Two-Dimensional Numerical Model for Urban Drainage System

Sudarshan Patowary and Arup K. Sarma

Abstract Modeling of urban drainage system is carried out for understanding and predicting the flow behavior in the drain, so that an adequate design of the drainage system can be achieved. However, many a time urban drainage models are developed without considering some important aspects like erosion and deposition processes in the drain. Though the erosion process can be neglected in the lined canal, deposition process has a major role in the urban drainage system. Therefore, study of sediment deposition in the urban drainage system is an essential aspect for minimization of artificial flood in the urban area. Sediment deposition in the drain depends upon various factors like particle size, settling velocity, critical shear stress, and bed shear stress. Therefore, in this paper an attempt has been made to develop a numerical model, which can identify the critical section of the urban drain where sediment starts to deposit. Two-dimensional continuity and momentum equations of unsteady free surface flow are solved by second order finite difference implicit scheme. To find the settling velocity of the particle Newton's law and Stock's law are used. After calculating the settling velocity of the particle, the position where the particle starts to settle down is determined. Critical sections of the drain from sediment deposition point of view are identified by comparing bed shear stress and critical shear stress computed by the Shields' equation for incipient motion. No deposition occurs if bed shear stress is greater than critical shear stress. Again, result obtained from the sensitivity analysis of average particles size has shown that with the increases of sediment size, problem of sediment deposition aggravates.

Keywords Settling velocity · Critical section · Unsteady flow · Critical shear stress

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1 Introduction

With the development of efficient numerical algorithm and the advent of high speed digital computer, solution of unsteady one-dimensional and two-dimensional hydrodynamic equation is becoming quite common for addressing various unsteady flow problem. Solution of unsteady flow in river is quite common. Though the basic equations for developing river model and urban drainage model are same but the river model cannot be applied directly to urban drainage model. In this paper, an attempt has been made to study the unsteady flow pattern considering some important aspect like sediment deposition in the urban drainage system. Keskin et al. (1997) developed a one-dimensional flood routing model using Saint-Venant equation. Hashemi et al. (2007) developed unsteady flow model and they had used differential quadrature method for simulation of the unsteady flow. Zhang and Shen (2007) developed steady and unsteady flow model for channel networks. Fang et al. (2008) developed one-dimensional numerical simulation of nonuniform sediment transport under unsteady flows using Saint-Venant equation. To minimize the artificial flood problem, critical section of the drain where the sediment starts to deposit is determined and if we can remove the sediment from that location, then it is possible to abatement the flood problem in urban area. Critical section of the drain depends upon various factors like particle size, settling velocity, critical shear stress, bed shear stress, and incipient motion of the particle. Sheilds (1936) had been the pioneer to establish a functional relationship between critical shear stress and boundary Reynolds number. Shields diagram has extensively used for determination of incipient conditions for sediment movement problems. Beheshti and Ataie-Ashtiani (2008) compared most widely used threshold curves presented to Shields diagram, a method based on the concept of probability of sediment movement and empirical methods based on movability number for predicting incipient motion. If the bed shear stress is greater than critical shear stress then incipient motion occurs. Many researchers developed some empirical formula to find the critical shear stress (Lane 1995; Highway Research Board 1970).

2 Mathematical Model

The proposed model has two components: (1) 2-D unsteady flow model for free surface flow computation, (2) Identify the critical section of the urban drain where sediment starts to deposit.

2.1 2-D Unsteady Flow Model

Two-dimensional continuity equation and fully dynamic form of the momentum equation in non-conservation form has been used as a governing equation for solution of unsteady flow in a hypothetical non-prismatic rectangular river reach considered in this study. The continuity and momentum equation are given in Eq. (1)

$$U_t + E_x + F_y + S = 0, \tag{1}$$

where

$$s_{f_y} = \frac{n^2 v\sqrt{u^2+v^2}}{1.49h^{1.33}} \quad E = \begin{pmatrix} uh \\ u^2h + \frac{1}{2}gh^2 \\ uvh \end{pmatrix} \quad F = \begin{pmatrix} vh \\ uvh \\ v^2h + \frac{1}{2}gh^2 \end{pmatrix}$$

$$S = \begin{pmatrix} 0 \\ -gh\left(S_{o_x} - S_{f_x}\right) \\ -gh\left(S_{o_y} - S_{f_y}\right) \end{pmatrix} \quad s_{f_x} = \frac{n^2 u\sqrt{u^2+v^2}}{1.49h^{1.33}} \quad s_{f_y} = \frac{n^2 v\sqrt{u^2+v^2}}{1.49h^{1.33}},$$

where h is the water depth, n is the manning's coefficient, u is velocity in the x direction, v is velocity in y direction, g is acceleration due to gravity, S_{o_x} and S_{o_y} are bed slope in x and y direction, s_{f_x} and s_{f_y} are friction slope in x and y direction.

