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Vulnerability analysis of water distribution networks to accidental pipe burst

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ABSTRACT

Even the best-maintained water distribution network (WDN) might suffer pipe bursts occasionally, and the utility company must reconstruct the damaged sections of the system. The affected area must be segregated by closing the corresponding isolation valves; as a result, the required amount of drinking water might not be available. This paper explores the behaviour and topology of segments, especially their criticality from the viewpoint of the whole system. A novel, objective, dimensionless, segment-based quantity is proposed to evaluate the vulnerability of both the segments and the whole WDN against a single, incidental pipe break, computed as the product of the probability of failure within the segment and the amount of unserved consumption. 27 comprehensive real-life WDNs have been examined by means of the new metric and with the help of complex network theory, exploiting the concept of the degree distribution and topology-based structural properties (e.g. network diameter, clustering coefficient). It was found that metrics based purely on topology suggest different network behaviour as vulnerability analysis, which also includes the hydraulics. The investigation of the global network vulnerabilities has revealed several critically exposed systems, and the local distributions unveiled new properties of WDNs in the case of a random pipe break.

1. Introduction

As clean drinking water is a fundamental demand of the population, the water distribution networks (WDNs) are one of the most elementary infrastructure of every modern settlement from a small village to a large metropolis. Proper operation of these systems is essential from the viewpoint of the inhabitants' health, living standards and industrial efficiency. Predicting the robustness or the vulnerability of a WDN during the design phase or in the case of working networks is still a challenging task nowadays. Even with the help of a large Geographic Information System (GIS) database and a detailed hydraulic model, the consequences of a pipe burst is difficult to predict accurately. However, the vast computational potential, which is provided by CPU or GPU clusters, does not redeem efficient and system-tailored algorithms (e.g. in the case of creating shutdown plans for a metropolis).

For the solution of the issues mentioned above, researchers nowadays are mostly applying two out of the novel mathematical tools developed in the last decades. On the one hand, machine learning is starting to penetrate into the analysis of the public utility networks, e.g. Zhou et al. (2019), Dini and Tabesh (2014), Romano, M. et al. (2014). These algorithms build a mathematical model based on sample data, known as "training data", in order to make predictions or decisions without being explicitly programmed to perform the task. On the other hand, complex network theory Barabási (2016) made an appearance, where large networks (e.g. internet: Albert et al., 1999, social networks: Liljeros et al., 2001, protein interactions: Jeong et al., 2001; Wagner, 2003) are studied with the novel toolbox of graph theory. As it will be presented later in details, this research belongs to the latter group; the presented results were achieved by borrowing concepts from complex network theory.

This study analyses the overall behaviour of the water distribution network in the case of a single, accidental pipe burst. Due to a pipe break, the corresponding utility company must depressurise the affected area, creating the possibility for the reconstruction works, but also depriving the drinking water from potential consumers. By applying a hydraulic model, the smallest island can be determined which necessarily needs to be segregated via the closing of the isolation (ISO) valves. Instead of the general pipe-node representation (e.g. Agathokleous et al., 2017; Shuang et al., 2014; Yazdani et al., 2011), the segment-valve approach (idea

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after Walski, 1993) is used, where the segment graph is built up using the ISO valves as the edges, and the nodes as the segregated islands between the valves. This means that, from the network theory point of view, the depressurisation of a segment can be modelled as the loss of one node from the segment graph. The segment graph itself is investigated borrowing complex network theory tools. The focus of the current studies: whether a specific graph type can describe the segment graph of a WDN universally; and how that affects its overall operation in the case of a single, accidental pipe break.

Also, we wish to explore the possibility of estimating the effect of a single, accidental pipe break using the hydraulic model from the viewpoint of the consumers. During the reconstruction, beyond the segregated area that is not fed, there might be consumers "accidentally" excluded, e.g. closing an ISO valve at the beginning of a linear pipeline, while there are several segments downstream, these are so-called outage segments, see Walski et al. (2006). On the other hand, there might be regions which are topologically connected, yet the system pressure is still insufficient to serve the demand entirely. We provide methods to analyse the overall behaviour of the system in the case of an accidental pipe break, and locate the most vulnerable areas. We are also interested in the differences in terms of vulnerability in the network, or in other words, how the vulnerability is distributed throughout the network. Furthermore, since utility companies are responsible for multiple independent networks, how can two different networks be compared to each other in terms of their vulnerability. The calculations of this paper could support decision making in the distribution of financial sources.

This paper is organised as follows. First, the essential mathematical tools (e.g. hydraulic model solver that includes pressure-dependent demands) are introduced in the following section. These methods are similar to the ones found in literature or in commercially available programs; however, we found it important to clarify every step in details due to the reproducibility of the current work, as for the analysis an in-house hydraulic solver was used. It is followed by presenting the results from the analysis of the topological attributes of the real-life WDNs segment graph in Section 3. After that, Section 4 introduces a new quantity to objectively measure the vulnerability of a WDN with respect to a segment in the case of a random pipe burst. The last section before the conclusions reveal the "scale-free" nature of WDNs. To test the ideas and new concepts in this paper, 27 comprehensive real-life WDNs of Hungary were used, ranging from relatively small WDNs with a few tens of segments up to larger ones with several hundreds of segments. Finally, due to the relatively wide range of fields covered in the paper (notably hydraulics of WDNs, vulnerability of WDNs and complex graph theory), we have found it more convenient to cite the relevant literature always at the beginning of the corresponding section.

2. Mathematical tools

Nowadays industrial and scientific hydraulic modelling is dominated by the EPANET solver (Rossman, 2000) which is an efficient 1D hydraulic solver, with numerous sophisticated features, e.g. the capability of the active element handling (e.g. Flow Control Valve) and determining chlorine concentration. Several commercially available software packages are built on this code, e.g. WaterGEMS (see Wu et al., 2007) or MIKE (see Ekklesia et al., 2015). Moreover, as the source code of EPANET is available, extensions are also possible in order to develop more complex tools, e.g. pressure-dependent consumptions (Muranho et al., 2014; Pathirana, 2012. However, as the numeric solver in EPANET is highly specialised, it might be cumbersome to implement new models or techniques; indeed in Abdy Sayyed et al. (2015) the authors needed to combine a check valve and a flow control valve to model pressure-dependent consumption.

The present study employs the in-house code `staci` available at Wéber et al. (2020), which is similar to EPANET apart from a few modelling and numeric issues described in this section. The full control over the source code made it more efficient and flexible for us to experiment with new ideas and techniques. Computationally `staci` it is not as efficient as the EPANET solver, but due to its modularity and general framework (there is no structural restriction on the modelling equations), the implementation efforts of new methods can be lower. Nevertheless, all of our hydraulic results reported in this paper were validated against EPANET. Beyond standard hydraulic simulations, this study also makes use of segmenting WDNs. Such tools can be found in commercially available programs as well, e.g. WaterGEMS (Abdel-Mottaleb and Walski, 2020; Trietsch and Vreeburg, 2005).

