is one of the most important factors influencing a supply chain, many studies have already been developed on it. The difference between this research and the past researches is that the bullwhip effect and inventory is influenced in non-linear complexity system. First of all, we adjusted the parameters to investigate the actions of the non-linear system. Secondly, we focused on what extent the parameters can reach while the system is in stability, bifurcation range, and chaos, which means the system's with different states of entropy. At last, we analyzed the variance of bullwhip effect and inventory variance when they intermediate under three situations in question. The suggestions for management discussed in this paper can help managers adjust their management strategies in competitive market and avoid the cost waste.

Figure 10. The inventory variance ration timing diagram (α=1.5, β=0.8)



Besides, this paper without considering some important factors and we hope it could be extended in the following directions. Firstly, we consider only two retailers, but in real market, there are three or more retailers. Besides, it is of great value to figure out whether it is feasible for the retailers to use products difference as the main consider factor. Perhaps we could turn to the products price factor. Secondly, we used the ES forecasting method in this paper. We can also apply other forecasting methods, such as MA, AR(1), AMRA(1,1), etc., to investigate the change of bullwhip effect in Cournot-Bertrand complexity model. Another limitation of this research is that we fail to consider the disturbance factors influencing this system. Anyway, there are always unpredicted factors in reality, which worth more study in the future researches.

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Figure 11. The order variance with parameter (α) adjustment

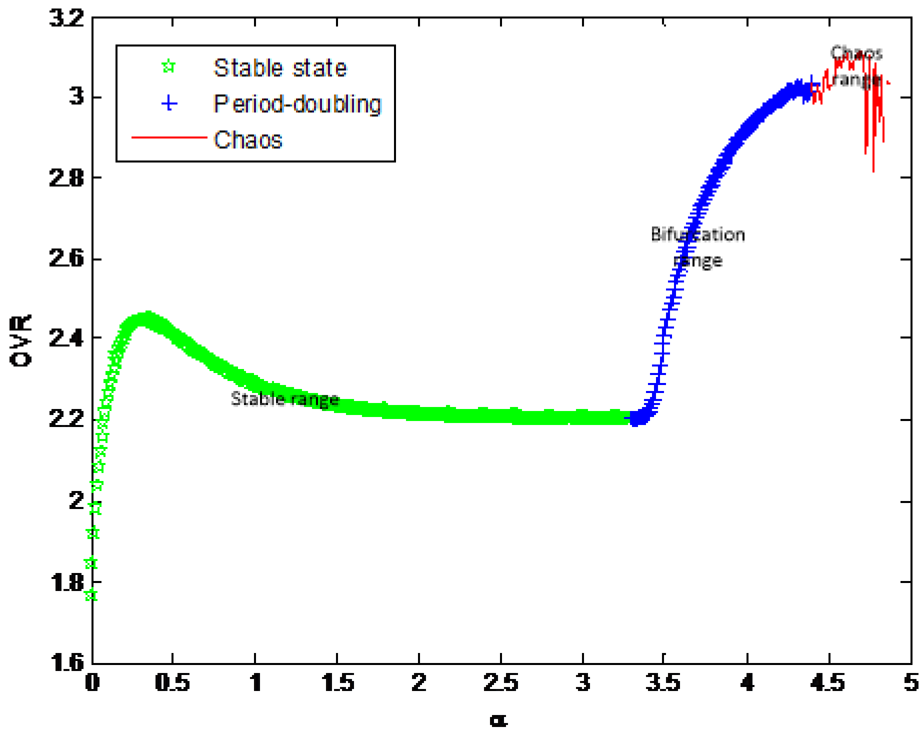
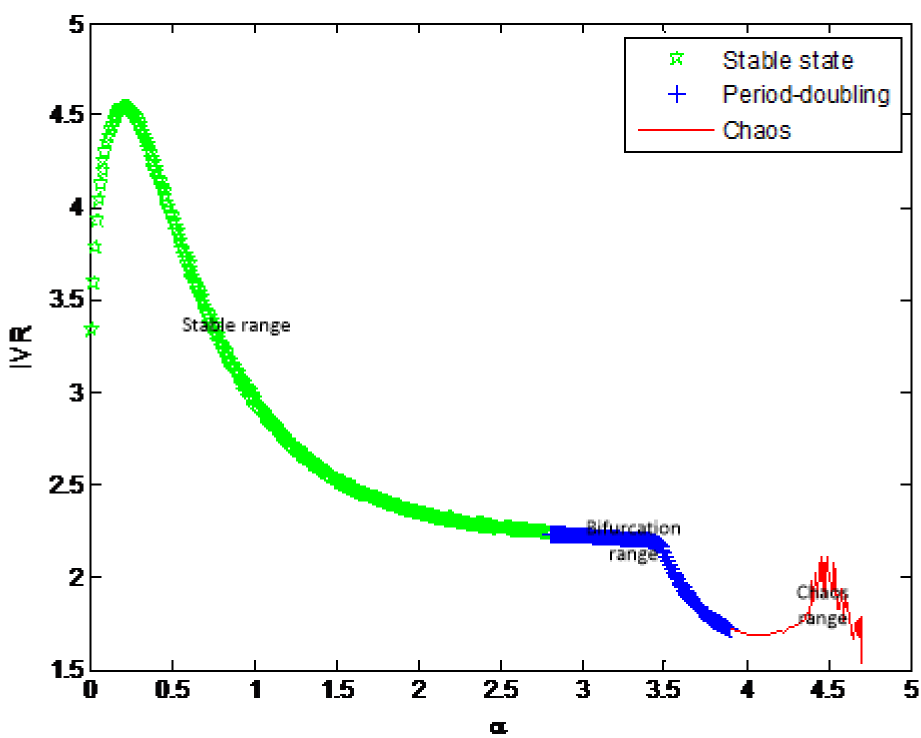


Figure 12. The inventory variance with parameter (α) adjustment



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