2.2 Sediment Deposition Model

Sediment deposition in drain depends upon various factors like particle size, settling velocity, critical shear stress and bed shear stress. To find the settling velocity of the particle Newton's law and Stock's law are used.

2.2.1 Newton's Law for Settling Velocity

Newton's law of settling velocity is given by Eq. (2)

$$v_s = \sqrt{\frac{4g}{3c_D}(s_s - 1)d}, \tag{2}$$

where v_s is settling velocity of the particle, s_s is specific gravity of the particle, c_D is the drag coefficient and d is the diameter of the particle.

Again drag coefficient c_D is related to Reynolds number R by following observational relationships.

For R between 0.5 and 10^4

$$c_D = \frac{24}{R} + \frac{3}{\sqrt{R}} + 0.34$$

For high Reynolds number ($R > 10^3$–10^4) $c_D = 0.4$.

For low Reynolds number $c_D = \frac{24}{R}$.

2.2.2 Stoke's Law

If the diameter of the particle less than 0.1 mm, involving value of R less than 1, Stoke's law is applied for finding the settling velocity of the particle.

$$v_s = \frac{g}{18\upsilon}(s_s - 1)d^2, \tag{3}$$

where v_s is settling velocity of the particle, s_s is specific gravity of the particle, d is the diameter of the particle, υ kinematic viscosity of water in centistokes.

After calculating the settling velocity of the particle, critical section of the drain where particle starts to deposit is calculated and then at that particular section critical shear stress and bed shear stress is determined.

Shields (1936) established a functional relationship based on experimental data

$$\frac{\tau_c}{(\gamma_s - \gamma)d} = F\left(\frac{U_{*c}d}{\upsilon}\right), \tag{4}$$

where $U_{*c} = \left(\frac{\tau_c}{\rho}\right)^{1/2}$ is the critical friction velocity. The left hand side of Eq. (4) is the dimensionless critical shear stress and the right hand side is called critical boundary Reynolds number.

The Shields diagram contains the critical shear stress as an implicit variable that cannot be obtained directly. To overcome this difficulty, the ASCE Sediment Manual (1975) utilizes a third dimensionless parameter

$$\frac{d}{\upsilon}\left[0.1\left(\frac{\gamma_s}{\gamma} - 1\right)gd\right]^{1/2} \tag{5}$$

From the value of the third parameter, the value of the critical Shields stress is obtained at an intersection with the Shields curve from which critical shear stress τ_c can be calculated.

3 Numerical Formulation

The Saint-Venant equation is not amenable to analytical solution except for a few special cases.

They are partial differential equation that, in general, must be solved using numerical methods. To approximate Saint-Venant equation many numerical schemes have been developed. Beam and Warming scheme, which basically uses implicit finite difference method has been implemented for solving the governing equation. The central finite difference method is derivative.

The final expression using Beam and Warming finite difference scheme is (Bean and Warming 1976)

$$\begin{aligned}&\left[I+\Delta t\frac{\theta}{1+\xi}\left(\frac{\partial}{\partial x}A^k+\frac{\partial}{\partial y}B^k+Q^k\right)\right]\Delta_t U^{k+1}\\&\quad=-\Delta t\frac{1}{1+\xi}\left(\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+S\right)^k+\frac{1}{1+\xi}\Delta_t U^k\end{aligned} \tag{6}$$

For Euler Implicit scheme $\theta = 1$ and $\xi = 0$.

The final expression after replacing $\theta = 1$ and $\xi = 0$ is as follows:

$$\begin{aligned}&\left[I+\Delta t\left(\frac{\partial}{\partial x}A^k+\frac{\partial}{\partial y}B^k+Q^k\right)\right]\Delta_t U^{k+1}\\&\quad=-\Delta t\left(\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+S\right)^k+\Delta_t U^k\end{aligned} \tag{7}$$

4 Initial Conditions and Bathymetry of the River

A hypothetical drain of 2 km length and 2 m width was considered for simulation of the model. Total number of grids in longitudinal and transverse direction of the drain is 200 and 10 with Δx = 10 m and Δy = 0.2 m, respectively. The longitudinal slope of the bed is 0.0005. Figure 1 shows the initial condition and bathymetry of the drain.

