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Using Accounting-based Information on Young Firms to Predict Bankruptcy

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This article has been accepted for publication and undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process which may lead to differences between this version and the Version of Record. Please cite this article as doi: 10.1002/for.2586

Abstract

This study analyzes the nonlinear relationships between accounting-based key performance indicators and the probability that the firm in question will become bankrupt or not. The analysis focuses particularly on young firms and examines whether these nonlinear relationships are affected by a firm's age. The analysis of nonlinear relationships between various predictors of bankruptcy and their interaction effects is based on a structured additive regression model and on a comprehensive data set on German firms. The results of this analysis provide empirical evidence that a firm's age has a considerable effect on how accounting-based key performance indicators can be used to predict the likelihood that a firm will go bankrupt. More specifically, the results show that there are differences between older firms and young firms with respect to the nonlinear effects of the equity ratio, the return on assets, and the sales growth on their probability of bankruptcy.

Keywords

Accounting-based key performance indicator · Bankruptcy prediction · Firm age · Nonlinear interaction effect · Structured additive regression model

1 Introduction

Accounting-based information can be very useful for predicting whether a firm will become bankrupt within a specific period. Particularly in cases where there is no readily available market-based information on a firm, to assess that firm as a going concern and predict the probability of its going bankrupt it is necessary to use accounting-based information instead.

The explanatory power of accounting-based key performance indicators that are obtained from a firm's annual financial statements has been studied extensively, including in the early literature (Martin, 1977; Ohlson, 1980). In addition to studying such indicators, researchers have sought to develop empirical and statistical methods and models for predicting as accurately as possible the probability of a firm, or a debtor in general, going bankrupt (for an overview see e.g., Altman & Saunders, 1997; Balcaen & Ooghe, 2006; Bellovary, Giacomino, & Akers, 2007; Dimitras, Zanahkis, & Zopounnidis, 1996; Scott, 1981). The present study contributes to this body of research by analyzing the nonlinear interactions between accounting-based key performance indicators and a firm's age and their effects on the probability of bankruptcy. For that purpose, this study will apply a structured additive regression model.

Several studies in the literature assume that there are non-monotonous relationships between the key accounting-based indicators of a firm's performance and predicting that firm's probability of bankruptcy (Atiya, 2001; Erlenmaier, 2011; Saunders & Allen, 2010). These assumptions are partially supported by a body of empirical evidence. Specifically, several studies have found that when a generalized linear model (GLM) or a generalized additive model (GAM) is applied, it is indeed possible to detect nonlinear relationships between accounting-based key performance indicators and the predictors that measure a firm's probability of going bankrupt. Escott, Glormann, and Kocagil (2001), Falkenstein, Boral, and

Carty (2000), and Sobehart, Keenan, and Stein (2000) have all provided such evidence with respect to a number of factors; namely, a firm's finance structure (see also Estrella, Park, & Peristiani, 2000), profitability (see also Estrella, Park, & Peristiani, 2000; van Gestel et al., 2005), liquidity (see also Serrano-Cinca, 1997), turnover, growth (see also Hayden, 2011), and size (see also Altman, Sabato, & Wilson, 2010). Statistically speaking, however, this empirical evidence is rather weak, because the respective works tend to apply univariate methods and analyze nonlinear effects only with respect to quantiles of classified data. Nevertheless, Lohmann and Ohliger (2017), who used a GAM to estimate nonlinear relationships as accurately as possible, have confirmed and described in more detail the existence of nonlinear relationships between accounting-based key performance indicators and the predictors used to predict whether a firm will go bankrupt or not.

The empirical methods and models available to researchers can estimate the direct effects of accounting-based key performance indicators on predicting the probability of a firm becoming bankrupt. A GLM strictly assumes a linear relationship between such an indicator and the predictor; in contrast, a GAM can estimate more accurately such relationships. Both models, however, rely on the assumption that there are no statistically significant interactions between the independent variables that they use. If in practice the independent variables in question do interact in a statistically significant manner, to estimate accurately whether a firm will go bankrupt or not, it is necessary to obtain detailed information on those interactions.

The objective of this study is to analyze whether the interactions between accounting-based key performance indicators and a firm's age affect the prediction of whether a firm will become bankrupt or not and, if so, to what extent. Research has shown that a firm's age, particularly in the case of young firms, determines to a considerable extent that firm's growth (e.g., Fort et al. 2013) and, therefore, its accounting-based key performance indicators. Our starting point is that accounting-based key performance indicators affect either linearly or nonlinearly the predictor that measures a firm’s latent probability of bankruptcy. On that basis, we hypothesize that a firm’s age affects the relationships between these factors. In particular, we expect that in the case of young firms the relationship between accounting-based key performance indicators and a firm’s probability of bankruptcy is more pronounced.

In this paper we use a structured additive regression model to analyze how a firm’s age interacts with its accounting-based key performance indicators and how this interaction affects the prediction of bankruptcy. A number of studies have applied GAMs to examine creditworthiness (Alp et al., 2011; Burkhard & de Giorgi, 2006) and bankruptcy prediction (Berg, 2007; Cheng, Chu, & Hwang, 2010; Dakovic, Czado, & Berg, 2010; Hwang, Cheng, & Lee, 2007; Lohmann, & Ohliger, 2017, 2018). However, these studies focus on comparing several empirical models from a strictly statistical perspective and do not analyze or describe existing interaction effects between a firm’s age and its accounting-based key performance indicators. A firm’s age, as we explain below, is nevertheless a potentially important factor when it comes to assessing its creditworthiness and predicting its probability of going bankrupt.

Given that young companies and older companies are at different stages of their life cycle, they are likely to differ with respect to their accounting-based key performance indicators and their rates of change. We expect that this is particularly true for indicators that relate to equity and firm growth. If we assume that a young firm’s opportunities for growth are much greater than those of an older firm, we can expect to see this difference reflected in the two firms’ financial structures. Both the signaling theory (Meyers, 1977) and the pecking-order theory (Myers, 1984; Myers & Majluf, 1984) argue that a younger company’s opportunities for growth are associated with lower equity and higher debt. In contrast, agency theory (Jensen &

Meckling 1976) argues that in younger firms these opportunities are associated with higher equity and lower debt. In either case, what matters from our point of view is that these differences may also affect a firm's probability of bankruptcy. If this hypothesis is correct, estimating this probability requires that we take into account the interaction effects between a firm's age and accounting-based key performance indicators.

To test whether a firm's age indeed affects the nonlinear relationship between that firm's accounting-based key performance indicators and probability of bankruptcy, we will apply structured additive regression models with a two-dimensional spline function that captures the interaction effects in question. To the best of our knowledge, this is the first study to do so. Our study furthermore evaluates the validity of the structured additive regression models that we apply. As our analysis is based exclusively on a set of data on German companies, our work also contributes to research on predicting the probability of bankruptcy specifically in the case of German companies (Anders & Szczesny, 1998; Kaiser & Szczesny, 2003; Schuhmacher, 2006).

The present paper is structured as follows: in the next section we will present our empirical data, the refined final sample, and the distributions of the independent variables we used. In the third section we will introduce our methodology and present in detail the structured additive regression model we applied. In the fourth section we will present the results of our empirical analysis with respect to the interaction between firm age and accounting-based key performance indicators and we will discuss how this interaction affects the accuracy with which we can predict whether a firm will become bankrupt or not. Finally, in the last section we will summarize the main results and discuss their implications for predicting bankruptcy.

2 Empirical Data

As already mentioned, our empirical analysis is based on a set of data on German firms. We extracted our data from the database of Bureau van Dijk Electronic Publishing, focusing on information on the structure, annual financial statements, and bankruptcies of German firms for the fiscal years 2000–2017. Our data show whether a firm in the sample became bankrupt before the end of 2017 and, for firms that did, when. The information we drew from annual financial statements includes balance sheets and profit and loss statements. We also collected information on the firms' legal form and industry.

