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The impact of corporate lifecycle on Fama–French three-factor model

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HIGHLIGHTS

- This paper develops a discrete-time asset pricing model under a framework of the partial equilibrium and analyzes how the firm lifecycle impacts on the relationship between two determinates and expected stock return.
- The negative impact of firm size on expected stock returns will weaken as corporate lifecycle processes.
- The positive impact of book-to-market ratio on expected stock returns is not changing over time.

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ABSTRACT

From the Fama–French three-factor model, the expected stock return is a negative function of market value of equity (i.e., firm size), and also positive of book-to-market ratio. In this paper, we develop a discrete-time asset pricing model under a framework of the partial equilibrium and analyze how the corporate lifecycle impacts on the relationship between them. The results show that as firms become mature, the negative impact of market value of equity, which reflects the relative importance of growth options, on expected stock returns will weaken. In contrast, the positive relationship between the book-to-market ratio and expected stock returns is not changing over time. The theoretical analysis is supported by the empirical results of A-share listed firms from 1998 to 2016 in China.

1. Introduction

On the basis of capital asset pricing model [1–4] point out that in addition to the market beta, book to market ratio and market value of equity (i.e., firm size) are also the two important determinates for the cross-section of expected return. By constructing a small-minus-big portfolio factor based on firm size and a high-minus-low portfolio factor based on book-to-market ratio, and combining a market portfolio factor, they proposed a well-known three-factor model [5]. However, the mechanism behind the relationship between them remains controversial, and two points of view are broadly discussed, such as risk-based rational pricing or behavioral-based mispricing.[1]

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1 For example, [6] pointed out that both the book-to-market ratio and the market value of equity (firm size) are related to the volatility of firm earnings. Therefore, the two determinates, especially the book-to-market ratio, reflect the financial distress risk. [7] argued that high book-to-market ratio stocks have higher expected stock return is due to the suboptimal behavior of the typical investor and not because of fundamentally risk. [8] argued that book-to-market ratio does not reflect risk but reflect investor's preferences for the "firm characteristic", investors prefer to hold growth stocks with good fundamentals, and not prefer to value stocks with poor fundamentals, resulting in high returns for high book-to-market ratio stocks.

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[9] decomposed firm assets into asset-in-place and growth options.[2] They established a discrete-time asset pricing model and pointed out that the dynamic exercising of growth options affects the risk and return of assets by changing the relative composition of assets. They also derived the theoretical relationship between expected return and book-to-market ratio and market value of equity. [10] constructed a dynamic general equilibrium production economy to explicitly link expected stock return to firm size and the book-to-market ratio. They pointed out that firm size and book-to-market ratio are correlated with the true conditional market beta and therefore appear to predict stock returns. [11] linked expected stock returns to firm size and book-to-market ratio by introducing operating leverage and finite growth opportunities. They pointed out that book-to-market effect is related to operating leverage, while firm size captures the residual importance of growth options relative to asset-in-place. In the neoclassical framework with rational expectations and competitive equilibrium, [12] demonstrated asset-in-place are much riskier than growth options, especially in bad times when the price of risk is high. Because high book-to-market stocks have more asset-in-place assets than those of low book-to-market stocks, thus investors require higher stock returns for compensation. Taken nonconvex adjustment costs and irreversibility of investment into consideration, [13] also provided a rational explanation for value premium.[3]

However, both the classical research of [9] and the various extensions of the above-mentioned research did not take the dynamic changes of the future growth opportunities and their uncertainties into consideration, thus it cannot reveal the dynamic changes of growth opportunities as well as the impact on the risk–return along with firm lifecycle. In fact, for a typical firm, the relative importance of growth options gradually declines over time, because when the firm is growing, the available growth opportunities of firms are decreasing, and asset-in-place is increasing due to the exercising of available growth opportunities. Further, since the exercising of growth opportunities convert the assets from option status with high systematic risk into underlying status with low systematic risk [14,15], the systematic risk of total assets decreases. More importantly, if the firm size reflects the relative importance of growth options, it can be expected that the impact of firm size on the expected stock return will also decline as the corporate lifecycle goes by.

Based on the model of [9], this paper expands the condition that the gradually decline of growth opportunities as corporate lifecycle progresses and studies how corporate lifecycle affects the relationship between firm size or book-to-market ratio and expected stock return theoretically. The paper also examines theoretical predictions by using a sample of Chinese listed firms during 1998 and 2016. The results show that the negative relationship between the market value of equity and the expected stock return will gradually weaken as firm become mature, and the positive relationship between book-to-market ratio and expected stock return is not change.