2.1. Hydraulic simulation

The main assumptions are the followings: 1D flow in the pipelines, incompressible fluid and steady-state operation. The applied modelling software was an in-house hydraulic toolkit, called `staci` (Wéber et al., 2020), that is implemented in C++ language. However, the underlying equations are the same as those in EPANET (Rossman, 2000). On the one hand, the conservation of mass is ensured by solving

$$\sum_{i\in\text{in}} Q_i - \sum_{i\in\text{out}} Q_i = c \tag{1}$$

for every node, where Q stands for the volume flow rate, "in" denotes the pipelines those are pointing inwards, while "out" refers to the outward directed pipes and the c indicates the nodal consumption. At this point, the consumption is considered to be independent of the pressure.

On the other hand, set of nonlinear equations is solved for the conservation of energy through the edges (mainly pipelines, but also pumps, valves, etc.) and formally, it is given by

$$p_s - p_e = f(Q) \tag{2}$$

where p_s denotes the pressure at the starting node of the edge, while p_e at the ending node and f is usually a nonlinear function of the volume flow rate, depending on the exact type of the edge. The most common element is a single pipeline, where function $f(Q)$ contains the geodetic height difference and the friction losses that can be modelled either by the Darcy-Weisbach or the Hazen-Williams model.

These equations form a system of $N_{\text{nodes}} + N_{\text{edges}}$ number of nonlinear equations (with N_{nodes} being the number of unknown nodal pressures and N_{edges} is the number of unknown edge flow rates), which can be conveniently rearranged as

$$\mathbf{F}(\mathbf{x}) = \mathbf{0} \tag{3}$$

where $\mathbf{x} = [\mathbf{p}, \mathbf{Q}]^T = \left[p_1, p_2, \ldots, p_{N_{\text{nodes}}}, Q_1, Q_2, \ldots, Q_{N_{\text{edges}}}\right]^T$ contains the unknown hydraulic variables. This set of equations is solved using Newton's technique that is a general algebraic equation solver:

$$\mathbf{J}(\mathbf{x}_n)(\mathbf{x}_{n+1} - \mathbf{x}_n) = -\mathbf{F}(\mathbf{x}_n), \tag{4}$$

that is a linear set of equations where $\mathbf{J}$ is the Jacobian matrix: $J_{i,j} = \partial F_i / \partial x_j$. Note that $\mathbf{J}$ is a sparse matrix, meaning that, there are only a few non-zero elements in each row. For solving sparse, linear equations efficiently in C++ environment, one can use several packages, e.g. `UMFPACK` (Davis, 2004); however, for this study, the `Eigen` library (Gael and Benoi, 2010) was implemented. This solution method differs from the EPANET, where a more efficient, special iteration is implemented. As a result of the generalised algebraic equation solver, the presented approach is more universal, and different extensions or models can be added in a straightforward way without the need for modifying the solver.

2.2. Modelling pressure-dependent consumptions

As it can be seen from the presented equations, basic hydraulic solvers work with constant consumption values, i.e. the amount of water leaving the WDN at the nodes are independent of the nodal pressure. The accuracy of this approach is usually acceptable since a properly operating WDN should provide enough pressure to serve every demand in the network. However, in some cases (e.g. damaged systems or in the case of large consumption e.g. hydrant operation) the pressure might drop drastically, and the consumption will be below the nominal demand. However, even if the network pressure is low, a large portion of the demands is volumetric and can be adequately supplied during outages (Schück and Lansey, 2018). Besides, even during normal operational circumstances, a certain part of the consumption depends on the nodal pressure, e.g. leakages. Based on the literature, several different approaches are available for the accurate modelling of such phenomena (e.g. Klise et al., 2017; Pathirana, 2012; Wagner et al., 1988); during this study, the following function is used, following Abdy Sayyed et al. (2015),

$$c(p) = \begin{cases} d & \text{if } p > p_{\text{des}} \\ d(p - p_{\min}/p_{\text{des}} - p_{\min})^{1/m} & \text{if } p_{\text{des}} > p > p_{\min} \\ 0 & \text{if } p < p_{\min} \end{cases} \tag{5}$$

where d denotes the nominal demand (when the consumer is fully served), $p_{\min}$ and p_{des} represents the lower and upper boundaries for pressure (in m.w.c.), respectively. while m is the exponent. Fig. 1 depicts the pressure-demand relationship with different exponent values. During this study, the following parameters were used: $p_{\min} = 10$ m, $p_{\text{des}} = 25$ m and $m = 2.5$, that is also highlighted in the Figure. Due to the general solution of the nonlinear, algebraic equation system, additional modifications are not necessary.

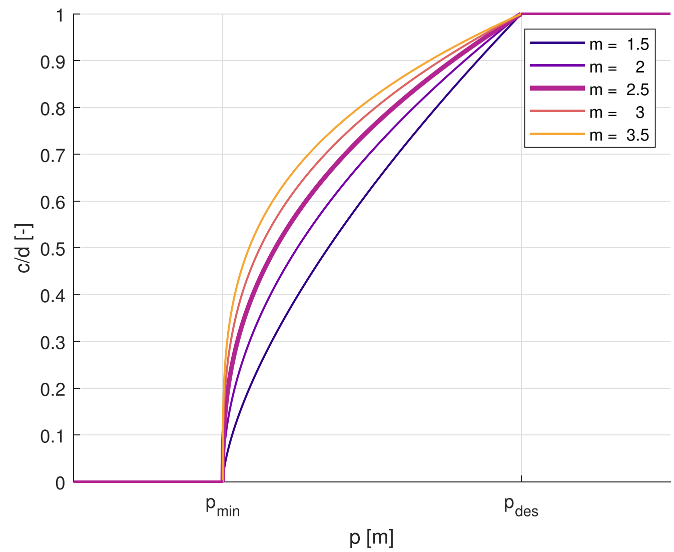


Fig. 1. Illustration of modelling the pressure-dependent demands with different exponents.