We subjected our initial sample to a number of selection criteria, as a result of which we excluded two categories of firms: firms whose financial reporting did not follow the German Commercial Code (HGB) and firms owned by a public institution or that were part of a private group. In addition, we excluded non-profit firms, firms that belonged to the finance and insurance industry, and firms that did not provide data based on the total cost method or whose annual financial statements did not provide sufficient details. The raw data we collected on the basis of these criteria amount to 96,772 annual financial statements that correspond to 34,598 German firms. The seven-step procedure we followed in order to collect and process these data is outlined in Table 1. From these raw data, we extracted a refined sample on which we based our empirical analyses.

The empirical model we applied predicts a firm's probability of bankruptcy. The dependent variable measures whether a firm defaulted within the period of coverage. We classified each defaulted firm according to the type of default (Dickerson & Kawaja, 1967; Erlenmaier, 2011; Schwarz & Arminger, 2010) and the period within which it defaulted. Specifically, we classified a firm as "bankrupt" if it had declared bankruptcy within three years after the annual financial statement that we selected (for a similar approach, see Dakovic, Czado, &

Berg, 2010). We should note that as a result of this approach, it was not possible to predict whether a firm would remain solvent in the three years following any date during the period spanning the fiscal years 2015–2017.

Having collected our data, we then proceeded to exclude all annual financial statements after the fiscal year 2014. This step reduced the empirical database to 64,911 annual financial statements drawn from 27,608 German firms. In the third and fourth steps we chose our metric accounting-based independent variables. We selected the equity ratio (*EQR*), the fixed assets ratio (*FAR*), the return on assets (*ROA*), the adjusted balance sheet total (*BST*), and the sales growth (*SG*). The metric independent variables we selected cover the main areas of balance sheet analysis and are presented in detail in Table A1 which is part of the appendix. It should be noted that in our final sample we only included annual financial statements that met all of our criteria. Consequently, all metric independent variables are complete. However, the division by zero (*EQR*, *FAR*, *ROA*) or the absence of the previous annual financial statement (*SG*) may explain why some variables appear to be missing. In any case, we eliminated all observations with missing and implausible negative values. Our refined sample, after these steps, was reduced to 30,406 annual financial statements drawn from 15,191 German firms.

In the fifth step, we examined the effect of a firm's industry on the metric independent variables. Specific industry characteristics can affect significantly a firm's accounting-based key performance indicators (Chava & Jarrow, 2004; Lev, 1969). This means that it is not sufficient to examine the absolute values of a firm's key performance indicators, as these do not reveal much about how a firm's industry may affect its probability of going bankrupt. Moreover, there is evidence that taking into account the effects of a firm's industry on that firm's key performance indicators has a positive effect on a model's stability (Ooghe, Joos, &

de Bourdeaudhuij, 1995). Our approach was to modify each accounting-based key performance indicator by calculating its relative deviation from the annual industry median for each year (Berg, 2007; Gordon, Horwitz, & Meyers, 1966; Izan, 1984; Platt & Platt, 1990, 1991). Consequently, each of the metric accounting-based independent variables we used can be interpreted independently of the type of industry, as these variables represent the relative deviation from the year-specific industry median.

One problem with the approach we describe above is that if the annual industry median is close to zero, it produces outliers (Hawkins, 1980). To avoid distorted estimations, it is necessary to identify outliers and eliminate the respective observations. To identify and eliminate outliers, in the fifth step we applied a box-plot approach. The thresholds we used to identify an outlier for each metric independent variable are given by: mean ± 1.5 · (75% quartile – 25% quartile). Applying this method, we eliminated from our analysis all annual financial statements that produced at least one outlier (Dakovic, Czado, & Berg, 2010). At the end of this process, the usable data were further reduced to 16,792 annual financial statements derived from 8,830 German firms.

It should be noted that we did not perform the process of elimination that we describe above for the metric independent variable that reflects a firm's age (*AGE*). The main reason for this decision was that the number of older firms (*AGE* > 30) was too low for the results to be valid. This was not entirely unexpected, however: when structured additive regression models with bivariate splines are applied, certain value ranges will contain only a very small number of observations (Fahrmeir et al., 2013, p. 507). In our case, to correct this problem, we decided to cap the variable *AGE* at 30 and did so in the sixth step of our analysis. This led us to eliminate another 1,305 firms, whose age exceeded 30 years. Although this step narrowed the value range ($0 \leq AGE \leq 30$), at the same time the distribution of the metric independent variable *AGE* became much denser.

One final question to resolve was whether we should use each firm's most recent available annual financial statement or time-coherent panel data. The advantage of the first method is that the observations it yields are independent of each other; the advantage of the second method is that it yields a higher number of annual financial statements. Initially we applied the first method, which reduced our final sample to 7,525 annual financial statements corresponding to 7,525 German firms. Finally, however, we decided to apply the second method to ensure that our observations were reliable. As a result of this decision, our final sample comprises 791 bankruptcies. The forecast horizon we chose, which partly coincides with the financial crisis that began around 2007, explains this rather high a priori bankruptcy rate.

The qualitative variable *INDUSTRY* is subdivided into ten categories. These categories correspond to the German classification of industries according to the WZ code ("Klassifikation der Wirtschaftszweige"; Statistisches Bundesamt, 2008). The largest industry categories in our final sample are retail & trading (26.0%) and manufacturing (20.6%). We also decided to use the dummy variable *LEGAL* to distinguish between firms with limited liability (87.0% of our sample) and firms with unlimited liability, as a firm's legal form may also affect its probability of going bankrupt. To make our database as comprehensive as possible, we also included categorical variables; however, these variables are not useful in the analysis of nonlinear relationships, so they are of secondary importance.

To examine the correlations between each pair of independent variables, we relied on the lowest scale of two independent variables. The measures we used are the Bravais–Pearson correlation coefficient and Cramér's V. The Bravais–Pearson correlation coefficient measures how metric independent variables are correlated, while Cramér's V measures how categorical independent variables are correlated and how a metric and a categorical independent variable are correlated. In the latter case, the metric independent variable has to be classified into five categories based on quantiles. In our analysis, the Bravais–Pearson correlations are below 0.3 and Cramér's V is below 0.22. When we compared solvent and bankrupt firms we identified statistically significant differences in the accounting-based metric independent variables. Overall, we found that our accounting-based key figures exhibits the expected data structures, which indicates that our database is valid.

The kernel density estimation of the metric independent variable *AGE* shows that the number of firms decreases in firm age. Furthermore, the kernel density estimations of the metric accounting-based independent variables show that these variables fluctuate around the value of 0 because we eliminated the industry effects. The fixed assets ratio (*FAR*) and the adjusted balance sheet total (*BST*) have natural lower limits in this case. The kernel density estimations of the metric independent variables are presented in detail in Fig. A1 which is part of the appendix. In Fig. 1 we present the two-dimensional kernel density estimations of *AGE* and of the metric accounting-based independent variables. In Fig. 1 the two-dimensional kernel density estimations exhibit sparsely populated periphery areas, as expected. However, the overall structure here is comparable to that displayed in Fig. A1.

