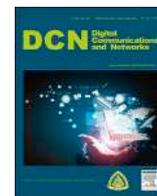




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## Weak harmonic signal detection method in chaotic interference based on extended Kalman filter

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## ABSTRACT

The traditional methods of weak harmonic signal detection under strong chaotic interference often suffer from high computational complexity and poor performance. In this paper, an Extended Kalman Filter (EKF) based detection method is proposed for the detection of weak harmonic signal. The EKF method avoids matrix inversion by iterating measurement equation and state equation, which simultaneously improves the robustness and reduces the complexity. Compared with the existing detection methods, the proposed method has the following advantages: 1) it has better performance than the neural network method; 2) it has similar performance with the optimal filtering method, but with lower computational complexity; 3) it is more robust compared with the optimal filtering method.

## 1. Introduction

Many engineering problems can be attributed to the detection of weak harmonic signals from strong chaotic interference [1–10]. For example, the mechanical fault diagnosis in Refs. [1,2], the ECG abnormal signal detection in Refs. [3,4], and the ocean clutter weak signal detection in Refs. [5–8]. In these engineering problems, clutter can be regarded as strong chaotic interference, and the target signal is very weak. Therefore, detecting the weak harmonic signals from strong chaotic interference has attracted wide attention.

In order to detect weak harmonic signals from strong chaotic interference, many different detection methods have been developed [11–21]. These methods can be classified into two categories: the phase space reconstruction methods [11–17] and the optimal filter methods [18–21]. The phase space reconstruction methods explore different geometric characteristics of the harmonic signals and the chaotic signals in the phase space to detect the weak harmonic signals [11–17]. Among them, the neural network methods predict the chaotic interference signals, and then subtract the predicted chaotic interference signals from the received signals to detect the weak signal [12–17]. However, due to the dissipation of chaotic systems, errors frequently occur when using the neural network to predict the chaotic interference, and large deviations will occur in the process of detecting the weak harmonic signal. To solve

the problem, some optimal filter methods [18–21] are then proposed. The optimal filter method detects the weak harmonic signal by relying on the characteristic that the second-order statistics of the chaotic signals are stationary [22,23]. Nonetheless, the optimal filter methods have high computational complexity, so the practical application of these methods is limited. Furthermore, matrix inversion is unavoidable in the optimal filtering methods, resulting in instability.

In this paper, a detection method relying on the Extended Kalman Filter (EKF) is proposed to detect the weak harmonic signals under strong chaotic interference. As the properties of the second-order statistics of the chaotic signals are stationary, the proposed method transforms the problem of weak harmonic signal detection into a problem of minimum variance undistorted response. By iterations in state equation and measurement equation, the proposed EKF method can avoid matrix inversion and produce lower complexity. It outperforms the neural network method in Ref. [15] and has a similar performance as that of the optimal filter method in Ref. [20]. But while the optimal filter method in Ref. [20] has a complexity of  $O(M^3)$ , the proposed method only has a complexity of  $O(M^2)$ . Furthermore, it is more robust than the optimal filter method.

The remainder of this paper is organized as follows: the problem analysis is presented in Section 2; in Section 3, we derive our method and discuss its computational complexity; Section 4 presents the simulation

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results by comparing our EKF method to the neural network method in Ref. [15] and the optimal filter method in Ref. [20]. Finally, conclusions are drawn in Section 5.

## 2. System model

The detection of the weak harmonic signals from strong chaotic interference can be formulated as

$$\begin{aligned} H_1 : y_D(n) &= c_1(n) + s(n), \quad 1 < n \leq M \\ H_0 : y_R(n) &= c_0(n), \quad 1 < n \leq L \end{aligned} \quad (1)$$

where  $y_D(n)$  denotes the sequence to be detected,  $y_R(n)$  denotes the reference sequence, and  $M$  and  $L$  are the number of sampling points in  $y_D(n)$  and  $y_R(n)$  respectively.  $c_1(n)$  and  $c_0(n)$  are different strong chaotic interference signals, while  $s(n)$  is the weak harmonic signal. We have to detect the weak signal  $s(n)$  from  $y_D(n)$ , only knowing the chaotic interference signal  $c_0(n)$  in the reference sequence  $y_R(n)$ .