2.3. Building the segment graph

A properly operating WDN (or its zones) are fully connected graphs; however, in some exceptional cases, it might occur that, some part of the system must be manually segregated with ISO valves, for example, if pipe failure emerges. As a result, an area might become detached and smaller, unconnected islands are formed. For handling such cases, one can build up the segment graph, where, instead of the general link-node representation, the smallest parts of the network, that can be isolated (i.e. segments) become the graph nodes, and the regarding ISO valves are the links between them. The importance of the analysis of the segment graph in terms of reliability was first presented in Walski (1993). There are two different approaches in the literature for building the segment graph: breadth-first search and depth-first search, see Barabási (2016), Li and Kao (2008) for details. Based on these ideas, several different algorithms were developed recently for WDNs (e.g., Alvisi et al., 2011; Jun and Loganathan, 2007). Giustolisi and Savic (2010), Creaco et al. (2010) showed different methods for the identification of segment graph (and also suggested methods for the optimal layout of the ISO valves). Since the size of the hydraulic models is continuously increasing, the efficiency of these algorithms might be a critical issue; thus, Gao (2014) presented

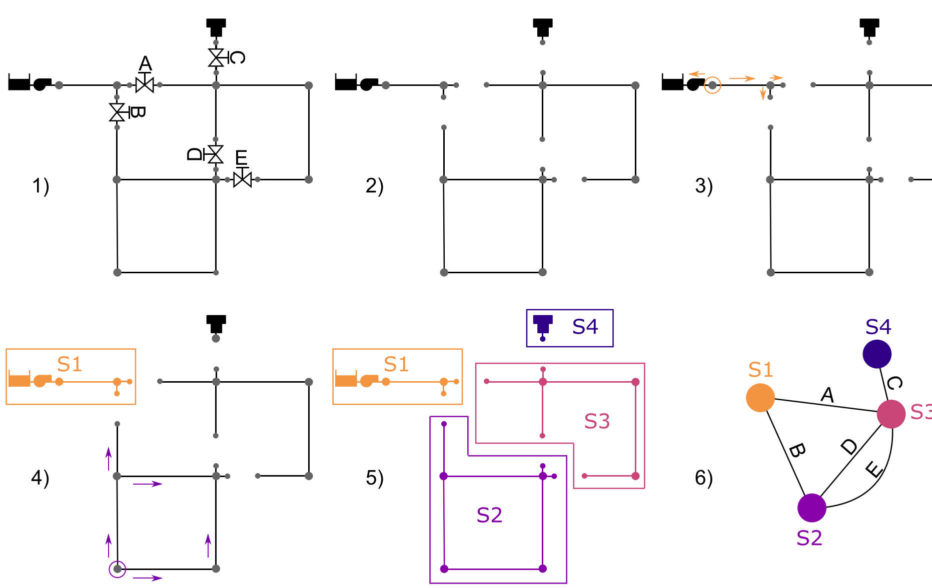


Fig. 2. Demonstrating the algorithm that builds the segment graph of a WDN.

an efficient method and demonstrated it using a WDN containing 70k+ pipelines.

The results in this paper were obtained by a depth-first search-based approach, which is similar to the one published in Huzsvár et al. (2019). Fig. 2 demonstrates the steps of the algorithm using EPANET's test network Net1, in which five ISO valves are present. First, the ISO valves are removed (see inset 2), then, starting from the first node of the system, the algorithm finds every connecting node (inset 3). Once it is unable to reach yet unvisited nodes, it verifies that every node of the current segment has been discovered, and the next node is selected that is not part of the already visited segments (inset 4). This mapping repeats until every component of the network has been assigned to a segment (inset 5). The last step is the identification of the ISO valve placements; the algorithm connects two segments with a link if the segments are having a mutual ISO valve, e.g. in Fig. 2 Valve A connects segments S1 and S2.

3. Topological properties

Complex network theory is a new research area born around the end of the last millennium, allowing scientists to analyse large graphs containing thousands, or even tens of thousands of edges or nodes, e.g. internet, actor-network or protein interactions, see Barabási (2016). A significant discovery in this field was that most of the real-life networks cannot be approximated accurately by the traditional random network models proposed by Erdős and Rényi (1959); but realistic networks are often "scale-free" (Barabási and Albert, 1999), meaning that most of the nodes are negligible (they can be removed without significantly affecting the behaviour of the network), but there also exist hubs where edges tend to accumulate rendering them critical in terms of the connectivity of the graph. Albert et al. (2001) analysed how these networks (both random and scale-free) behave against errors and attacks. They have found that the latter one (scale-free) is significantly more vulnerable, since the loss of hubs could easily provoke the topological detachment of the network into several subgraphs. Such features are also important from the practical point of view: the loss of some critically important parts of the electrical network caused the New York City blackout in 1977.

During the investigation of the behaviour of real networks (e.g. Albert et al., 1999; Estrada, 2006), numerous methods and algorithms were developed, which are used in the area of utility systems as well. Based on the topology structural properties are evaluated for real-life or artificial WDNs (e.g. KY* networks Jolly et al., 2014) to understand their overall operation (Yazdani and Jeffrey, 2011), to design extensions (Yazdani et al., 2011), to quantify the robustness (Yazdani and Jeffrey, 2012) or to identify the critical components (Diao et al., 2014). Some papers are attempting to discover a quantitative relationship between a purely topological parameter and the hydraulics, see Pandit and Crittenden (2016), Meng et al. (2018), Torres et al. (2016). These techniques are valuable for the designers of WDNs, since some aspects of the network can be approximated as early as during the planning phase, without a detailed hydraulic model; as building such models require an enormous amount of data (e.g. GIS data) and engineering work. Most of the works in the literature are using the general pipe-node representation as a graph for the WDN, even in the case of analysing the robustness, resilience or vulnerability. However, this study follows the idea from Walski (1993) and analyses the segment-valve approach, since the focus of the paper is to investigate the general effect of a single, accidental pipe break, that is, a loss of a node in the segment graph. The main goal is to explore the general behaviour of the real-life WDNs in terms of their segment graph.

Table 1
General information about the examined real-life WDNs, representing the comprehensiveness of the networks.

Property	Min	Max
Number of nodes	301	7188
Number of segments	12	717
Overall pipe length, km	4.59	112
Total nominal consumption, m^3/h	0.4	540

3.1. Degree distribution of the segment graph of real-life WDNs

In the field of graph theory, the degree of a node is the number of edges connecting trough that node. Node degree can also be defined for directed graphs; however, in this study, we are considering only undirected networks as water can flow in both directions of a pipe. The degree distribution p_k provides the probability that a randomly selected node in the network has degree k. Since p_k is a probability, it must be a normalized quantity, i.e. $\sum p_k = 1$. Traditionally, real networks were assumed to be random graphs (which degree distribution follow Poisson distribution); however, in recent years it was revealed that the degree distribution of certain real-life networks is following power law (see Barabási, 2016), and this behaviour is called scale-free. The main difference is that the latter ones (scale-free) have a relatively high probability for containing nodes with a high degree (called hubs), that are critical in terms of the connectivity of such networks. The degree distribution of a random graph is typically modelled using the Poisson distribution. At the same time, a scale-free network follows a power law distribution, i.e. it is linear in the case of a log-log scale, see Fig. 3.