3 Methodology

The dependent variable is firm bankruptcy. This variable takes the value 0 for companies in the class "solvency" and the value 1 for companies in the class "bankruptcy," which had defaulted in the period of interest. We used this variable to transform the information we had on qualitative bankruptcy into a metric Bernoulli-distributed measure. This metric measure, which we used in our subsequent regression analysis, can be interpreted as the probability π_i

of firm i being in the class “bankruptcy.” In a GAM, the probability π_i depends on two things: first, on a set of independent variables that take the values x_{i1}, x_{i2},..., x_{ip} and second, on the applied response function $h(\cdot)$, which transforms the results of the linear function with the coefficients β_0, β_1,..., β_p (Nelder & Wedderburn, 1972). When the dependent variable follows a Bernoulli distribution (Rauhmeier, 2011), the response function $h(\cdot)$ needs to be a distribution function $F(\cdot)$. For example, in a probit GLM the distribution function of the standard normal distribution is used, whereas in a logit GLM, the distribution function of the logistic distribution is used (on the choice of distribution functions, see Amemiya, 1981; Fahrmeir & Tutz, 2001; Porath, 2006). The probability $\pi_i = F(\eta_i)$ retains the constraint $\pi_i \in [0,1]$ because of the slope of the distribution function. The predictor η_i is still a linear function; however, the relationship between each independent variable and the probability π_i is no longer linear. This results from the conjunction with the distribution function $F(\cdot)$. Each independent variable, however, has a monotonic effect on the probability π_i because every distribution function is strictly increasing (Hosmer, Lemeshow, & Sturdivant, 2013). In Equation 1 we apply a GLM to calculate the probability π_i of firm i being in the class “bankruptcy.”

$$\pi_i = h(\eta_i) = F\begin{pmatrix} \beta_0 + INDUSTRY + LEGAL + \beta_1 \cdot EQR_i + \beta_2 \cdot FAR_i \\ + \beta_3 \cdot ROA_i + \beta_4 \cdot BST_i + \beta_5 \cdot SG_i + \beta_6 \cdot AGE_i \end{pmatrix} \quad (1)$$

To examine the nonlinear relationships between each independent variable and the predictor, we can apply a GAM and replace the linear predictor η_i with the additive predictor η_i^{add}. The additive predictor η_i^{add} consists of the functions $f_1(\cdot)$, $f_2(\cdot)$,..., $f_p(\cdot)$, which follow an unspecified form. In Equation 2 the GAM is expressed in concrete terms. The same equation calculates the probability π_i of firm i being in the class "bankruptcy."

$$\pi_i = h\left(\eta_i^{add}\right) = F\begin{pmatrix} \beta_0 + INDUSTRY + LEGAL + f_1\left(EQR_i\right) + f_2\left(FAR_i\right) \\ + f_3\left(ROA_i\right) + f_4\left(BST_i\right) + f_5\left(SG_i\right) + f_6\left(AGE_i\right) \end{pmatrix} \tag{2}$$

The unspecified function $f(\cdot)$ is modeled by polynomial splines. In polynomial splines the range of the independent variables is split at intervals delimited by knots k_j, with $j = 1, \ldots m$. The lower limit of the range $[x_{\min}, x_{\max}]$ is k_1, while the upper limit is k_m. Here, we estimated the polynomial of rank g for every interval, as this approach results in a better fit to the data than a polynomial model without the split would. The unspecified function $f(\cdot)$, which is characterized by a number of polynomial splines, also has to be $(g-1)$-times continuously differentiable. As a result, the function is smooth. Furthermore, this differentiability prevents jump discontinuity at the interval limits (Kneib, 2006).

In practical terms, it is possible to model the splines by using the base functions that relate to either the truncated power series or to the B-spline-base (Kneib, 2006). In both approaches the spline function $f(\cdot)$ can be modeled as a linear combination of so-called basis functions $B(\cdot)$. Equation 3 puts the spline function $f(\cdot)$ in concrete terms for the independent variable x.

$$f(x) = \gamma_1 B_0(x) + \gamma_2 B_1(x) + \ldots + \gamma_{g+m-1} B_{g+m-2}(x) \tag{3}$$

To distinguish between the coefficients of the global regression model and those of the individual spline functions, in Equation 3 the coefficients are denoted as follows: $\gamma_0, \gamma_1, \ldots, \gamma_{g+m-2}$. Our approach follows the approach of Hastie and Tibshirani (1990), who applied a truncated power series to model splines with the desired attributes. Here, we used a global polynomial of rank g (see first part of Equation 4) and took into account the change in the coefficients at every knot k_j, with $j = 2,\ldots m-1$ (see second part of Equation 4). We did not apply this modification at knots k_1 and k_m, however, because they coincide with the lower and upper limits of the range of values that the metric independent variable x can take.

$$\begin{aligned} f(x) &= \gamma_1 x^0 + \gamma_2 x^1 + \gamma_3 x^2 + \ldots + \gamma_{g+1} x^g + \gamma_{g+2}(x-\kappa_2)_+^g + \ldots + \gamma_{g+m-1}(x-\kappa_{m-1})_+^g \\ &= \sum_{j=1}^{g+1} \gamma_j x^{j-1} + \sum_{j=2}^{m-1} \gamma_{g+j}(x-\kappa_j)_+^g \end{aligned} \tag{4}$$
$$\text{with} \quad (x-\kappa_j)_+^g = \begin{cases} (x-\kappa_j)^g & x \geq \kappa_j \\ 0 & else \end{cases}$$

Both approaches involve two subjective design elements: choosing the number m and the position of the knots is subjective, although knots are usually arrayed equidistantly or on the basis of the quantiles. To solve this problem, we used penalized splines. A polynomial spline with a large number of knots can be easily used to approximate function $f(\cdot)$. The large number of knots means that the approximation will be flexible, so how the knots are arrayed becomes less important. The second problem we had to solve was to find a way to achieve a balance between flexibility and smoothing. For that purpose, we used an additional penalty term for every spline function in the maximum likelihood estimation of the GAM. This term penalizes highly different interval-specific polynomials.

A third problem we had to solve was that of likelihood maximization. Our solution was to weight the penalty term with a smoothing parameter λ. As a result of this method, the variability of a penalized spline can be controlled by a single parameter λ (Eilers & Marx, 1996). Higher values of λ decrease the variability of function $f(\cdot)$ and increase the smoothness of function $f(\cdot)$. However, increasing simultaneously both smoothness and adaption to the data is not feasible. For that reason, it is necessary to objectify the smoothing parameter λ by applying the restricted maximum likelihood (REML) criterion (Wood, 2017, pp. 276–278). This means that it is necessary to optimize the restricted maximum likelihood criterion in order to determine the smoothing parameters.

To examine the interaction effects, we chose to introduce two-dimensional polynomial spline functions. Specifically, we used a two-dimensional polynomial spline to model the bivariate relationship between two metric independent variables. However, we did not use separate spline functions to estimate the individual effects of each independent variable on the predictor. Consequently, here the two-dimensional polynomial spline function captures both the main effects and the interaction effect of the two metric independent variables. The estimated spline enables us to illustrate its functional form. In Equation 5 we apply a structured additive regression model with a two-dimensional polynomial spline function that simultaneously captures the direct effects of *EQR* and *AGE* as well as the interaction effect between *AGE* and *EQR*.

$$\pi_i = h\left(\eta_i^{add}\right) = F\begin{pmatrix} \beta_0 + INDUSTRY + LEGAL + f_2\left(FAR_i\right) + \beta_3\left(ROA_i\right) \\ + f_4\left(BST_i\right) + f_5\left(SG_i\right) + f_6\left(AGE_i, EQR_i\right) \end{pmatrix} \tag{5}$$

To estimate the two-dimensional polynomial spline function, we chose the tensor product approach. More specifically, to model the two-dimensional polynomial spline function $f(x_1, x_2)$ we composed pairwise the products of the univariate basis functions. From this calculation we obtained a two-dimensional basis whose linear combination we used to create a two-dimensional spline function. Equation 6 demonstrates our approach, which is comparable to modeling a univariate spline function. The modeling approach we chose enabled us to vary smoothly the univariate spline function $f(x_1)$, depending on x_2.

$$f(x_1, x_2) = \sum_{j=1}^{g+m-1} \sum_{\kappa=1}^{g+m-1} \gamma_{j\kappa} B_{j\kappa}(x_1, x_2) \tag{6}$$

Although it is possible to choose individually the rank g of the polynomial and the number m of knots k_j, we chose to apply the same values that we used in univariate modeling (Fahrmeir et al., 2013, pp. 503–511; Wood, 2017, pp. 228–232).