This paper contributes to two streams in the existing literature. First, we theoretically extend the study of the impact of book-to-market ratio and firm size on expected return from the perspective of corporate lifecycle, while other papers from the dynamic general equilibrium [10], fixed operating leverage and finite growth opportunities [11,16,17], competitive equilibrium [12], nonconvex adjustment costs [13]. Second, our paper supports for the first time a new possible mechanism about the negative relationship between firm size and expected stock return from the perspective of corporate lifecycle, although plenty of empirical research investigate the impact of firm size on the expected stock return for the Chinese stock market [18–23].

The remaining of the paper proceeds as follows: First, we construct a discrete-time asset pricing model by considering the growth opportunities decease over time and obtain the testable theoretical prediction; In Section 3, empirical design and empirical results are represented.

2. Model

Our model is related to Berk et al. [9] who developed an investment-based asset-pricing model under a framework of the partial equilibrium to analyze how firm's collection of projects determines its risk and expected return change over time. Our model is directly to analyze how the decreasing of growth options over firm lifecycle determines the changes of the explanatory power of two pricing factors, i.e. book-to-market ratio and firm size, on the risk premium of total assets.

2.1. Assumption

Following the framework of [9], we assume that a firm operates with an infinite horizon in discrete time, at each date $t \in \{0,1,2,\ldots\}$ a possible project becomes available. The key assumption of our paper which different from [9] is the project arrives in each date with a probability and the probability present a decrease function over time. This assumption is motivated by [24] and [25] which shows a decline trend of firm growth opportunities over time,[4] for any time k, we define $p(k)$ as the

[2] Asset-in-place are the projects which have already invested and can generate cash flows in the future, while growth options are the investment opportunities which have not yet been invested, this paper cross use of the term of growth options or growth opportunities .

[3] Value premium refers to the high book-to-market ratio stocks have higher expected stock return than those of low book-to-market ratio stocks.

[4] There are a plenty of papers supported the evidence that the growth opportunities will decrease over firm lifecycle [26,27]. [26] found that the newly listing firms (relatively young) have more growth opportunities than non-newly listing firms (relatively mature). [27] pointed out that young firms face relatively abundant investment opportunities with limited resources, whereas mature firms have higher profitability and fewer attractive investment opportunities. Moreover, By using Tobin Q as the proxy of growth opportunities and *RT/TA* or *RT/TE* as the corporate lifecycle proxies, we find there is a negative relation between Tobin Q and *RT/TA* or *RT/TE*, it predicts that the growth opportunities show a downward trend with corporate lifecycle.

kth project arrival probability with $p(k+1) < p(k)$ and $\lim_{k\to+\infty} p(k) = 0$, thus, the possible growth opportunity arrived at time k can be expressed as an indicative function

$$h(k) = \begin{cases} 1 & \text{probability is } p(k) \\ 0 & \text{probability is } 1 - p(k), \end{cases} \tag{1}$$

where $h(k) = 1$ and $h(k) = 0$ represent growth opportunity arrival and non-arrival, respectively.

At each date, if the growth opportunity arrives and the NPV (net present value) is positive, the firm need to pay a one-time investment cost immediately since the project cannot be postponed or preserved, and the investment required to undertake a project is denoted as I. After the investment, the firm will continue to receive cash flow from the next date. At date t, the cash flows from a project that was undertaken at date k $(k < t)$ is $C_k(t)$, which satisfies the following process

$$C_k(t) = I \exp\left[\overline{C} - \frac{1}{2}\sigma_k^2 + \sigma_k \varepsilon_k(t)\right], \tag{2}$$

where $\overline{C}$ and σ_k denote the mean and the variance of cash flows, respectively, and the innovations $\{\varepsilon_k(t), t > k\}_{i=1}^{\infty}$ are serially independent standard normal.

We take the process of pricing kernel to be exogenous which consistent with aggregated cash flows consequent to the investment decisions made at the firm level rather than consumption allocation, and this also gives us the tractability we need to focus on the dynamics for the relative risks of individual firms. Assume the exogenous pricing kernel $\{z_t\}_{t=1}^{\infty}$ satisfies the following process

$$z(t+1) = z(t) \exp\left[-r - \frac{1}{2}\sigma_z^2 - \sigma_z \zeta(t+1)\right], \tag{3}$$

where: $z(0) = 1$, r is the riskless interest rate, σ_z is the standard deviation of the pricing kernel and $\{\zeta(t)\}_{t=1}^{\infty}$ are serially independent standard normal.

For the project arrived at date k, "systematic risk" or the beta of cash flows from this project can be defined as

$$\beta_k \equiv \sigma_k \sigma_z \operatorname{cov}(\varepsilon_k(t), \zeta(t)), \tag{4}$$

For simplicity, we assume that the processes of cash flows from each project are independent and the beta of each project is randomly determined by a distribution $F(\beta)$.