For the purposes of this study, we collected 27 comprehensive, real-life WDNs from Western-Hungary, the main properties can be observed in Table 1. As it can be seen, they cover a wide range from the smallest one with only twelve segments, to the largest one that provides hundreds of cubic meter per hour of drinking water. The average number of the isolation valves in one kilometre is 5.45, while the median is 5, which seems low compared to 12 that was found by Walski (2011) for 13 real-life WDNs. This implies that the isolation valves are much sparser located throughout the networks. In the case of the segment graph of a WDN, the degree of a segment indicates how many ISO valves are required to segregate that specific part of the network. By applying the algorithm presented in the previous section, the segment graphs were built for the WDNs; then the degree distributions of segment graphs were analysed. In the case of an isolation, it might happen that further segments of the network are segregated unintentionally, e.g. closing a dead-end line at the beginning, while there are more segments downstream, i.e. outage segments (Walski, 1993). It can happen that not every isolation valve must be closed for proper segregation due to the connection of an outage segment. However; during this study, it is still considered that all of the ISO valves have to be closed for a proper segregation, since it might also happen as a result of the geodetic differences that water is flowing backwards from an originally downstream segment.

Fig. 4 shows the results of the degree distributions of the segment graph of the 27 real-life WDNs. As it can be seen, the characteristics of these distributions are more similar to the Poisson distribution. The mean degree is 2.25 ($\pm$ 0.19) which means that on average to isolate a segment, 2.25 number of ISO valves must be closed. The explanation behind this surprisingly low number (compared to 2.55 found in Walski et al. (2006)) lies behind two facts: first, a large amount of the segments are dead-ends (see Fig. 4) which only need one valve for shutdown, and second, the N-rule (Liu et al., 2017) is applied at most of the junctions. This also seems contradictory to the low density of isolation valves per

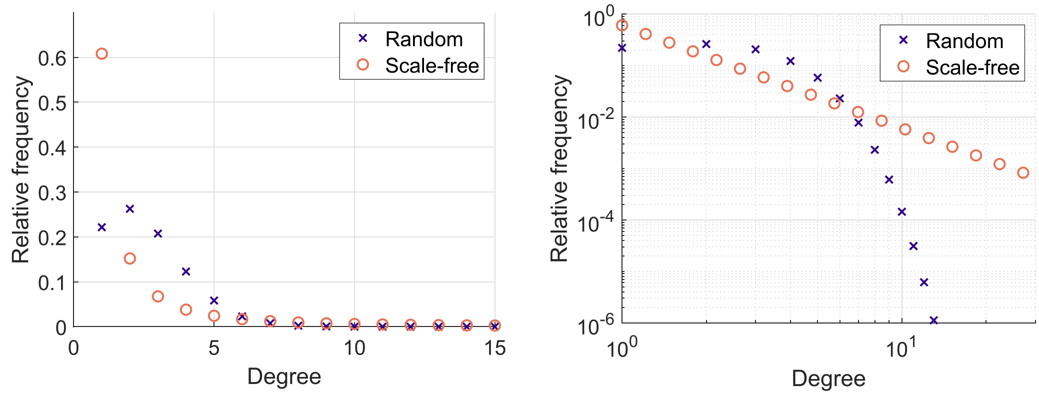


Fig. 3. Typical degree distribution of random and scale-free graph on log-log scale.

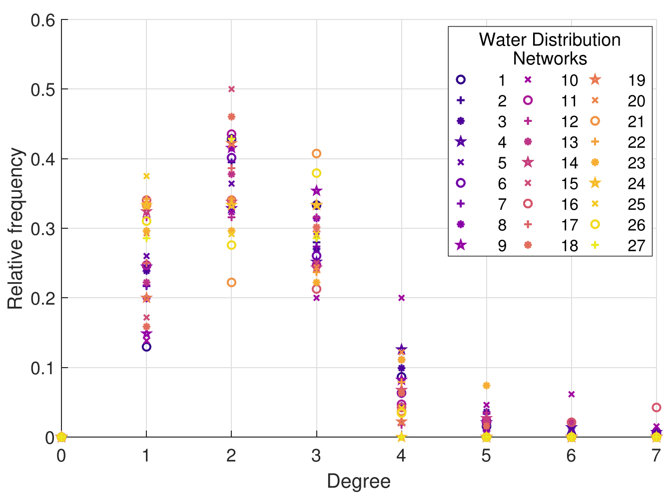


Fig. 4. Degree distributions of the segment graphs of 27 real-life WDNs.

kilometre, but the explanation is simply that our networks contain long straight pipelines and also in branches the N-rule is followed. Analysing the segments with high degrees, as it can be seen in the Figure, the maximum degree is seven and the numbers of segments with at least 4 degrees is less than 10%. Based on these results, it can be summarised that these networks do not contain hubs which would be critically important in terms of the topological connectivity. Furthermore, it seems that based on the degree distribution, in terms of the topology, the segment graph of real-life WDNs are behaving as robust random graphs, rather than vulnerable scale-free networks, because random graphs does not contain nodes with high degree i.e. the segments in WDNs can be isolated using only several valves.

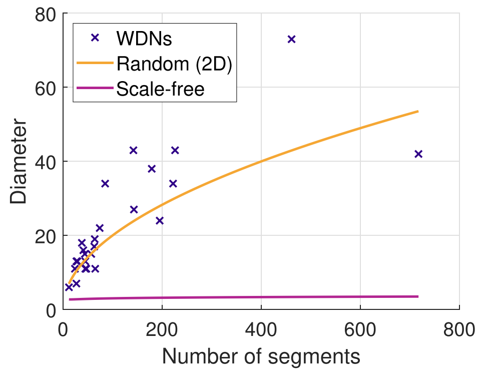
3.2. Diameter and clustering coefficient

The last decades brought numerous topological indicators for quantifying different behaviours of networks (again, we refer to Barabási, 2016) which are already exploited in the field of WDNs, e.g. Yazdani et al. (2011). In this paper, we are focusing on two of them, namely the diameter and the clustering coefficient. For the efficient calculation of these quantities, the `igraph` toolbox (Csardi and Nepusz, 2012) was employed. A path between two distinct nodes in a graph is the sequence of edges. Since typically for two nodes, there are numerous different paths, the shortest path is considered, where the number of edges along the path is the smallest. In the case of a network, between every pair of nodes, the shortest paths can be determined, and the length of the longest shortest path is called the diameter of the graph. This means that from any node of the network, every other node can be reached, while the number of edges along the path included is below (or in some specific case equal to) the diameter.