4 Results

In this analysis of empirical data on German firms, we applied a GLM according to Equation 1 (Model 1), a GAM according to Equation 2 (Model 2), and five structured additive regression models according to Equation 5, which capture the common effects of *AGE* and of each of the five accounting-based independent variables on the probability of default (models 3a–3e).

We expected that the estimation models we used would exhibit a satisfactory degree of external validity and would be usable both with existing data and with new data from the same population. In addition, we expected that the models would allow us to make high-quality predictions. Initially, one potential concern was that taking into account the nonlinear effects would increase the complexity of the models, which might have diminished their external validity. To address this concern, we assessed the models' validity by splitting randomly our sample of 7,525 observations into a training sample (70%, 5,267) and a validation sample (30%, 2,258). The training and the validation subsamples originate from the same population but are independent of each other. To test for differences between the subsamples, we ran means comparison tests and chi-square homogeneity tests. The results of these tests do not reveal any structural and statistically significant differences between the subsamples. Consequently, the results of the correlation analysis also apply to the two subsamples.

To estimate the models, we used the accounting-based independent variables after adjusting the industry effects. This allowed us to use in our calculations the relative deviation of the independent variable from the corresponding year-specific industry median. To model the nonlinear effects of the independent variables in the GAMs, we applied penalized splines and used basic functions of rank $g = 3$ and 12 equidistant intervals to put the univariate and bivariate spline functions in concrete terms. The smoothing parameter is determined by the restricted maximum likelihood criterion. We also used splines to model the independent integer variable firm age (*AGE*), following Beck and Jackman (1998). We treated the categorical independent variables as dummy variables. As our reference category we used limited liability corporations that belong to the retail and trading industry.

We present all model estimations in Table 2. We have included the regression coefficients in the results we obtained for both the GLM and for the qualitative variables of the GAMs. The asterisks denote the level of significance on the basis of the likelihood ratio test. The results we obtained from the GAMs on the basis of the metric independent variables we used show the equivalent degrees of freedom df_f. These represent the variability of the estimated splines of the metric independent variables. The value $df_f = 1$ shows that the estimated spline corresponds to a linear function, while the increasing degrees of freedom indicate that the level of nonlinearity increases. Here too, the asterisks denote the level of significance on the basis of the likelihood ratio test (Wood, 2017, p. 411). In the training sample the share of bankrupt firms is less than 50%. This is reflected in the constant, which in all seven model estimations includes the joint effect of the reference categories and assumes a statistically significant negative value.

Our results also show that the probability of real-estate firms, utilities, and unlimited liability firms going bankrupt is considerably lower than the probability of bankruptcy of their reference categories and that the probability of manufacturing, construction, and transportation industries going bankrupt is considerably higher than the probability of bankruptcy of their reference category. These figures are statistically significant (see Table 2). All model estimations we performed yield the same results. In Model 1 we found that five of the six metric independent variables we used have a statistically significant effect on the probability of bankruptcy. Specifically, we found that, ceteris paribus, when the independent variables *AGE*, *EQR*, *FAR*, and *ROA* increase, the estimated probability of bankruptcy is lower. Conversely, when the independent variable *BST* increases, the estimated probability of bankruptcy is higher. The effect of the independent variable *SG* on the probability of bankruptcy is not statistically significant.

The estimation of Model 1 corresponds to the GAM estimation of Model 2 with respect to the level of significance of the metric independent variables, except the variable *SG*, which reflects sales growth. On the basis of the equivalent degrees of freedom in the GAM estimations, we conclude that there are nonlinear relationships between the metric independent variables and the predictor, but none in the case of the fixed assets ratio *FAR* ($df_f \approx 1.00$). The other metric independent variables, however, do exhibit nonlinear relationships, as $df_f > 1.00$ indicates. From the same GAM estimation, we concluded that the independent variable *SG* does have a nonlinear and statistically significant effect on the predictor when the estimation model takes into account nonlinear relationships.

We will now examine the spline patterns in more detail, to discuss the direction of the nonlinear effects. The spline patterns of the significant metric independent variables we used for the estimation of Model 2 are displayed in Fig. 2, where the black bold line represents the estimated spline and the value of the independent variable is plotted on the x-axis. The independent variable represents the relative deviation of the absolute value of the key performance indicator from the year-specific industry median. The effect on the predictor is plotted on the y-axis. Higher values on this axis indicate a higher probability of bankruptcy. It should be noted, however, that these probabilities also depend on the values of the other variables. The gray-shaded area represents the 95% confidence band. The dashed line represents the corresponding linear estimator of Model 1 for the purposes of comparison. As both the linear function and the nonlinear spline are centered, we can compare directly the linear and the nonlinear univariate estimations. Finally, the dotted line represents the kernel density estimations of the corresponding independent variable with regard to the training sample.

The effects of the independent variables *EQR*, *FAR*, *ROA*, and *BST* on the predictor are in line with the empirical findings of previous research on the probability of bankruptcy (e.g., Altman, Sabato, & Wilson, 2010; van Gestel et al., 2005). Specifically, we found that as low and high values of *ROA* and *SG* increase the predictor, the probability of bankruptcy also increases, as the U-shaped relationship between the return on assets (*ROA*) and sales growth (*SG*) shows.

Our results also indicate that there is a nonlinear relationship between a firm's age (*AGE*) and the probability of that firm going bankrupt. Interestingly, in our models the spline of *AGE* does not indicate that newly founded German firms go through a "honeymoon period" during which the probability of bankruptcy is relatively low. More precisely, our models show that the probability of bankruptcy is strictly decreasing in *AGE* and reaches a largely constant level for older firms. However, these findings partly contradict the findings of previous studies. For example, Everett and Watson (1998) and Hudson (1987) found that a firm's probability of going bankrupt increases after the first two years following its launch, while according to Honjo (2000) the "honeymoon period" spans the first six years following a firm's foundation. Altman, Sabato, and Wilson Altman et al. (2010), who used a dummy variable for firms between three and nine years old, also obtained similar results.

Our next question is whether the nonlinear relationships between the metric independent variables and the probability of bankruptcy that we identified differ between younger and older firms. To answer that question, we need to evaluate models 3a–3e according to Equation 5. We see that the common effects of *AGE* and of every metric accounting-based independent variable on the probability of bankruptcy are statistically significant. Fig. 3 displays the effects of these variables on the predictor (estimated in models 3a–3e) and Fig. 4 displays their effects on the probability of bankruptcy.

The two-dimensional spline patterns show that younger and older firms indeed differ with regard to the nonlinear relationships between accounting-based independent variables and either the predictor or the probability of bankruptcy. In the case of younger firms, when the value of the accounting-based independent variable deviates from the year-specific industry median, this effect can be observed more clearly. Specifically, a younger firm's probability of going bankrupt is higher when the values of *EQR* and *FAR* are lower. When *EQR* and *FAR* increase, the probability of bankruptcy decreases. Although firm age does not change the effect's direction, in the case of younger firms this effect is much more pronounced. The probability of bankruptcy also reacts more sensitively when *BST* of a younger firm is lower or higher than the year-specific industry median.