2.2. Asset-in-place value

Asset-in-place value is the value of future cash flows of all projects that have been already invested. At time t, the ex-dividend value of the kth $(k \le t)$ invested project can be expressed as the present value of its future cash flows[5]

$$V_k(t) = \mathrm{E}_t\left[\sum_{s=t+1}^{\infty} \frac{z(s)}{z(t)} C_k(s)\right] = I e^{(\overline{C}-\beta_k)} \frac{1}{e^r - 1}, \tag{5}$$

where: $\frac{1}{e^r-1}$ is the value of a risk-free bond, where the payments on this risk-free bond depreciate at a constant rate 1-π. Appendix gives the derivation of Eq. (5) in detail. Eq. (5) indicates the lower the beta is, the higher the value of the project is.

Further, considering the project arrival probability at each date, we sum up all the projects which have arrived and been invested in and before date t and obtain the value of asset-in-place

$$\begin{aligned} V^{AIP}(t) &= \sum_{k=0}^{t} V_k(t)\, h(k) \\ &= \sum_{k=0}^{t} I e^{(\overline{C}-\beta_k)} \frac{1}{e^r - 1} h(k) \\ &= I \sum_{k=0}^{t} h(k) \frac{1}{e^r - 1} e^{\overline{C}} \sum_{k=0}^{t} \frac{e^{-\beta_k} h(k)}{\sum_{k=0}^{t} h(k)} \\ &= I e^{\overline{C}} \sum_{k=0}^{t} e^{-\beta_k} h(k) \frac{1}{e^r - 1}, \end{aligned} \tag{6}$$

5 In fact, the cash flows of any investment project may distinguish due to product market competition or product update. We can describe this by assuming the future cash flows have a fixed depreciation rate over time [28]. For simplicity, this paper assumes the depreciation rate is zero, if we also assume that the cash flows are depreciate at a certain probability, the relevant conclusions of this paper do not change substantially.

where: $h(k)$ is an indicative function of whether the kth growth opportunity is arrived or not, if arrived, $h(k)$ equal 1, or else $h(k)$ equals 0. From Eq. (6), we can see the value of asset-in-place is not only determinate by the arrival probability at each date, but also determinate by the beta of each project.

Note $n(t) \equiv \sum_{k=0}^{t} h(k)$, $b(t) \equiv n(t) I$ and $e^{-\beta(t)} \equiv \sum_{k=0}^{t} e^{-\beta_k} \frac{h(k)}{n(t)}$, Eq. (6) can be expressed as

$$V^{AIP}(t) = b(t) e^{\overline{C} - \beta(t)} \frac{1}{e^r - 1}, \tag{7}$$

where: $n(t)$ represents the number of projects that have arrived in and before date t and have been already invested; $b(t)$ is the cumulative investment amount of these invested projects, which is the book value of the asset-in-place; $\beta(t)$ summarize the average systematic risk of all the firm's asset-in-place. From Eq. (7) we can see the value of the asset-in-place is determinate by four parameters: projects cumulative investment $b(t)$, average systematic risk $\beta(t)$, mean cash flows ($\overline{C}$) and the risk-free rate (r). The larger the cumulative investment amount, the mean cash flows or the lower the average systematic risk, risk-free rate, the higher the value of asset-in-place.

2.3. Growth option value

At date t, growth option value is the discounted value of all the future growth opportunities. For a particular growth opportunity that arrived at date s, since the investment decision cannot be postponed, the value at date s is the maximum the NPV of the project or zero

$$\max\left[V_s(s, \beta_s) - I, 0\right], \tag{8}$$

Combine with Eq. (5), and take arrival probability into consideration, the value of growth opportunity at date t is

$$\begin{aligned} G_t(s, \beta_s) &= p(s) \mathrm{E}_t \left[\frac{z(s)}{z(t)} \max\left(V_s(s, \beta_s) - I, 0\right) \right] + [1 - p(s)] \cdot 0 \\ &= p(s) e^{-r(s-t)} \mathrm{E}_t \left[\max\left(I e^{(\overline{C} - \beta_s)} \frac{1}{e^r - 1} - I, 0 \right) \right], \end{aligned} \tag{9}$$

Further, since the betas of the underlying project of each growth opportunity at each date are from the same distribution F(β), with the expectation on β, we can finally get the growth opportunity that arrives at date s in date t is

$$G_t(s) = \mathrm{E}_{\beta_s}\left[G_t(s, \beta_s)\right] = p(s) e^{-r(s-t)} K, \tag{10}$$

where: $K \equiv \int_{\beta_s} \mathrm{E}_t \left[\max\left(I e^{(\overline{C} - \beta_s)} \frac{1}{e^r - 1} - I, 0 \right) \right] dF(\beta)$, denote the value of the growth opportunity at time s, we can see that the value of growth opportunity is decreasing with the arrival time.