5 Boundary Condition

Numerical form of the Saint-Venant equation is used in the interior grid points to compute the unsteady flow and velocity. At the boundaries, however, we cannot use this equation, since there is no grid point outside flow domain. Therefore, for

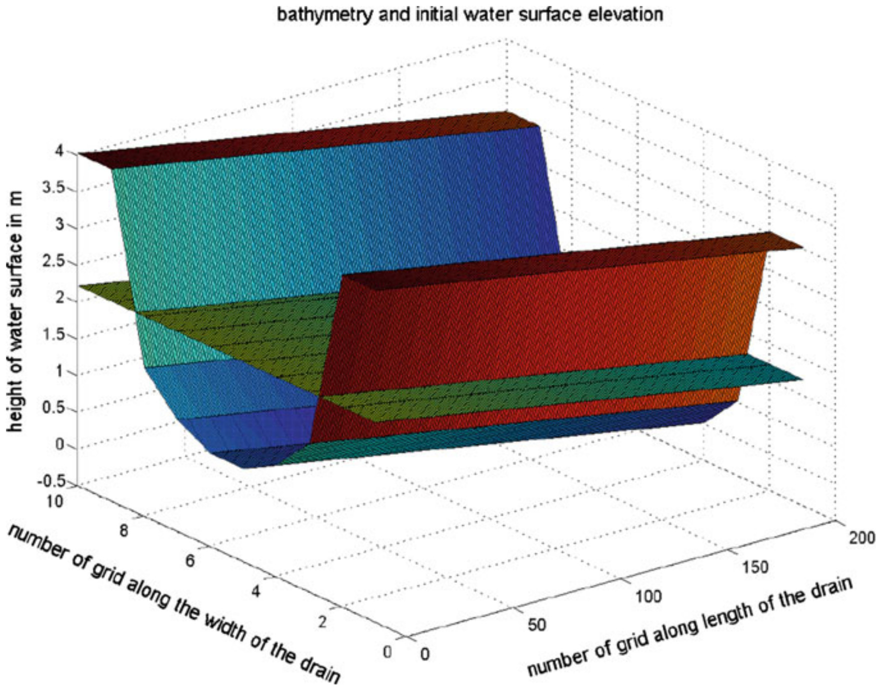


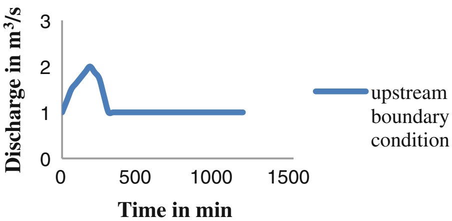
Fig. 1 Bathymetry and initial condition

two-dimensional unsteady flow, we need two boundary conditions. One is upstream boundary condition and the other is downstream boundary condition.

5.1 Upstream Boundary Condition

The discharge hydrograph as shown in Fig. 2 has been taken as the upstream boundary condition.

Fig. 2 Upstream boundary condition



5.2 Downstream Boundary Condition

To calculate the flow parameter at the downstream boundary we have used two equations, positive characteristic equation [Eq. (8)], and Manning's equation [Eq. (9)]

$$u_{i,j-1}^{k} + 2c_{i,j-1}^{k} = u_{i,j}^{k+1} + 2c_{i,j}^{k+1} \tag{8}$$

For

$$\frac{dx}{dt} = u + c$$

$$u = \frac{1}{n} y^{\frac{2}{3}} s_f^{\frac{1}{2}} \tag{9}$$

Celerity c of shallow water wave is given by $c = \sqrt{gh}$.

Reflective boundary condition technique was applied as the solid boundary. For this dummy cells are created outside the bank line and the flow variables at those cells are made equal to the inner line to make the bank behave like a wall. The normal velocity component at the dummy cell is just opposite to the velocity at the inner line of bank. The values of the variables at the dummy cells for left bank are as follows:

$$u_{i,j}^{k} = u_{i+2,j}^{k}$$
$$h_{i,j}^{k} = h_{i+2,j}^{k}$$
$$v_{i,j}^{k} = -v_{i+2,j}^{k}$$

6 Result and Discussion

The elevation at different time, discharge hydrograph and the depth hydrograph at downstream section are presented in this section. The data used in the model are hypothetical. After running the unsteady flow model, the critical section of the sediment deposition is calculated.