## 3. The EKF based detection method

### 3.1. Harmonic signal detection based on minimum variance

Since the second-order statistics of chaotic signals are stationary, we can use the reference sequence that only contains the chaotic interference signal to design a filter. The weak harmonic signal can pass the filter without distortion, and the chaotic interference signal can be suppressed as much as possible, which can be written as

$$\min_{\{w\}} E \|w c_1\|^2, \quad \text{st. } w^H s = 1 \quad (2)$$

where  $w$  is the filter coefficient,  $s$  is the steering vector for the weak harmonic signal.  $s(\omega_l) = [1, e^{-j\omega_l}, \dots, e^{-j\omega_l(M-1)}]^T$ ,  $\omega_l$  is the frequency of the weak harmonic signal.  $\min_{\{w\}} \|w c_1\|^2$  indicates that the filter can suppress the strong chaotic interference in  $y_D$ .  $w^H s = 1$  ensures that the weak harmonic signal in  $y_D$  can go through the filter without distortion.

It can also be written as  $E \|w c_1\|^2 = w^H R w$ , where  $R$  represents the covariance matrix of the chaotic interference signal. As the second-order statistics of the chaotic interference signal are stationary, we can use  $c_0$  to estimate the covariance matrix  $R$  of  $c_1$ . Construct the data matrix by reference sequence  $y_R$ : first, assume the length of reference sequence  $y_R$  is  $L$ ; then divide  $y_R$  into  $N$  parts, the  $i$ -th part being  $y_i$  and the length of sequence to be detected being  $M$  ( $M = L/N$ ); finally, we can estimate the covariance matrix  $R$  of  $y_D$  from  $y_R$ :

$$R = \frac{1}{N} \sum_{i=1}^N y_i y_i^H \quad (3)$$

Combining (3) with (2), we can obtain the weight vector  $w$ :

$$w = \frac{R^{-1} s}{s^H R s} \quad (4)$$

From (4), we can derive the output Signal to Interference plus Noise Ratio (SINR):

$$\text{SINR} = \frac{|w^H s|^2}{|w^H R w|} \quad (5)$$

### 3.2. The proposed robust weak harmonic signal detection method

In (2), the constraint condition  $w^H s = 1$  ensures the weak harmonic signal to pass the filter without distortion. But in practical applications, there may exist some unknown mismatches between the actual steering vector  $s$  and the presumed  $s$ , as indicated in

$$d = s + \delta \quad (6)$$

where  $\delta$  is the mismatches between the actual steering vector  $d$  and the presumed steering vector  $s$ ,  $\|\delta\| \leq \varepsilon$ . Therefore, the actual signal steering vector  $d$  belongs to the set:

$$\mathcal{A}(\varepsilon) \triangleq \{u = s + e, \|e\| \leq \varepsilon\} \quad (7)$$

In (7),  $u = d$ , when  $e = \delta$ . As  $\delta$  is unknown,  $d$  can be any vector in  $\mathcal{A}(\varepsilon)$ . So, we modify the constraint condition as follows:

$$|w^H u| \geq 1, \quad \forall u \in \mathcal{A}(\varepsilon) \quad (8)$$

Then (2) can be rewritten as

$$\min_{\{w\}} w^H \hat{R} w, \quad \text{st. } |w^H u| \geq 1, \quad u \in \mathcal{A}(\varepsilon) \quad (9)$$

As for the optimal problem described in (9), the actual steering vector  $d$  belongs to the set  $\mathcal{A}(\varepsilon)$ . Therefore, the constraint condition ensures that the target signal passes through the filter without distortion. However, the constraint condition transforms the optimization problem into a nonlinear and non-convex problem. The following is the derivation of the constraint condition for transforming a nonconvex problem into a convex problem: first, the constraint condition in (9) is equivalent to the following equation:

$$\min_{\{w\}} w^H \hat{R} w, \quad \text{st. } |w^H u| = |w^H s + w^H e| \geq 1 \quad (10)$$

By applying the Cauchy-Schwarz's inequality and using the condition  $\|e\| \leq \varepsilon$ , we obtain

$$|w^H s + w^H e| \geq |w^H s| - |w^H e| \geq |w^H s| - \varepsilon \|w\| \quad (11)$$

When  $\varepsilon$  is so small that  $|w^H s| > \varepsilon \|w\|$ , we have

$$|w^H s + w^H e| \geq |w^H s| - \varepsilon \|w\| \quad (12)$$

Then, (10) can be rewritten as

$$\min_{u \in \mathcal{A}(\varepsilon)} |w^H u| \geq |w^H s| - \varepsilon \|w\| \quad (13)$$

Furthermore, the optimization problem in (9) can be represented as

$$\min_{\{w\}} w^H \hat{R} w, \quad \text{st. } |w^H s| \geq \varepsilon \|w\| + 1 \quad (14)$$

Note that (14) simplifies the problem in (9) and makes it convex. But (14) is still a nonlinear problem.