Finally, by adding up the values of all growth opportunities after date t, the total value of growth options at date t would be

$$V^{GO}(t) = \sum_{s=t+1}^{\infty} G_t(s) = K \sum_{s=t+1}^{\infty} p(s) e^{-r(s-t)}, \tag{11}$$

Since $p(t+1) < p(t)$ and $\lim_{t \to +\infty} p(t) = 0$, we can see growth options value is a decreasing function with time t.[6]

Combine Eqs. (7) and (11), we can obtain the total value of assets at date t

$$\begin{aligned} P(t) &= V^{AIP}(t) + V^{GO}(t) \\ &= b(t) e^{\overline{C} - \beta(t)} \frac{1}{e^r - 1} + K \sum_{s=t+1}^{\infty} p(s) e^{-r(s-t)}. \end{aligned} \tag{12}$$

2.4. Expected return

This section is to derive the expression for the conditional expected return of total assets. By decomposing the conditional expected return into market value of equity and book-to-market ratio, we investigate how the impact of corporate lifecycle on the relationship between the market value of equity, the book-to-market ratio and conditional expected return. At the date t +1, the total value of the assets consists of three parts.

[6] Since $p(s) \leq 1$, then $V^{GO}(t) \leq K \sum_{s=t+1}^{\infty} e^{-r(s-t)} = \frac{K}{e^r - 1}$.

First, the conditional expectations of the value next period of existing projects which have been invested in and before date t

$$\begin{aligned} \mathrm{E}_t\left[\sum_{k=0}^{t} V_k\left(t+1\right)h\left(k\right)\right] &= \mathrm{E}_t\left[\sum_{k=0}^{t} Ie^{\left(\overline{C}-\beta_k\right)}\frac{1}{e^r-1}h\left(k\right)\right] \\ &= \mathrm{E}_t\left[I\sum_{k=0}^{t} e^{\left(\overline{C}-\beta_k\right)}h\left(k\right)\right]\frac{1}{e^r-1} \\ &= b\left(t\right)e^{\overline{C}-\beta(t)}\frac{1}{e^r-1}, \end{aligned} \tag{13}$$

Second, the conditional expectations of next period's cash flow of all the existing projects which have been invested in and before date t

$$\begin{aligned} &\mathrm{E}_t\left[\sum_{k=0}^{t} C_k\left(t+1\right)h\left(k\right)\right] \\ &= \mathrm{E}_t\left\{\sum_{k=0}^{t} I\exp\left[\overline{C}-\frac{1}{2}\sigma_k^2+\sigma_k\varepsilon_k\left(t+1\right)\right]h\left(k\right)\right\} \\ &= e^{\overline{C}}b\left(t\right), \end{aligned} \tag{14}$$

Third, the conditional expectations of the value of growth opportunities at $t+1$

$$\begin{aligned} &\mathrm{E}_t\left\{p\left(t+1\right)\max\left[V_{t+1}\left(t+1\right)-I,0\right]+V^{GO}\left(t+1\right)\right\} \\ &= \mathrm{E}_t\left\{\sum_{s=t+1}^{\infty} p\left(s\right)\frac{z\left(s\right)}{z\left(t+1\right)}\max(V_s\left(s\right)-I,0)\right\} \\ &= \sum_{s=t+1}^{\infty} p\left(s\right)e^{-r(s-t-1)}\int_{\beta}\mathrm{E}_t\left\{\max\left(Ie^{\left(\overline{C}-\beta_s\right)}\frac{\pi}{e^r-\pi}-I,0\right)\right\}dF\left(\beta\right) \\ &= \sum_{s=t+1}^{\infty} p\left(s\right)e^{-r(s-t-1)}K, \end{aligned} \tag{15}$$

The conditional expected return of total assets is defined as the sum of these three parts divided by the value of total assets $P(t)$

$$\begin{aligned} \mathrm{E}_t\left(1+R_{t+1}\right) &= \frac{b\left(t\right)e^{\overline{C}-\beta(t)}\frac{1}{e^r-1}+e^{\overline{C}}b\left(t\right)+\sum_{s=t+1}^{\infty}p\left(s\right)e^{-r(s-t-1)}K}{P\left(t\right)} \\ &= \frac{\left[P\left(t\right)-\sum_{s=t+1}^{\infty}p\left(s\right)e^{-r(s-t)}K\right]+e^{\overline{C}}b\left(t\right)+\sum_{s=t+1}^{\infty}p\left(s\right)e^{-r(s-t-1)}K}{P\left(t\right)} \\ &= 1+e^{\overline{C}}\frac{b\left(t\right)}{P\left(t\right)}+K\left[\left(e^r-1\right)\sum_{s=t+1}^{\infty}p\left(s\right)e^{-r(s-t)}\right]\frac{1}{P\left(t\right)}, \end{aligned} \tag{16}$$