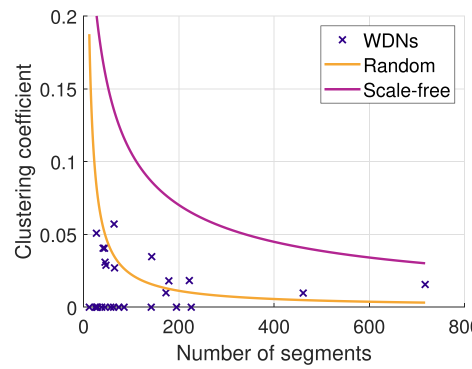
If the diameter of the network is small for a large network, then the system might have a critical part in terms of the connectivity (see Yazdani and Jeffrey, 2011); a hub which connects a large part of the nodes. Since this indicator is able to predict the possible hubs in the network (where connections tend to accumulate), this quantity represents a strong relation with the network robustness and vulnerability. For some special graph, the diameter can be estimated; e.g. in the case of a 2D grid-like random network, it is $\approx 2\sqrt{N}$ where N is the number of nodes (Barabási, 2016); however for a scale-free system, it can be approximated using $\log(N)/\log(\log(N))$ according to Bollobás and Riordan (2004a). The left side of Fig. 5 depicts these correlations, and also the data points of real-life WDNs. As it shows, even the prediction of the random graph is slightly underestimates the diameter of real-life WDNs.

The second commonly analysed parameter is the clustering coefficient of a graph, which characterises the average density of the graph, i.e. closeness to the complete graph (where every distinct pair of nodes is directly connected with a unique edge). This quantity can be defined as the average number of the connections between the neighbours of an arbitrarily selected node in the network, see Meng et al. (2018). Therefore, the clustering coefficient tend to be high in the case of a highly looped network, while zero for a tree-like system. For a scale-free network, the clustering coefficient is expected to be high (due to the hubs), while for random networks it is expected to be close to zero. This quantity can also be approximated for special cases; for a random network we have $\approx \bar{k}/N$ where $\bar{k}$ is the average degree (that is 2.25 for our WDNs), and for a scale-free network $\approx \log(N)^2/N$, see Bollobás and Riordan (2004b). The right side of Fig. 5 presents the results for the approximations with the results from the real-life WDNs. The scale-free estimation is clearly incorrect, while the prediction from the random graph is significantly closer. It also important to highlight that the clustering coefficient of 14 networks (out of 27) equals to zero.

Based on the results of diameter and clustering coefficient, strictly from the topological point of view, the segment graph of real-life WDNs behave similarly to random graphs, rather than scale-free networks. This indicates that, based on using only the previously described "traditional" complex network theory concepts, the real-life WDNs are robust against random pipe failures.



(a) Diameter



(b) Clustering coefficient

Fig. 5. Diameter and clustering coefficients of real-life WDNs compared to random and scale-free network approximations.

However, there are also differences. On the one hand, a segment graph of a WDN is necessarily connected, i.e. there cannot be nodes (segments) separately from the main body of the system. On the other hand, it is essential, that a WDN is a planar network (i.e. edges cannot cross each other without a node), which is not valid for random graphs in general. In overall, based on the degree distributions and the structural properties, the segment graph of a WDN behaves like a connected, planar random graph. The corresponding literature on complex graph theory generally affirms that scale-free networks are much more sensitive than random networks to attacks indicating that random networks are more robust (see e.g. Guillaume et al., 2005). Our result (that is, the WDN analysed here are similar to random graphs) is counterintuitive in the sense that WDNs are known to be vulnerable in the hydraulic sense even though there is an intentionally engineered redundancy in them. For example, isolation of central segments (in the downtown) change the system hydraulics completely or multiple pressure zones connected in series at the outer verges of the network might also be weak points.

3.3. Universal degree distribution function for real-life WDNs

The purpose of the current subsection is to create a universal function, which is able to approximate the degree distribution of the segment graph of a general real-life WDN. Based on the degree distribution and the structural properties, it seems that the segment graphs of WDNs behave as random graphs; thus, one might fit the Poisson distribution for describing the degree distribution in general. However, the fact that the topology of a WDN is planar by nature and every node is necessarily connected; thus there are no nodes with zero edges, i.e. there are no segments without ISO valves; these properties make this approach unsuitable. For example, if one would like to describe a WDN using a Poisson distribution with average degree of 2.25, more than 10% of the nodes would have zero degrees. Due to these difficulties, another idea came up to create a general function for the approximation of the degree distributions. The function was defined by taking the average of the relative frequencies at each degree (for numerical values, see Table 2), that is still a distribution function, i.e. the sum is equal to one. This fitted function (also the Poisson distribution) can be observed in Fig. 6 with the real-life WDNs.

A statistical hypothesis test, namely chi-squared test (see Montgomery and Runger, 2003), was performed for the investigation of the results' homogeneity, i.e. it was analysed whether the degree distributions of the real-life WDNs follow the same distribution as the fitted function or as the Poisson. In the case of 95% of significance level, 85% (precisely 23 out of 27) of the WDNs follows the fitted function, while that is only 51% (14 out of 27) for the latter one. Based on these calculations, the degree distributions of real-life WDNs' segment graph follow a unique distribution; and most of the analysed WDNs, within these ranges of system properties (see Table 1), can be described by this fitted function.

The practical benefit of this degree function is twofold. On the one hand, this could be a basis for comparison for different WDNs, especially coming from outside of Hungary or even outside of Europe; it would be interesting to see if other WDNs share the same property. On the other hand, this could be a guideline for WDN planner for judging the location of isolation valves. If the

Table 2
Universal degree distribution function numerically for describing the segment graphs of real-life WDNs.

Degree	Rel. freq.
0	0
1	0.2538
2	0.3690
3	0.2846
4	0.0682
5	0.0160
6	0.0055
7	0.0029

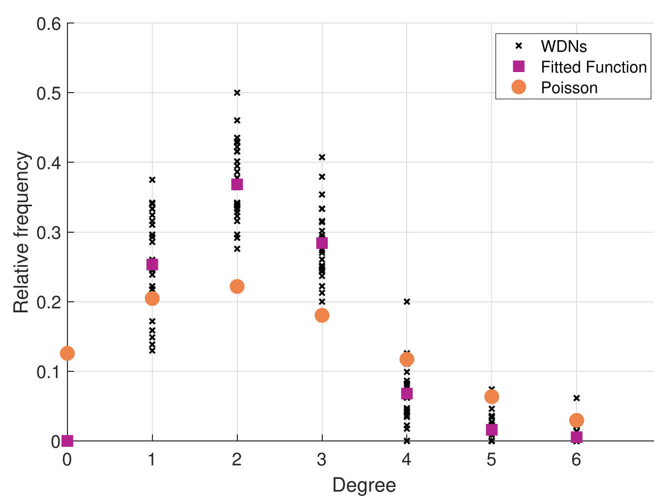


Fig. 6. Universal degree distribution function (squares), data points of real-life WDNs (crosses) and the Poisson distribution (circles). Average degree of WDNs: 2.25.

distribution for a WDN is dominated by low degree (that is, most of the segments can be isolated by one or two ISO valves), the system is more robust; but if there are numerous segments with high degree, operating problems might arise due to the need of shutting several ISO valves (see also Huzsvár et al., 2019).