The general effects of *ROA* and *SG* on the probability of bankruptcy are comparable to the effects that the univariate splines of Model 2 indicate. However, the effects of *ROA* and *SG* on the probability of bankruptcy seem to vary, depending on a firm's age. A younger firm's probability of going bankrupt increases when the values of *ROA* and *SG* are either particularly low or particularly high. While the probability of going bankrupt increases substantially when a young firm exhibits above-average sales growth or outstanding profits, the U-shaped relationship that reflects this probability is much less pronounced in older firms. This empirical finding indicates that young firms often achieve outstanding profits at the expense of economic stability and that above-average sales growth is often not sustainable in the longer term.

Models 3a–3e show that there are clear interaction effects between a firm's age and the accounting-based independent variables. Nevertheless, the likelihood ratio test does not indicate that the difference between Model 2 and models 3a–3e is statistically significant. Furthermore, in these models the validity measures that are based on likelihood or classification are on the whole comparable. Applying the statistical test that DeLong et al. (1988) recommend, we found that when we use the training sample only the validity measure AUC, which is based on classification, is significantly higher at the 10% level for the bivariate Model 3c, which displays the interaction between *AGE* and *ROA*, and Model 3e, which displays the interaction between *AGE* and *SG*. The comparisons of these models demonstrate that the effects of *ROA* and *SG* on the probability of a firm defaulting differ depending on a firm's age. With regard to Akaike's information criterium (AIC), which takes into account a model's complexity, Model 2 is preferable over the bivariate models.

Overall, our empirical analysis shows that taking into account the age-specific effects of accounting-based independent variables does not significantly increase the validity of the models used to predict whether a firm is likely to go bankrupt or not. This preliminary finding is based on our estimations of the smoothing parameters in the univariate and bivariate models. As the smoothing parameters of the univariate splines are considerably lower than the corresponding smoothing parameters of the bivariate splines, the likelihood-based validity of both model classes reach comparable levels.

When the applied smoothing parameters are equal in both the univariate and the bivariate models (see Table 3), the differences in the models' validity become more obvious. The smoothing parameters are equal when we regard the REML-optimized smoothing parameters of the univariate splines as given and apply them to the bivariate splines. In that case, the differences in the likelihood-based validity between Model 2 and models 3a–3e are statistically significant at the 10% level. The smoothing parameters may also be equal when we regard the REML-optimized smoothing parameters of the bivariate splines of models 3a–3e as given and transfer them to the univariate Model 2. As models 3a–3e exhibit different smoothing parameters for the selected combinations of the independent variables (i.e., for *AGE* and the selected accounting-based independent variable) the transfer of the smoothing parameters produces the models 2a–2e which are equivalent to the structure of Model 2. In this case, the differences in the likelihood-based validity between Model 2a and 2c–2e and models 3a and 3c–3e are statistically significant at the 10% level. However, we found that the likelihood ratio test does not indicate a statistically significant difference between Model 2b and Model 3b. We also found that when we apply bivariate models 3a–3e to the training sample, the validity measure AUC, which is based on classification, is always significantly higher at the 10% level (with regard to the statistical test, see DeLong, DeLong, & Clarke-Pearson, 1988). This indicates that taking into account the interaction effects between accounting-based variables and a firm's age tends to increase a model's validity if we keep the smoothing parameters constant.

We also applied our model estimations to the validation sample. In that case, however, we obtained different results with regard to the models' validity. Specifically, when we used in other models the smoothing parameters that we applied in the univariate Model 2, we found that the AUC validity measure was higher. Consequently, the univariate splines indicate a higher external validity. However, we only observed statistically significant differences at the

10% level when we compared Model 2 with models 3a–3d. Furthermore, we did not find any statistically significant differences in the AUC validity measure when we regarded the REML-optimized smoothing parameters of the bivariate splines of models 3a–3e as given and transfer them to the univariate models 2a–2e. A plausible explanation for this observation is that a model's external validity may decrease when the model's complexity increases, as the partly increased AIC value indicates.

5 Conclusion

In order to predict reliably whether a firm is likely to go bankrupt, it is necessary to take into account the statistically significant nonlinear relationships between accounting-based independent variables and the probability of bankruptcy. Our analysis shows that a firm's age partly determines these relationships and is therefore a factor that needs to be taken into account when assessing a firm's probability of going bankrupt. Our main finding is that in younger firms changes in the accounting-based independent variables have a more pronounced effect on predicting the probability of bankruptcy. From this finding we can draw two main conclusions. First, when we consider young firms, omitting the interaction effects between accounting-based independent variables and a firm's age distorts the estimates of that firm's probability of going bankrupt. Second, if we omit these effects, we cannot estimate correctly the equity base that is required for a particular level of risk and the standard risk costs associated with lending to that company.

The five accounting-based key performance indicators we selected are based on the analysis of the annual financial statements of the firms in our sample. Our findings show that three of these indicators (*EQR, ROA, SG*) have economically plausible nonlinear effects on a firm's probability of going bankrupt and that there are statistically significantly differences in these effects between younger and older firms when the smoothing parameters of the univariate and bivariate models are kept constant. More specifically, we showed empirically that when the equity ratio, the return on assets, and sales growth take low values, these accounting-based key performance indicators have a greater impact on a younger firm's probability of going bankrupt than they do in the case of older firms. This suggests that in the case of young firms, when we observe a low equity ratio, a low return on assets, and low sales growth, we may underestimate their probability of going bankrupt. Furthermore, our empirical evidence shows that a high return on assets and a high sales growth also increase a young firm's probability of going bankrupt. We furthermore found that a firm's balance sheet total, which is a proxy for firm size, has a more pronounced effect on the probability that a young firm will become bankrupt.

We should note that the structured additive regression models that we have applied are more complex than a GLM or a GAM; therefore, the interpretation of the different validity measures is inconclusive. Nevertheless, the qualitative insight we derive from our analysis shows unambiguously that similar accounting-based key performance indicators lead to different conclusions when we estimate a firm's probability of going bankrupt, depending on the firm's age. More generally, our results show that particularly where young firms are concerned it is advisable to take into account the accounting-based key performance indicators, especially the return on assets and sales growth.

Our analysis has two main limitations. The first limitation results from the failure criterion we chose to apply and the second results from the low number of observations in the peripheral areas of the independent variables we examined. We based our failure criterion on the definition of default and on the prediction horizon. Further research will determine whether our results can be replicated on the basis of different criteria of failure or not. The second limitation of our study is that we based our analyses on a small number of observations. Specifically, although our study suggests that a firm's age has an impact on the nonlinear relationships between accounting-based independent variables and that firm's probability of bankruptcy, this finding is based on relatively few observations at the peripheral areas of the accounting-based independent variables. For that reason, further research is needed to investigate whether our observations and conclusions hold when different databases are used.

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Table 1. The seven-step procedure for collecting and processing the raw data.

	Number of annual financial statements	Number of bankruptcies within a triennial period	Number of firms
1. Collected data on German firms for the fiscal years 2000–2017, derived from the Database of Bureau van Dijk Electronic Publishing	96,772		34,598
2. Processed the collected data to derive a bankruptcy prediction for the triennial period following the reporting date	64,911	4,211 (6.49%)	27,608
3. Eliminated missing variables	31,044	2,345 (7.55%)	15,488
4. Eliminated implausible negative variables	30,406	2,327 (7.65%)	15,191
5. Adjusted the metric variables to evaluate the industry effects and eliminated outliers	16,792	1,023 (6.09%)	8,830
6. Capped firm age at 30 years	13,745	931 (6.77%)	7,525
7. Compiled each firm's profile on the basis of the most recent available annual financial statement	7,525	791 (10.51%)	7,525

Table 2. Model estimations of the training sample with significant dummy variables and metric independent variables and validity measures.