From Eq. (16), we can see the second term $\frac{b(t)}{P(t)}$ and the third term $\frac{1}{P(t)}$ are related to book-to-market ratio and market value of equity, respectively. Our model is different from [9] in the third term, we find the coefficient of $\frac{1}{P(t)}$ is related to t, Therefore, the impact of the market value of equity on the expected return presents lifecycle characteristics. Next, we demonstrate the coefficient is a decreasing function with time t.

Define $F\left(t\right)=K\left[\left(e^r-1\right)\sum_{s=t+1}^{\infty}p\left(s\right)e^{-r(s-t)}\right]$ is the coefficient of $\frac{1}{P(t)}$, since $p\left(k\right)\le 1$,

$$F\left(t\right)=K\left[\left(e^r-1\right)\sum_{s=t+1}^{\infty}p\left(s\right)e^{-r(s-t)}\right]\le K\left(e^r-1\right)\sum_{s=t+1}^{\infty}e^{-r(s-t)}=K \tag{17}$$

Thus, for any t, $F(t)$ is coverage to K.

Then, we calculate the difference between $F(t)$ and $F(t-1)$,

$$\begin{aligned}\Delta F(t) &= F(t) - F(t-1)\\ &= K\left[\left(e^{r}-1\right)\sum_{s=t+1}^{\infty} p(s)\, e^{-r(s-t)}\right] - K\left[\left(e^{r}-1\right)\sum_{s=t}^{\infty} p(s)\, e^{-r(s-t+1)}\right]\\ &= K\left(e^{r}-1\right)\left[\sum_{s=t+1}^{\infty} p(s)\, e^{-r(s-t)} - e^{-r}\sum_{s=t+1}^{\infty} p(s)\, e^{-r(s-t)} - e^{-r}p(t)\right]\\ &= K\left(e^{r}-1\right)\left[\left(1-e^{-r}\right)\sum_{s=t+1}^{\infty} p(s)\, e^{-r(s-t)} - e^{-r}p(t)\right]\end{aligned} \tag{18}$$

For any $s > t$, since $p(s) < p(t)$, then $\Delta F(t) < K(e^{r}-1)\left[\left(1-e^{-r}\right)p(t)\sum_{s=t+1}^{\infty} e^{-r(s-t)} - e^{-r}p(t)\right] = 0$, therefore the coefficient of $\frac{1}{P(t)}$ decreases with t monotonically. This shows that the impact of the market value of equity on the expected return will gradually decreases as the corporate lifecycle goes by.

From Eq. (16), we can see the coefficient of $\frac{b(t)}{P(t)}$ is $e^{\overline{C}}$ and it does not change with time t. Therefore, the impact of book-to-market ratio on the expected return will not change as the firm's corporate lifecycle goes by. In fact, [9] and [10], among others, both point out that the book-to-market ratio reflects the systematic risk of asset-in-place. In our model assumption, because the average systematic risk of the projects at each date are all the same, thus, the systematic risk of the asset-in-place is unchanged, therefore the impact of book-to-market ratio on the expected return does not change over time.

3. Empirical design

3.1. Data source, sample and variable definition

We collect data of all Chinese A-share listed firms from the databases of CSMAR (China Stock Market and Accounting Research database) between 1998 and 2016. We exclude observation as follows. First, we exclude firms in the financial industry, since firm's leverage ratio as well as the type of risk is quite different from firms in the other industry [29]. Second, we exclude firm observations with "PT" (particular transfer) status, which are stocks under financial distress and lack of market liquidity. Third, we exclude firms with negative book value of equity and negative sales to omit distressed firms and firm–year observations with one or more missing value. To reduce the influence of outliers, we winsorize all variables at the 1st and 99th percentiles. Our final sample consists of 2416 firms, 216 months and 267,540 firm–month observations.

The dependent variable is the individual stock excess expected stock return, which is defined as the monthly return of cash dividend reinvestment minus the monthly risk-free interest rate from January to December in the next financial year. The monthly risk-free interest rate is the 3-month deposit interest rate divided by 3. The independent variables include *Beta*, firm size (ln(*Size*)), and book-to-market ratio (ln(*B/M*)), where beta is the capital asset pricing model beta estimated using three-year monthly return data prior to return measurement in month t, the regression coefficient is the individual stock beta. The firm size is measured by the natural logarithm of the total market value of equity at the end of each year. The book-to-market ratio is the book value of equity to the total market value of equity at the end of each year and is then taken natural logarithm.