4. Vulnerability of WDNs

In the previous decades, numerous publications focused on the quantification of the resilience (Todini, 2000), reliability (Zhuang et al., 2013) or vulnerability (Shuang et al., 2014) of different type of networks. The main goal in the literature is to properly define a parameter that describes these aspects of wastewater (Sweetapple et al., 2018) networks, urban drainage systems (Wang et al., 2019), water resources infrastructure (Porse and Lund, 2016) or (as in our case) WDNs (Diao et al., 2016). The idea is to define a quantity, which is able to characterise the behaviour of networks from a selected aspect, hence it could support decision making in the case of new network design or during network condition analysis. Even if these networks are serving different purposes, analogous ideas are applied in the literature, e.g. the resilience analysis was used in Mugume et al. (2015) for the investigation of urban flooding and in Diao et al. (2016) for WDNs. A review article was presented in Hosseini et al. (2016) about the resilience analysis of several different types of networks. These publications strengthen the attitude that applying ideas from neighbouring research areas might lead to novel results.

While numerous studies were dealing with the question of resilience, reliability, or risk analysis of the different type of utility networks; this paper analyses the vulnerability of the segment graphs of WDNs. The goal of our study is to discover the average behaviour of WDNs in a single, accidental pipe break. Therefore, in this approach, it is considered that the probability of a possible pipe burst is equal throughout the system, i.e. every meter of pipelines has the same chance to break. This methodology can be extended by weighting the probability using data of the WDNs, e.g. if there is a network specific connection between the frequency of pipe bursts and the material of the pipeline, a case study can be performed with increased accuracy. During this study, we assumed that every isolation valve is properly operating, thus for the isolation of a single pipe break, only one segment must be depressurised. Important to mention that for the analysis of a specific real-life network, this hypothesis might not be thorough, and the topic of improperly working isolation valves can also be a research area (Abdel-Mottaleb and Walski, 2020; Liu et al., 2017). Although some papers (Gheisi and Naser, 2014; Jacobs and Goulter, 1991; Sweetapple et al., 2019) are suggesting that pipe or node failures tend to accumulate, this study focuses on a single segment loss (that is originating from a single pipe break). Finally, the nominal demands of the real-life WDNs are calculated based on the billing system, and yearly averages are defined. Again, the purpose of the study is to discover the overall behaviour of a WDN in an accidental pipe break; the method can be extended in this aspect as well by taking into consideration, e.g. adjusting the demands using specific pattern (daily, weekly or monthly), thus performing a detailed analysis on certain cases.

4.1. Definition of vulnerability

Maiolo et al. (2018) reviewed a few different definitions of vulnerability from the literature, and most of them are based on topological approaches. The goal of the vulnerability, according to the definition in this paper, is to catch the general hydraulic behaviour of WDNs in a single pipe break, thus it is based on the hydraulic model. Important that most of the vulnerability definitions from the literature belongs either to pipelines or nodes, while this paper is focusing on segments, similarly to Abdel-Mottaleb and Walski (2020). As a results, every element from a segment is having the same vulnerability, since every failure is causing the loss of the whole segment. The quantity is also dimensionless, thus it can be applied for comparing WDNs with different sizes. First, the length of every pipeline in the ith segment is denoted by L_i (typically there are several pipelines in one segment); also, it is normalized using the overall pipe length throughout the network (ΣL_i), thus $\lambda_i = L_i / \sum L_i$. This λ_i quantity denotes the relative pipeline length, and it also indicates the failure rate of the ith segment. Second, the amount of drinking water, that cannot be provided in the case of the segregation of the ith segment, is calculated by the 1D hydraulic solver, i.e. $b_i = \sum d_i - \sum c_i$, where d_i indicates the nominal demand and c_i represents the actual amount of served water according to the model. For calculating this, one must be able to cope with a) demands cannot be served inside the isolated segment, b) there might be some parts of the network that are segregated unintentionally i.e. outage segments. Furthermore, c) even in the case if a segment is topologically connecting to the main network, the hydraulic conditions might change on a scale, and the pressure of a segment drops drastically, that it is not capable of providing enough water to fulfil the nominal demand, i.e. the hydraulic solver must be able to handle pressure-dependent consumptions. The b_i variable is also normalized using the overall nominal demand, i.e. $\beta_i = b_i / \sum d_i$. This shows the relative loss in the overall nominal demand which is caused by the isolation of the ith segment. This quantity is similar to the "demand shortfall" in WaterGEMS, see Liu et al. (2017), but analogous parameters can be determined with other commercially available software as well (e.g. MIKE URBAN pipe criticality).

Using the previously defined quantities, the local vulnerability (γ_i) of the network with respect to the ith segment is

$$\gamma_i = \lambda_i \beta_i. \tag{6}$$

This parameter is dimensionless and segment-specific, moreover it is a product of the failure rate (λ_i) and the relative loss (β_i) caused by the ith segment failure. According to this definition, the vulnerability is high if a segment both contains a significant amount of pipelines, and causes a considerable amount of loss in the provided drinking water in the case of its isolation. If there is a segment, e.g. close to the major input point (where most of the drinking water is supplied, i.e. β_i is high), but there are only a few meters of pipes (λ_i is small), the vulnerability might be lower compared to a segment with hundreds meters of pipelines (λ_i is high) and with an average loss in consumption (β_i is moderate). Also in the opposite case, if there are long pipelines in a segment, but the loss caused to the consumptions is negligible (e.g. this part is a dead-end of the network), the vulnerability might be still small. To perform a thorough analysis for the behaviour of WDNs, every possible scenario was simulated, i.e. the affect of isolation of every segment was individually calculated.

Applying this definition for the vulnerability is already a valuable tool in order to unveil the critical areas of a WDN, and the network can be colourised according to local vulnerabilities. Fig. 7 shows an example of this, where several critically vulnerable segments are revealed. For the improvement of the WDN, one might consider increasing the number of ISO valves in these areas to reduce the local vulnerabilities.