	Model 1	Model 2	Model 3a	Model 3b	Model 3c	Model 3d	Model 3e
Intercept	–1.787***	–2.539***	–2.542***	–2.537***	–2.542***	–2.541***	–2.535***
Manufacturing	0.244*	0.327**	0.327**	0.326**	0.324**	0.327**	0.319**
Construction	0.800***	0.744***	0.744***	0.741***	0.744***	0.746***	0.746***
Real estate	–2.263***	–2.201***	–2.182***	–2.203***	–2.203***	–2.188***	–2.204***
Transportation	0.648***	0.714***	0.710***	0.713***	0.712***	0.721***	0.710***
Utilities	–0.769*	–0.740*	–0.733*	–0.738*	–0.737*	–0.746*	–0.738*
Unlimited liability firms	–1.085***	–1.165***	–1.167***	–1.168***	–1.171***	–1.160***	–1.163***
AGE	–0.051***	2.708***					
EQR	–0.350***	3.576***		3.584***	3.582***	3.577***	3.590***
FAR	–0.135**	1.001**	1.001**		1.001**	1.002**	1.001**
ROA	–0.199***	2.895***	2.894***	2.888***		2.899***	2.886***
BST	0.026**	3.283***	3.283***	3.284***	3.288***		3.277***
SG	–0.014	2.846***	2.843***	2.849***	2.829***	2.834***	
AGE, EQR			9.907***				
AGE, FAR				4.831***			
AGE, ROA					8.763***		
AGE, BST						8.993***	
AGE, SG							9.387***
Nagelkerke R^2	0.139	0.163	0.164	0.163	0.165	0.164	0.165
AIC	3,166.05	3,126.27	3,135.47	3,130.51	3,131.03	3,133.70	3,133.94
AUC training sample	0.740	0.760	0.760	0.759	0.762	0.760	0.761
AUC validation sample	0.715	0.721	0.723	0.722	0.720	0.722	0.722

*** *p*-value < 0.01, ** *p*-value < 0.05, * *p*-value < 0.1

Table 3. Validity measures obtained by varying the REML-optimized smoothing parameter.

REML-optimized smoothing parameters of the univariate splines					
	Model 2				
Nagelkerke R^2	0.163				
AIC	3,126.27				
AUC training sample	0.760				
AUC validation sample	0.721				
	Model 3a	Model 3b	Model 3c	Model 3d	Model 3e
Nagelkerke R^2	0.182	0.166	0.186	0.177	0.181
AIC	3,152.24	3,124.77	3,133.67	3,129.34	3,146.49
AUC training sample	0.772	0.762	0.776	0.768	0.774
AUC validation sample	0.709	0.716	0.712	0.712	0.713
REML-optimized smoothing parameters of the bivariate splines					
	Model 2a	Model 2b	Model 2c	Model 2d	Model 2e
Nagelkerke R^2	0.155	0.160	0.156	0.157	0.155
AIC	3,132.89	3,125.20	3,133.91	3,131.24	3,135.62
AUC training sample	0.753	0.757	0.752	0.754	0.753
AUC validation sample	0.727	0.724	0.721	0.724	0.720
	Model 3a	Model 3b	Model 3c	Model 3d	Model 3e
Nagelkerke R^2	0.164	0.163	0.165	0.164	0.165
AIC	3,135.47	3,130.51	3,131.03	3,133.70	3,133.94
AUC training sample	0.760	0.759	0.762	0.760	0.761
AUC validation sample	0.723	0.722	0.720	0.722	0.722

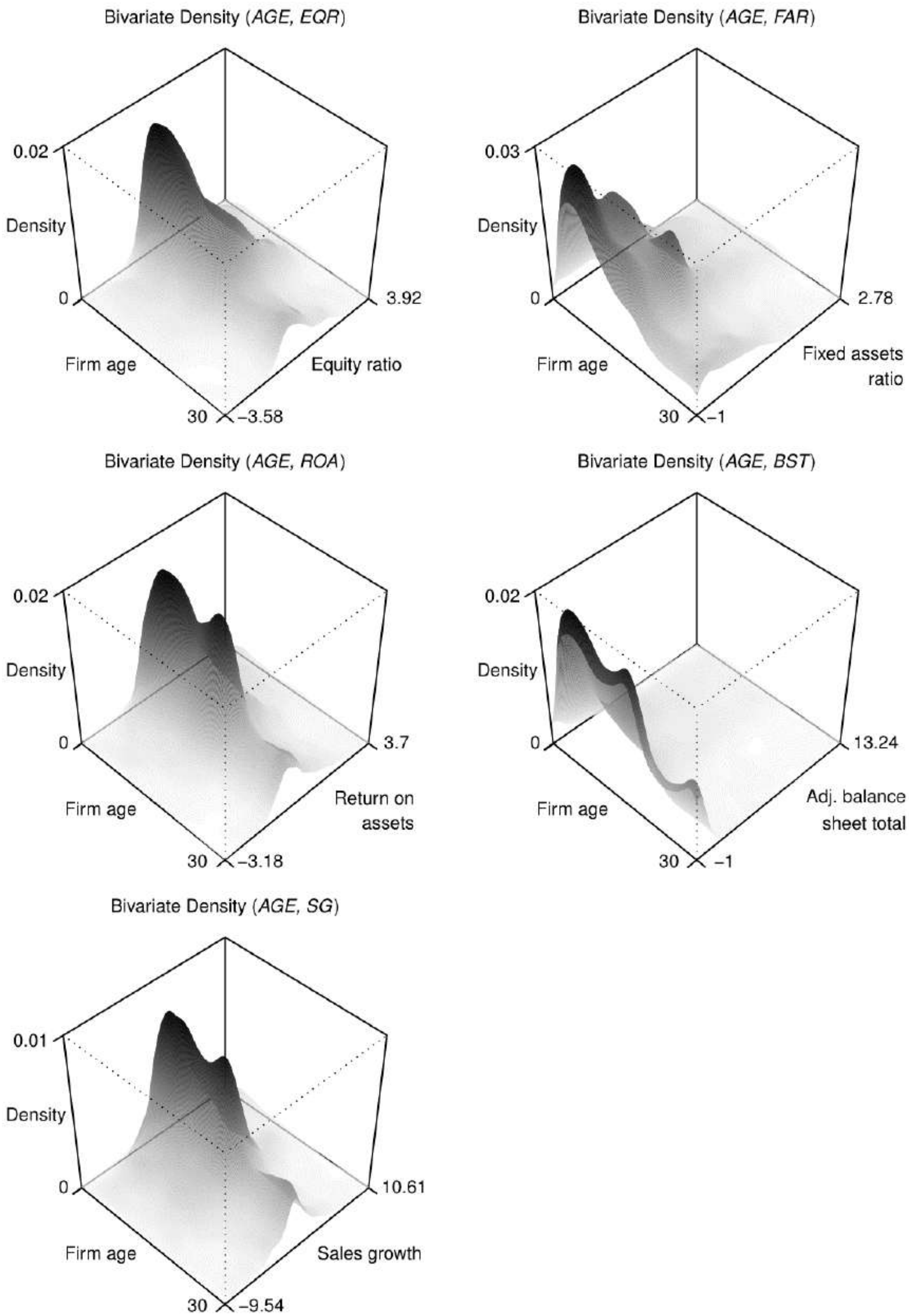

Fig. 1. Two-dimensional kernel density estimations of *AGE* and of the metric accounting-based independent variables.