For the corporate lifecycle variable, [27,30] use the ratio of earned equity to total common equity (*RT/TE*) and earned equity to total assets (*RT/TA*) to proxy corporate lifecycle, respectively.[7] The higher the *RE/TE* (*RE/TA*) is, the mature the firm is. The definitions of all variables are detailed in Table 1.

Table 2 reports summary statistics for our sample firm–year observations from 1998 to 2015. All variables used in the empirical research are defined in Table 1. The average *Beat* is appropriate 1 and the average *ROA* is 0.040. The average natural logarithm of market capitalization and natural logarithm of book-to-market ratio are 21.998 and −1.105, respectively. For the lifecycle proxies, the average RT/TE and RT/TA is 0.336 and the standard deviation is 0.200.

3.2. Fama–MacBeth regression result

We conduct Fama & MacBeth cross-section regression model at the individual stock level [31]. At each month t, we regress monthly excess stock return on beta, book-to-market ratio, market value of equity and other two interaction terms:

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7 According to the corporate lifecycle theory, when a company is young, its retained earnings are low due to abundant growth opportunities and less profit, thus, *RT/TA* or *RT/TE* is low. While, when a company is mature, its retained earnings are high due to less growth opportunities and high profit, thus, *RT/TA* or *RT/TE* is high. This means *RT/TE* and *RT/TE* will increase with firm age. According to the corporate lifecycle theory from dividend, mature firms are more likely to pay dividend because of lacking good growth opportunities. Our evidence shows that *RT/TE* and *RT/TE* are increasing with firm age and firms with higher *RT/TA* or *RT/TE* is more likely to pay dividends. Thus, these two proxies are proper to reflect corporate lifecycle in China.

Table 1
Definition and description of variables.

Variable	Symbol	Description
Excess expected stock return	*ER*	The monthly return of cash dividend reinvestment minus the risk-free interest rate for each month from January to December in the next financial year. The risk-free interest rate is the 3-month deposit interest rate divided by 3
Individual stock beta	*Beta*	Beta is computed as the slop coefficient of the regression of excess return on market excess return, both using the prior three-year monthly return
Market value of equity	ln(*Size*)	The natural logarithm of the total market value of equity at the end of each year
Book-to-market raito	ln(*B/M*)	Book value of equity to the total market value of equity at the end of each year, and then take natural logarithm
Investment	*CAPEX*	Capital expenditure divided by total assets.
Profitability	*ROA*	Net profits scaled by total assets.
Corporate lifecycle proxy	*RT/TE*	Retained earnings to book value of equity
	RT/TA	Retained earnings to total assets

Table 2
Summary statistics. This table reports the summary statistics of each variable and the correlation matrix of the main variables, including number of observations (*N*), mean (*Mean*), standard deviation (*SD*), minimum (*Min*), 25th percentile (*P*25), median (*P*50), 75th percentile (*P*75) and maximum (*Max*) of various firm characteristics, including the gross *Beta*, ln(*Size*), ln(*B/M*), *CAPEX*, *ROA*, *RT/TE*, *RT/TA* over the full sample period. See Table 1 for the detailed definitions of these variables. The sample period of other variables is from 1998 to 2015.

	N	*Mean*	*SD*	*Min*	*P25*	*P50*	*P75*	*Max*
Beta	236 123	1.001	0.251	0.368	0.842	1.003	1.158	1.724
ln(Size)	26 093	21.998	1.025	19.998	21.274	21.883	22.613	25.002
ln(B/M)	26 093	−1.105	0.708	−3.361	−1.540	−1.054	−0.607	0.329
CAPEX	26 093	0.059	0.058	0.000	0.016	0.041	0.082	0.275
ROA	26 093	0.034	0.058	−0.216	0.012	0.034	0.062	0.190
RT/TE	26 093	0.083	0.782	−5.648	0.111	0.225	0.346	0.707
RT/TA	26 093	0.093	0.201	−1.019	0.052	0.115	0.186	0.477