4.2. "Scale-free" nature of WDNs

During the examination of the spatial maps of local vulnerabilities, it was revealed that in each WDN there are only a few critical segments, while the rest of the networks is far less exposed. Therefore, a detailed analysis was performed on the distributions of local

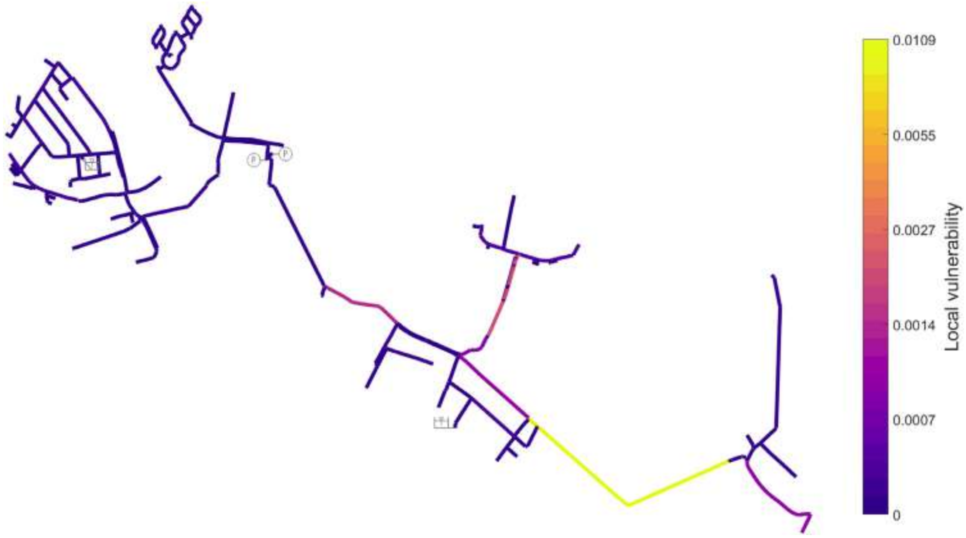


Fig. 7. Vulnerability map of a real-life WDN with logarithmic scale.

vulnerabilities by drawing the probability density function PDF, (Montgomery and Runger (2003)), or also called frequency distribution. This representation is similar to the histogram, but there are also some differences as we are plotting sampled data instead of a continuous variable. The main advantage of this graphical illustration (compared to the traditional statistical metrics, e.g. average, standard deviation or box plot) that it depicts the small details by indicating the dispersion between the minimum and maximum values of the data structure. Moreover, it is capable of visualising the number and the location of stagnation points (where the data points tend to accumulate), and it also unveils the elementary tendencies of the data and highlights the connections of the values. The steps of creating such a graph are the following. First, the whole range between the minimum and the maximum values were split into r number of bins, where $r = \sqrt{N_{seg}}$ if $N_{seg} < 100$ and $r = \log_2(N_{seg}) + 1$ otherwise, where N_{seg} is the number of segments. This means that each network is represented using r number of points in the diagram, that is between four and ten for these WDNs. Second, the boundaries of the bins were defined by sorting the same amount of data into each bin, i.e. the frequencies were equally distributed. Third, while determining the Y coordinate of each point, it is considered that the product of the horizontal width of the interval and the height of the point is equal to the relative frequency; i.e. these points share the same size of areas, which ensures that the cumulative distribution function, which is the integral of the PDF, tends to 1 as x tends to infinity.

Fig. 8 depicts the sampled probability density function of local vulnerabilities (γ_i, see Eq. (6)) for 27 real-life WDNs using log-log scale. Note that almost every WDN contains some segments with zero vulnerabilities, which means that the isolation of these parts will not affect the consumption at all, because there is no demand within the segment, and the looped network can fulfil the water need of downstream segments. These are not interpreted during the analysis of the distribution of local vulnerabilities, because it is not possible to visualise the zero values on a log-log scale. We define highly vulnerable segments as those ones whose local vulnerability exceeds 10^{-2}, meaning that e.g. probability failure is 10% and the segment 10% of the demands is lost if the segment needs to be isolated. (Another segment with 50% of failure probability and 2% of demand loss would also be deemed as vulnerable.) By definition, a segment is moderately vulnerable if its local vulnerability is above than 10^{-3} (that is, in average, it includes 3.15% of pipes and supplies 3.15% of the overall consumption). Obviously, these thresholds are artificial and might be argued.

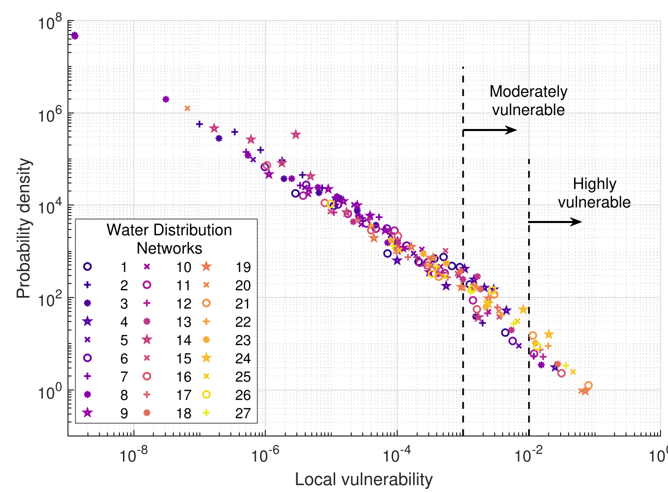


Fig. 8. Probability density functions (PDF) of the local vulnerability values of 27 real-life WDNs. Systems including segments with higher than 1% vulnerability (e.g. 10% of failure probability and 10% of lost demand) are deemed to be highly vulnerable. Note that the symbols are the data points of the PDF and not the segments themselves, however, a data point means that there is at least one segment within the bin range.

The points are located along a linear line that implies power law probability distribution, which indicates similar behaviour to the scale-free networks. Due to the nature of a scale-free network, it contains critical nodes with high degree (hub), which might be significantly higher than the average, and the loss of a hub could lead to the detachment of the network. As Fig. 8 depicts the local vulnerabilities, WDNs also include segments with high vulnerability (see highly vulnerable region), that implies that during the loss of such a segment, serious outage in water service might occur. Besides the highly vulnerable segments, some parts of the networks can be found in the moderately vulnerable region, and most segments can be considered negligible in terms of vulnerability.