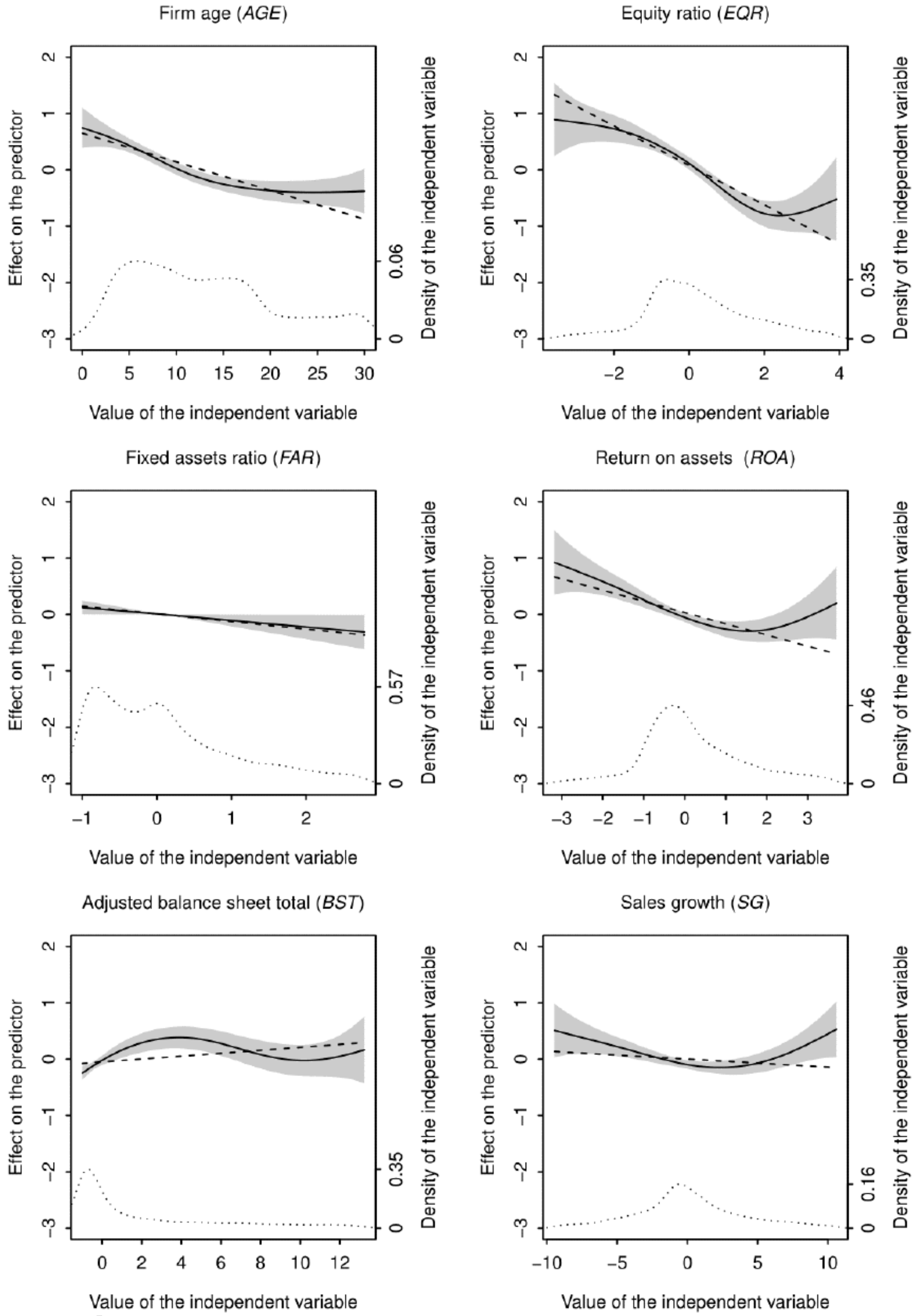

Fig. 2. Spline patterns for the estimation of Model 2.

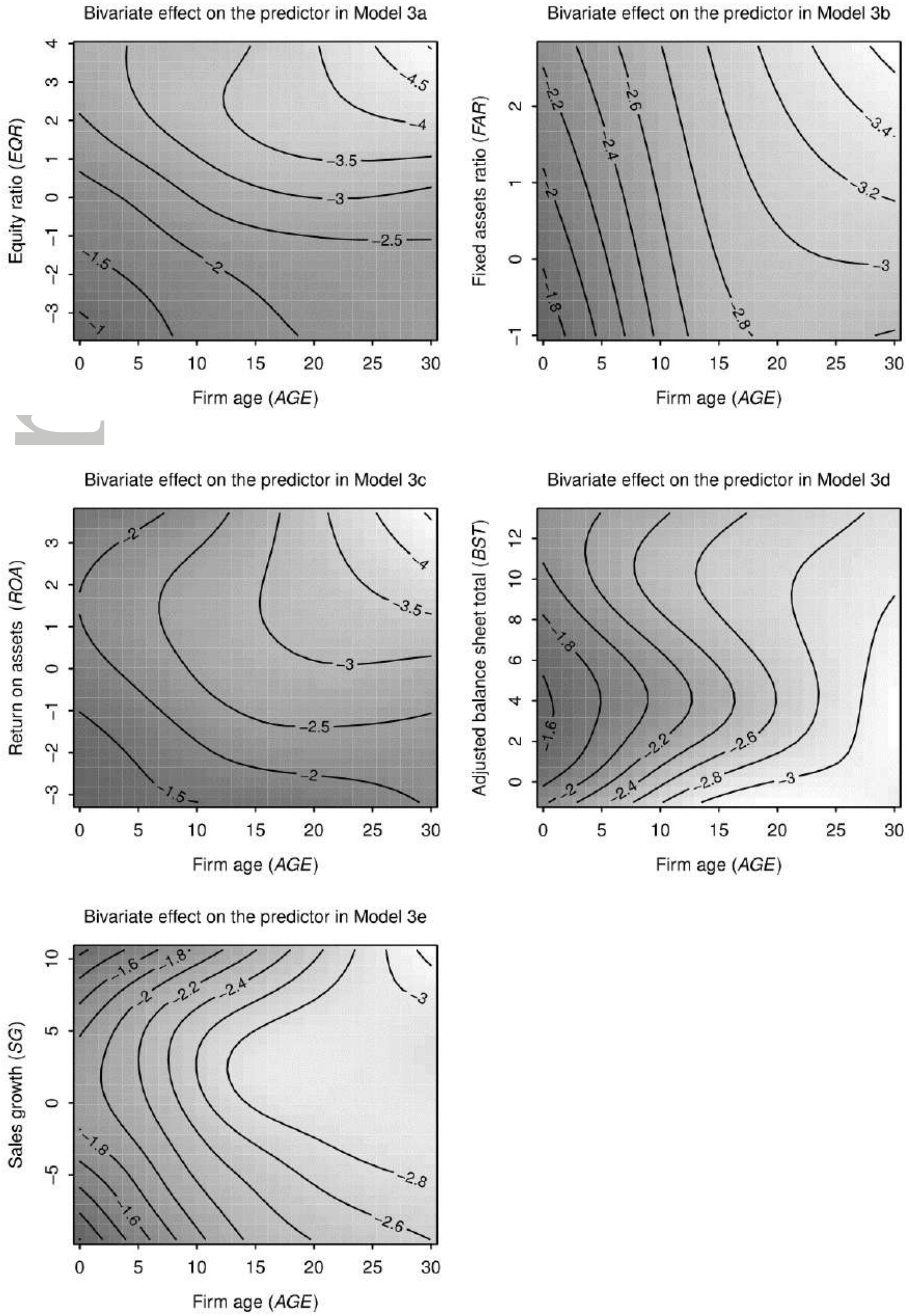

Fig. 3. Two-dimensional contour patterns that display the effects on the predictor in models 3a–3e.