RT/TE×ln(*Size*), *RT/TE*×ln(*B/M*) or *RT/TA*×ln(*Size*), *RT/TA*×ln(*B/M*), The regression models are as following[8]:

$$\begin{aligned} ER_{i,t+1} &= \beta_1 + \beta_2 Beta_{i,t} + \beta_3 ln\,(Size)_{i,t} + \beta_4 ln\,(B/M)_{i,t} + \beta_5 \times RT/TE_{i,t} \\ &\quad + \beta_6 \times RT/TE_{i,t} \times ln\,(Size)_{i,t} + \beta_7 \times RT/TE_{i,t} \times ln\,(B/M)_{i,t} + \varepsilon_{i,t+1}, \end{aligned} \tag{19}$$

$$\begin{aligned} ER_{i,t+1} &= \beta_1 + \beta_2 Beta_{i,t} + \beta_3 ln\,(Size)_{i,t} + \beta_4 ln\,(B/M)_{i,t} + \beta_5 \times RT/TA_{i,t} \\ &\quad + \beta_6 \times RT/TA_{i,t} \times ln\,(Size)_{i,t} + \beta_7 \times RT/TA_{i,t} \times ln\,(B/M)_{i,t} + \varepsilon_{i,t+1}, \end{aligned} \tag{20}$$

where, $\hat{\beta}_j$ (j = 1,2,3, … ,7) is the regression coefficient. We estimate $\hat{\beta}_j$ as the average of the cross-section regression estimates and use the standard deviations of the cross-sectional regression estimate to generate the sampling errors for these estimates.

$$\overline{\beta}_j = \frac{1}{T}\sum_{t=1}^{T}\hat{\beta}_{ji}, \sigma_j = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{\beta}_{jt} - \overline{\beta}_j\right)^2}, t_j = \frac{\overline{\beta}_j}{\sigma_j/\sqrt{T}} \tag{21}$$

All the t-statistics (in parentheses) are computed using the Fama and MacBeth procedure and the Newey and West formula with a lag of 3.

Table 3 gives the Fama & MacBeth regression results. From the regression results of Model 1, we can see that the market value of equity is significantly negative, which means 1 unit increases of market value of equity will lead to a decrease of 0.005 units of expected stock return. While the interaction between the ratio of retained earnings to equity and market value of equity (*RT/TE*×ln(*Size*)) is significantly positive, which means the relation between the market value of equity and expected return is affecting by *RT/TE* and 1 unit increases of *RT/TE* will lead to a decrease of 0.005 units of the decease speed of expected return on the market value of equity. The regression results of Model 3 show that the market value of equity is significantly negative, while the interaction between the ratio of retained earnings to assets and market value of equity (*RT/TA*×ln(*Size*)) is significant positive. These results show that the negative relationship between market value of equity and expected return will weaken as firms become mature.

Further, both from model 1 and model 3, we can see the regression coefficient between the ratio of retained earnings to equity(assets) and book-to-market ratio is negative, but it is not significant. These results indicate that the corporate

[8] In order to rule out the effects of multicollinearity, we decentralize all variables.

Table 3

Fama–MacBeth cross-section regression results. This table reports Fama–MacBeth estimates from monthly cross-sectional regressions of individual stock returns on the *Beta*, ln(B/M), ln(*Size*), *CAPEX* and *ROA*, the interaction term between ln(*Size*) and firm lifecycle proxy (*RETE* or *RETA*), the interaction term between ln(B/M) and lifecycle proxy (*RETE* or *RETA*). The mean of the monthly estimate of the coefficients are reported. The variables of interest are the interaction terms. The sample period is from 1998 to 2016. All the *t*-statistics (in parentheses) are computed using the **Fama and MacBeth** procedure and the **Newey and West** formula with a lag of 3. The significance levels 1%, 5%, and 10% are denoted by ***, **, and *, respectively.

	Constant	Beta	ln(B/M)	ln(Size)	CAPEX	ROA	RT/TE	RT/TE × ln(Size)	RT/TE × ln(B/M)
Model 1	0.110***	0.016*	0.004**	−0.005***			0.002	0.005***	0.000
	(3.381)	(1.689)	(2.288)	(−3.404)			(1.162)	(5.349)	(−0.141)
Model 2	0.125***	0.017*	0.005***	−0.006***	−0.006	0.056***	−0.001	0.004***	−0.001
	(3.959)	(1.781)	(2.831)	(−4.025)	(−0.820)	(4.408)	(−0.625)	(4.992)	(−1.341)
		Beta	ln(B/M)	ln(Size)	CAPEX	ROA	RT/TA	RT/TA × ln(Size)	RT/TA × ln(BM)
Model 3	0.106***	0.016*	0.004**	−0.005***			0.004	0.015***	0.000
	(3.202)	(1.687)	(2.460)	(−3.213)			(0.817)	(4.784)	(−0.029)
Model 4	0.120***	0.016*	0.005***	−0.005***	−0.006	0.056***	−0.006	0.013***	−0.001
	(3.742)	(1.753)	(2.826)	(−3.786)	(−0.792)	(4.325)	(−1.399)	(4.616)	(−0.294)

Note: Numbers in the parentheses are t-values.
*Denote significance at the 10% level.
**Denote significance at the 5% level.
***Denote significance at the 1% level.

lifecycle proxies have no impact on the positive relationship between book-to-market ratio and expected return. Therefore, the theoretical prediction of Eq. (16) is well verified.