6.1 Elevation

The elevation of water surface profile at different time step is shown in Fig. 3. From this figure it has been observed that with the increases of time the water surface

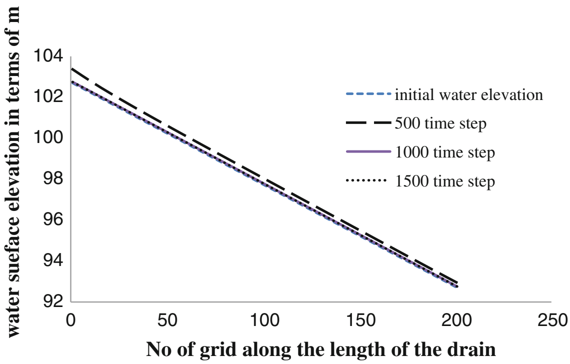


Fig. 3 Water surface elevation at different time step

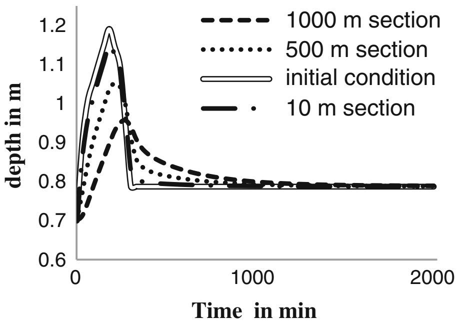


Fig. 4 Depth hydrograph at different section

elevation is increases up to a certain time step (500 time step). After that water surface elevation again starts to decreases with time.

6.2 Depth Hydrograph

The depth hydrograph at different section of the drain is shown in Fig. 4. Depth hydrograph is considered at section 10, 500, 1000, and 1500 m of the drain. From this result, it has been observed that as the distance of the observed section from upstream boundary of the drain increases, the peak of the depth hydrograph decreases and time required to attain peak increases.

6.3 Settling Velocity and Sediment Deposition

Figure 5 shows the bed shear stress at different time. From this figure it has been observed that bed shear stress increases with time up to a certain limit (198 min),

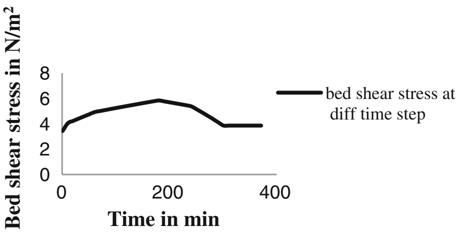


Fig. 5 Bed shear stress at different time

after it decreases and then it become constant. This implies that with the increases of water depth, bed shear stress increases.

6.4 *Critical Section*

Figure 6 represents the critical section of different particles at different time. Critical section of 0.0001, 0.0002, 0.001 m diameter particle are considered. From this figure, it has been seen that as the diameter of the particle increases the location of critical section from the upstream boundary decreases.

Table 1 shows different parameters of the particle like settling velocity, critical shear stress, critical section of drain, and bed shear stress. From this table it has been observed that with the increases of particle diameter settling velocity of the particle increases and larger sizes particles settle down at a nearest location from the upstream. If the bed shear stress of the particle is more than critical shear stress of the particle, incipient motion of the particle occurs. From the table below it has been observed that critical shear stress of 0.008 and 0.01 m particles are more than bed shear stress of the drain. Therefore, these particles start to deposit in their respective critical section of the drain. Smaller particles (<0.008 m) are washed away from the drain because critical shear stress of those particle are smaller than bed shear stress.

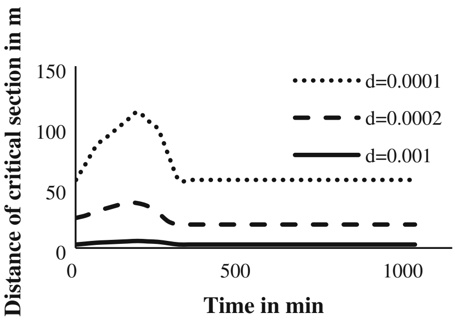


Fig. 6 Critical section of different particle with time

Table 1 Different parameters of the particle

Diameter of the particle (m)	Settling velocity (m/s)	Critical section of the drain (m)	Critical shear stress (N/m^2)	Bed shear stress (N/m^2)	Remarks
0.0001	0.0089	112–56	0.49	3.4–5.8	No sediment deposition
0.0002	0.0265	24–37	0.32	3.4–5.8	No sediment deposition
0.001	0.175	2.85–5.7	0.61	3.4–5.8	No sediment deposition
0.005	0.45	1.5–3.7	3.89	3.4–5.8	No sediment deposition
0.008	0.66	1.3–3	7.7	3.4–5.8	Sediment starts to deposition
0.01	0.75	1–2.5	9.7	3.4–5.8	Sediment starts to deposition

7 Conclusion

A two-dimensional numerical model considering the sediment deposition in the drain is developed. From the above results it has been observed that, particle size diameter has a great influence in determining the settling velocity of the particle and critical section of sediment deposition. In the proposed model particle sizes between 0.008 and 0.01 m are started to deposit in their respective critical location of the drain. Therefore, to minimize the artificial flood, it is necessary to remove the deposited sediment from the critical section of the drain.

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