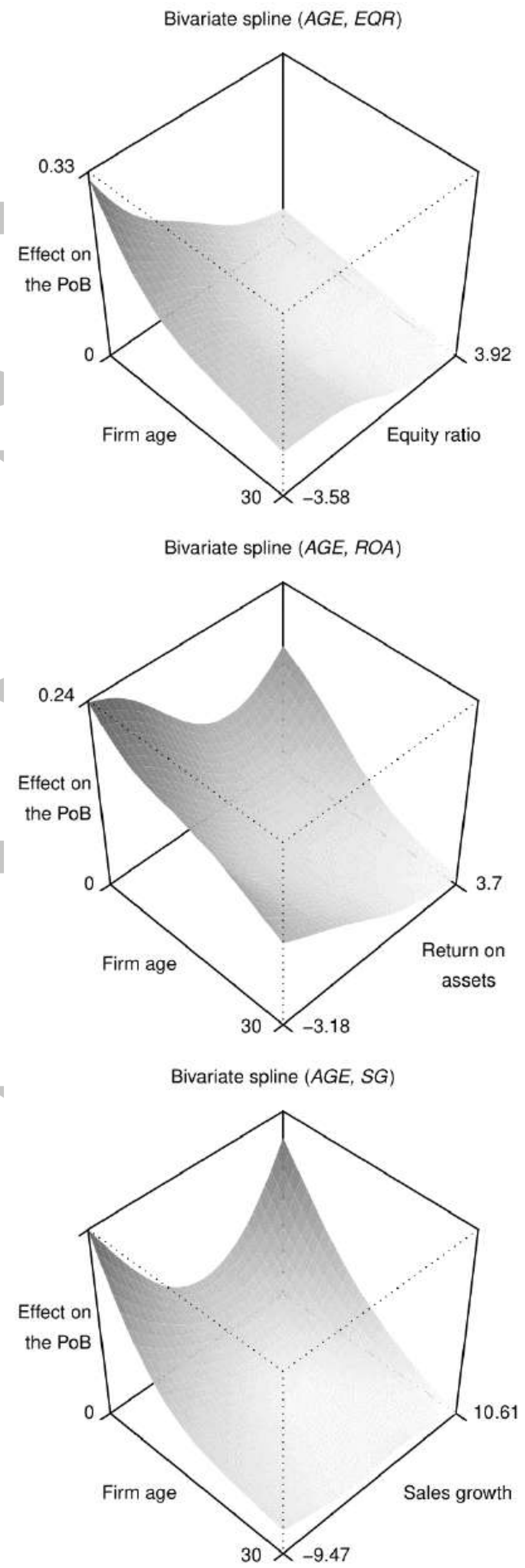

Fig. 4. Two-dimensional spline patterns that display the effects on the probability of bankruptcy (PoB) in models 3a–3e.

Appendix

Table A1. Metric accounting-based independent variables.

Metric accounting-based independent variable	Numerator	Denominator
Equity ratio (*EQR*)	Equity – Outstanding contributions to subscribed capital – Capitalized start-up and business-expansion expenses – Intangible assets + 0.7 · special items with an equity portion	Adjusted balance sheet total (*BST*)
Fixed assets ratio (*FAR*)	Fixed assets – Intangible assets	Adjusted balance sheet total (*BST*)
Return on assets (*ROA*)	Operating income	Balance sheet total – Outstanding contributions to subscribed capital – Capitalized start-up and business-expansion expenses – Intangible assets – Financial assets
Adjusted balance sheet total (*BST*)	Balance sheet total – Outstanding contributions to subscribed capital – Capitalized start-up and business-expansion expenses – Intangible assets – Securities allocated to current assets – Cash and cash equivalents	None
Sales growth (*SG*)	Sales – Sales previous year	Sales previous year

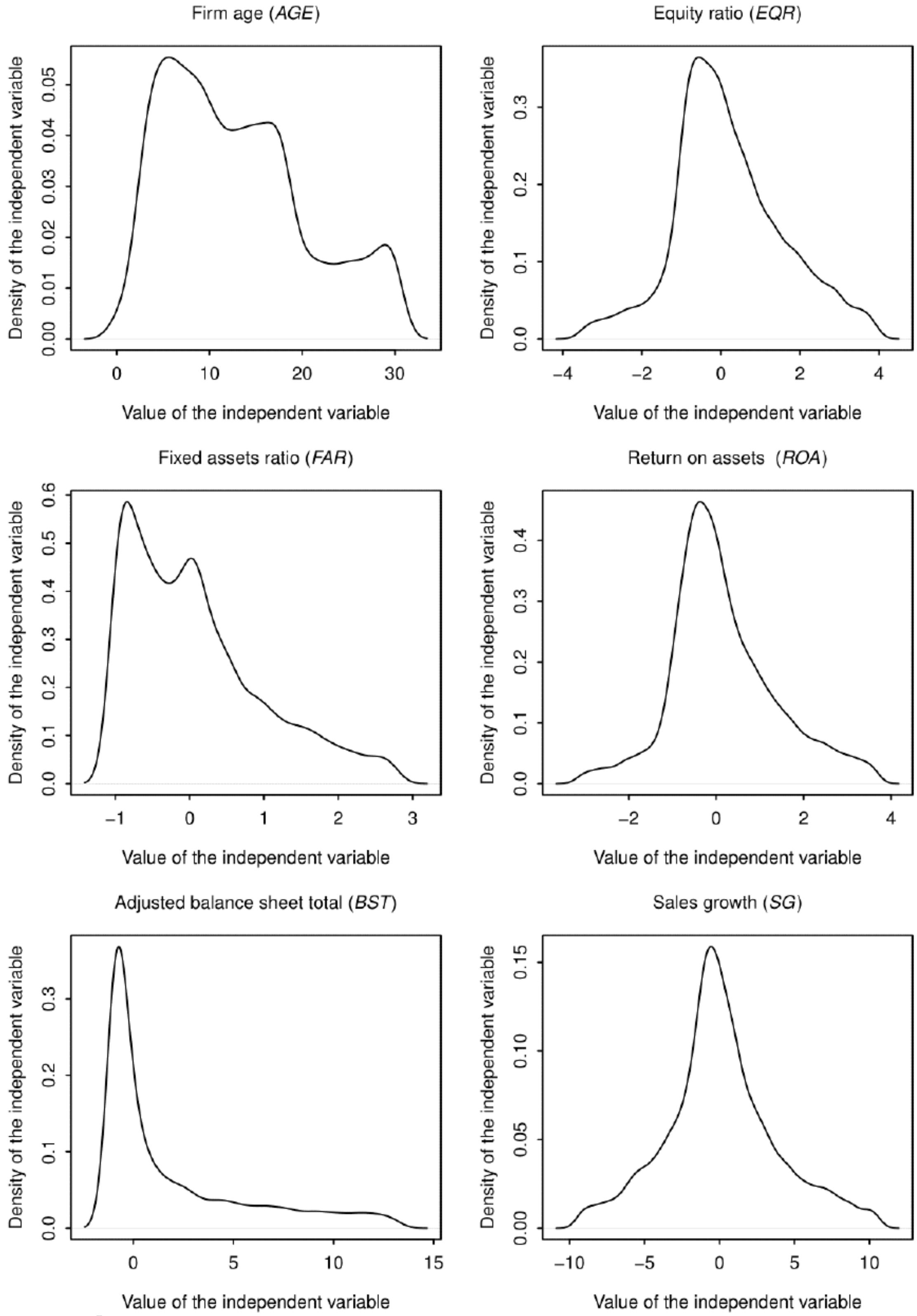

Fig. A1. Kernel density estimations of the metric independent variables.