Fama–French five-factor model has been widely used in practice. The Fama–French five-factor model contain the other two factors, investment factor and profitability factor, we add two variables into three-factor model, and find the results are still robust for model 2 and model 4.

This table reports Fama–MacBeth estimates from monthly cross-sectional regressions of individual stock returns on the *Beta*, ln(B/M), ln(*Size*), *CAPEX* and *ROA*, the interaction term between ln(*Size*) and firm lifecycle proxy (*RETE* or *RETA*), the interaction term between ln(B/M) and lifecycle proxy (*RETE* or *RETA*). The mean of the monthly estimate of the coefficients are reported. The variables of interest are the interaction terms. The sample period is from 1998 to 2016. All the *t*-statistics (in parentheses) are computed using the **Fama and MacBeth** procedure and the **Newey and West** formula with a lag of 3. The significance levels 1%, 5%, and 10% are denoted by***, **, and*, respectively.

4. Conclusion

In this paper, we study the impact of corporate lifecycle on the relationship between expected stock return and two determinants, i.e., market value of equity and book-to-market ratio. In our model, the arrival possibility of future growth opportunities gradually declines over time, which extends the framework of [9]. The analytical results show that the negative effect of market value of equity on the expected stock returns is weakened as firm becomes mature, while the positive effect of book-to-market ratio on the expected stock return is not changed as firm becomes mature. The results are supported by the empirical evidence of the A-share listed firms in Shanghai and Shenzhen of China in 1998–2016.

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Appendix. Derivation for the value of the *k*th invested project in Eq. (5)

At date t, the market value of the kth invested project ($k \leqslant t$) is the sum of all discounted values of future cash flows since $t+1$

$$V_k(t) = \mathrm{E}_t\left[\sum_{s=t+1}^{\infty} \frac{z(s)}{z(t)} C_k(s)\right] \tag{A.1}$$

For the cash flow at date $t+1$, we have

$$\begin{aligned} &\mathrm{E}_t\left[\frac{z(t+1)}{z(t)} C_k(t+1)\right] \\ &= I\mathrm{E}_t\left\{\exp\left[-r - \frac{1}{2}\sigma_z^2 - \sigma_z \zeta(t+1) + \overline{C} - \frac{1}{2}\sigma_k^2 + \sigma_k \varepsilon_k(t+1)\right]\right\} \\ &= I e^{(\overline{C}-\beta_k)} e^{-r} \end{aligned} \tag{A.2}$$

and the discounted value of cash flow at date $t+2$ is

$$\begin{aligned}
&\mathrm{E}_t\left[\frac{z(t+2)}{z(t)}C_k(t+2)\,\chi_k(t+2)\right]\\
&=\mathrm{E}_t\left\{\frac{z(t+1)}{z(t)}\mathrm{E}_{t+1}\left[\frac{z(t+2)}{z(t+1)}C_k(t+2)\right]\right\}\\
&=I\mathrm{E}_t\left\{\frac{z(t+1)}{z(t)}\exp\left(\overline{C}-r-\beta_k\right)\right\}\\
&=Ie^{\left(\overline{C}-\beta_k\right)}e^{-2r}
\end{aligned} \tag{A.3}$$

Similarly, the discounted value of cash flow from date $t+\tau$ can be generally expressed as

$$\mathrm{E}_t\left[\frac{z(t+\tau)}{z(t)}C_k(t+\tau)\right]=Ie^{\left(\overline{C}-\beta_k\right)}e^{-\tau r} \tag{A.4}$$

Finally, by summing up all the discounted values of future cash flows, at date t, the value of the kth invested project expressed by Eq. (5) would be

$$\begin{aligned}
V_k(t)&=\mathrm{E}_t\left[\sum_{s=t+1}^{\infty}\frac{z(s)}{z(t)}C_k(s)\,\chi_k(s)\right]=\sum_{s=t+1}^{\infty}Ie^{\left(\overline{C}-\beta_k\right)}e^{-(s-t)r}\\
&=Ie^{\left(\overline{C}-\beta_k\right)}\frac{e^{-r}}{1-e^{-r}}\\
&=Ie^{\left(\overline{C}-\beta_k\right)}\frac{1}{e^{r}-1}.
\end{aligned} \tag{A.5}$$

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