## 4. The proposed EKF detection method

To solve the nonlinear problem in (14), we can utilize the EKF as a solution. For the objective equation  $w^H R w$  in an optimal filter, we can represent it as the Mean Square Error (MSE) between the output of the optimal filter and the zero signal, i.e.,

$$\text{MSE} = E[|0 - y^H w(n)|^2] = w^H R w \quad (15)$$

We can transform the optimization problem in (14) into a further step,

$$\min_w \text{MSE}, \quad \text{st. } h_2(w(n)) = 1 \quad (16)$$

where the constraint condition  $h_2(w(n)) = 1$  can be derived from (14) as

$$\begin{aligned} h_2(w(n)) &= \varepsilon^2 w^H(n) w(n) - w^H(n) s s^H w(n) \\ &\quad + w^H(n) s + s^H w(n) \\ &= 1 \end{aligned} \quad (17)$$

According to (16) and (17), we can obtain the state equation and

measurement equation of EKF. The weight vector of the filter can be modeled as a dynamic system, and it undergoes a first-order Markov process. Thus, the state equation can be written as

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mathbf{v}_s(n) \quad (18)$$

where  $\mathbf{v}_s(n)$  is the noise vector, assumed to be white Gaussian with zero mean and the covariance matrix  $\mathbf{Q} = \sigma_s^2 \mathbf{I}$ , and the subscript “s” refers to the state equation. From (17) and (18), we can establish the measurement equation as

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{y}^H \mathbf{w}(n) \\ h_2(\mathbf{w}(n)) \end{bmatrix} + \begin{bmatrix} v_1(n) \\ v_2(n) \end{bmatrix} \quad (19)$$

where  $v_1(n)$  and  $v_2(n)$  are the residual errors and the constraint errors, respectively. Minimizing  $v_1(n)$  also minimizes the output energy of the filter, but minimizing  $v_2(n)$  makes the target signal pass the filter with no distortion.  $v_1(n)$  and  $v_2(n)$  can be modeled by zero-mean independent white noise sequences with the covariance matrix

$$\mathbf{R}_g = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \quad (20)$$

(19) can also be written in the matrix format

$$\mathbf{z} = \mathbf{h}(\mathbf{w}(n)) + \mathbf{v}_m(n) \quad (21)$$

where the subscript “m” refers to the measurement equation. Due to the non-linearity of the measurement equation (21), the second-order EKF can be used to find a recursion for the estimated weight vector  $\mathbf{w}$ . Calculating the second-order Taylor series expanding at  $\mathbf{w}(n)$  in  $h_2(\cdot)$ , we get the Jacobian  $\mathbf{J}_w(n, \mathbf{w}(n))$

$$\begin{aligned} \mathbf{J}_w(n, \mathbf{w}(n)) &= \left\{ \nabla_{\mathbf{w}} \mathbf{h}^T(\mathbf{w}(n)) \right\}^T \\ &= \begin{bmatrix} \mathbf{y}^H \\ \varepsilon^2 \mathbf{w}^H(n) - (s s^H \mathbf{w}(n))^H + s^H \end{bmatrix} \end{aligned}$$

To take the derivative for  $\mathbf{J}_w(n, \mathbf{w}(n))$ , we can gain the first derivative matrix  $\mathbf{J}_{\mathbf{w}\mathbf{w}}^{(1)}$  and the second derivative matrix  $\mathbf{J}_{\mathbf{w}\mathbf{w}}^{(2)}$ , respectively.

$$\mathbf{J}_{\mathbf{w}\mathbf{w}}^{(1)} = \nabla_{\mathbf{w}} \nabla_{\mathbf{w}}^H \mathbf{y}^H \mathbf{w}(n) = 0 \quad (22)$$

$$\mathbf{J}_{\mathbf{w}\mathbf{w}}^{(2)} = \nabla_{\mathbf{w}} \nabla_{\mathbf{w}}^H h_2(\mathbf{w}(n)) = \varepsilon^2 \mathbf{I} - s s^H \quad (23)$$

The recursion for the estimated weight vector starts with an initial weight vector estimate  $\mathbf{w}(0)$  with the associated covariance matrix  $P(0|0)$  and updates the weight vector estimate as

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mathbf{K}(n)(\mathbf{z} - \hat{\mathbf{z}}(n|n-1)) \quad (24)$$

where the predicted measurement  $\hat{\mathbf{z}}(n|n-1)$  and the filter gain  $\mathbf{K}(n)$  are given by

$$\hat{\mathbf{z}}(n|n-1) = \begin{bmatrix} \mathbf{y}^H \mathbf{w}(n-1) \\ h_2(\mathbf{w}(n-1)) + \frac{1}{2} \text{Tr}\{\mathbf{J}_{\mathbf{w}\mathbf{w}}^{(2)} P(n|n-1)\} \end{bmatrix} \quad (25)$$

and

$$\mathbf{K}(n) = \mathbf{P}(n|n-1) \mathbf{J}_w^H(n, \mathbf{w}(n-1)) \mathbf{V}^{-1}(n) \quad (26)$$