The linear trend (that is, power law probability distribution) also highlights that a large part of the network is slightly (or not at all) vulnerable, some regions are moderately vulnerable, but, most

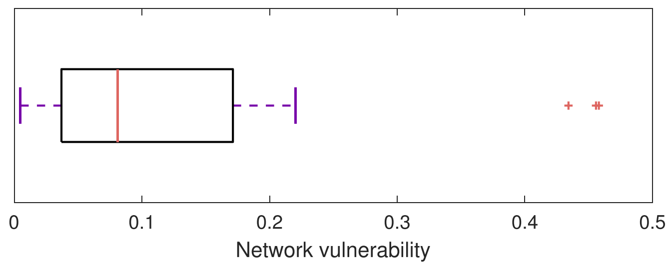


Fig. 9. Box plot of network vulnerability (Γ) of the 27 real-life WDNs, revealing 3 highly exposed (outlier) systems. (Red line: mean value, left side and right side of the box: 25th percentile and 75th percentile, i.e. the box includes 50% of all WDNs.) (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

importantly, there will be a few segments that are highly vulnerable. Distribution of household incomes (there are very few billionaires but the bulk of the population holds modest financial resources) or magnitude of earthquakes follow similar distribution. For example, in the case of Network #8, the probability of a randomly picked segment having larger vulnerability than 10^{-2} (i.e. it is highly vulnerable) is more than 5%, meaning that more than 5% of the segments are highly exposed. In overall, this method is able to emphasise the magnitude differences in the vulnerability of segments, i.e. it can highlight the most exposed parts of the network, due to the highly inhomogeneous distribution of local vulnerabilities.

4.3. Analysing the network vulnerabilities

Besides the importance of the local distribution of vulnerabilities inside a network, it is also useful to evaluate the overall quality of a network from the viewpoint of vulnerability for comparison with other WDNs. Thereby, the decision making of the utility company can be supported in the optimal allocation of reconstruction and development resources. Therefore, the network vulnerability is introduced, which is the sum of local vulnerabilities, that is

$$\Gamma = \sum_i \beta_i L_i / \sum_i L_i = \sum_i \beta_i \lambda_i = \sum_i \gamma_i. \quad (7)$$

This is the weighted average of the local consumption outages, i.e. Γ is the expected value of the amount of water loss in the case of a single, accidental pipe break according to the hydraulic model.

Fig. 9 provides the result obtained by the vulnerability analysis of the 27 WDN via box plot. As it can be seen, most network vulnerability values are under 0.2; however, there are three networks where the Γ value exceeds 0.4 (“outliers”). This means that these networks are highly vulnerable (compared to the rest of the sample) and during a random segment loss (all over the system), the expected value of unserved demand is above 40%. Obviously, this is a statistical analysis and does not mean that if a pipe breaks anywhere in the system, 40% will be lost, but gives an aggregated expected value of the severeness of a random pipe failure. These outlier networks need a scrutiny to unveil the origin of this high, possible outage in the service of demands.

5. Conclusions

This paper presented the analysis of the vulnerability of WDNs with respect to a single, random pipe burst. An algorithm was introduced, that is capable of creating the segment graph of a WDN efficiently, where the edges are the isolation (ISO) valves, and the segments (containing several pipes) are the nodes. A segment-wise measure of local vulnerability was defined as the product of the relative pipeline length (that is proportional to the probability of pipe burst) and the relative amount of unfulfilled demand in the case of the shutdown of the segment. Thus, those segments are considered as vulnerable that have a high probability of failure and the amount of lost demand is high. The concept of network vulnerability was also introduced, that quantifies the expected value of relative consumption that cannot be served due to a single, accidental pipe break. To test these novel concepts, 27 real-life WDNs of Hungary were used with a wide range of pipeline lengths, overall demands and topology. Based on the results, the following conclusions can be drawn.

- The analysis of the degree distributions of the segment graphs unveiled that the number of hubs (nodes with high degree) is negligible, most parts of the WDNs can be segregated by closing not more than 3 ISO valves, and the average is 2.25. Based on this, and the fact that the diameters (in the sense of graph theory) of these networks are large, and the clustering coefficients are almost zero, the behaviour of the segment graph of real-life WDNs is close to a random graph, if only the topology is considered. In other words, strictly from the topological point of view, the segment graph of a WDN is a planar, connected, random graph. Naively, this would suggest that such networks are robust against random pipe burst.
- An approximating degree distribution for the segment graph of real-life WDNs was provided, and a statistical hypothesis test (chi-square test) was performed in order to investigate whether the real distributions could be originated from the fitted one. Based on the test, 85% of the WDNs were accepted (23 out of 27), which means most of the degree distributions of the segment graph of WDNs can described with this general function (at least for the sample used in this study). From the practical point of view, this distribution can be utilised for comparing other networks, especially during the design phase of new WDNs, and it can support decision making in terms of the number or location of isolation valves.
- Beyond purely topological analysis, the proposed new measure for vulnerability exploits also the hydraulic properties of the network via pressure-dependent demands. Our vulnerability indicator is objective (free of user-defined parameters), dimensionless (allows the comparison of different networks), segment-based and can be extended straightforward way towards non-uniform failure distribution over the network. By the detailed investigation of the probability density functions of these local vulnerabilities, it was revealed that the behaviour of WDNs is similar to the scale-free networks in terms of its vulnerability. That is, most segments are slightly vulnerable, while some parts (segments) are critical in terms of lost demand, similar to hubs in a scale-free network. A pipe burst in one of these highly vulnerable segments of a WDN might lead to a serious outage in water service, because either they contain a significant amount of demands, or they transfer a large quantity of water.
- The proposed network vulnerability parameter revealed that 3 WDNs (out of 27) have serious issues in terms of vulnerability, because in the case of a random pipe break, more than 40% of the demands cannot be served according to their hydraulic model. As this parameter characterises the overall quality of the network, it can support the decision making process in the distribution of reconstruction resources or upgrade capacities.

Besides these conclusions, there are some open questions and directions for further analysis and research. In terms of the degree distributions, it would be useful to test more real-life WDNs against our proposed function and explore whether they follow the same distribution. This would be especially interesting in the case of WDNs outside of Hungary or Europe. As the results showed, traditional graph tools (such as degree distribution, diameter, clustering coefficient) cannot reveal the scale-free nature of WDN, thus it would be useful from the practical point of view to find a purely topological parameter (that does not require detailed hydraulic model) that is capable of unveiling this aspect (if such measure exists), because setting up the hydraulic simulation requires significant effort. In terms of the design of a new WDN, one might consider building a new algorithm that is able to create a more favourable distribution of local vulnerabilities to avoid the highly exposed areas. This might also be interesting for already existing systems by recreating the layout of the isolation valves. It would also be straightforward to use the system vulnerability parameter as the objective function of an optimisation process to improve the layout of WDNs.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary material

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