Here, the innovation covariance  $\mathbf{V}(n)$  and the predicted weight vector covariance  $P(n|n-1)$  are given by

$$\begin{aligned} \mathbf{V}(n) &= \mathbf{J}_w(n, \mathbf{w}(n-1)) \mathbf{P}(n|n-1) \mathbf{J}_w^H(n, \mathbf{w}(n-1)) \\ &+ \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \text{Tr}\{\mathbf{J}_{\mathbf{w}\mathbf{w}}^{(2)} P(n|n-1) \mathbf{J}_{\mathbf{w}\mathbf{w}}^{(2)} P(n|n-1)\} \\ &+ \mathbf{R}_g \end{aligned}$$

and

$$\mathbf{P}(n|n-1) = \mathbf{P}(n-1|n-1) + \mathbf{Q} \quad (27)$$

The updated weight vector covariance and the predicted weight vector covariance can be expressed as

$$\begin{aligned} \mathbf{P}(n|n) &= [\mathbf{I} - \mathbf{K}(n) \mathbf{J}_w(n, \mathbf{w}(n-1))] \mathbf{P}(n|n-1) \\ &\times [\mathbf{I} - \mathbf{K}(n) \mathbf{J}_w(n, \mathbf{w})] (n-1)^H \\ &+ \mathbf{K}(n) \mathbf{R}_g \mathbf{K}^H(n) \end{aligned} \quad (28)$$

$$\mathbf{P}(n+1|n) = \mathbf{P}(n|n) + \mathbf{Q} \quad (29)$$

Substitute (26)–(31) into (25) by iteration until the weight vector converges, and then send it back to (5), we can obtain the output SINR.

### The computational complexity

The evaluation of the Jacobian matrix in (22) has the complexity of  $O(M^2)$ . The weight vector update (25) has the complexity of  $O(M)$ , and the complexity of the evaluation of the predicted measurement  $\hat{\mathbf{z}}(n|n-1)$  in (26) is  $O(M^2)$ . Additionally, the computation of the filter gain  $\mathbf{K}(n)$  in (27) has the complexity of  $O(M^2)$ . The innovation covariance matrix  $\mathbf{V}(n)$  can be simplified as

$$\begin{aligned} \mathbf{V}(n) &= \mathbf{J}_w(n, \mathbf{w}(n-1)) \mathbf{P}(n|n-1) \mathbf{J}_w^H(n, \mathbf{w}(n-1)) \\ &+ \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} (\varepsilon^4 \|P(n|n-1)\|_F^2 \\ &- 2\varepsilon^2 \|P(n|n-1)s\|^2 + |s^H P(n|n-1)s|^2) + \mathbf{R}_g \end{aligned} \quad (30)$$

which requires only  $O(M^2)$  computational complexity. The computational complexity of the updated weight vector covariance in (30) requires  $O(M^3)$  multiplications. However, this equation can be replaced by

$$\mathbf{P}(n|n) = \mathbf{P}(n|n-1) - \mathbf{K}(n) \mathbf{V}(n) \mathbf{K}^H(n) \quad (31)$$

whose complexity is  $O(M^2)$  as  $\mathbf{K}(n) \in \mathbb{C}^{M \times 2}$  and  $\mathbf{V}(n) \in \mathbb{C}^{M \times 2}$ . Therefore, we can draw a conclusion that the proposed EKF method has the computational complexity of  $O(M^2)$  per iteration, whereas the Second-Order Cone Programming (SOCP) in Ref. [24] has the complexity of  $O(M^3)$ .

## 5. Simulation results

We use the Lorenz system to generate the chaotic interference signal. The nonlinear state equations are as follows:

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = -xz + rx - y \\ \dot{z} = xy - bz \end{cases} \quad (32)$$

where  $\sigma = 10$ ,  $r = 28$ ,  $b = 8/3$ , the step length is set to 0.01, and the initial value is  $x_0 = y_0 = z_0 = 0.1$ . Randomly take 5 sections of chaotic series as the strong chaotic interference signals, with the length of each signal being 2000. Then use the weak harmonic signal in one of the chaotic sequences as the detection sequence, and use the other 4 sequences as the reference sequences. Let  $s(n) = \alpha e^{j2\pi f n}$  denote the weak harmonic signal, where the normalized frequency is  $f = 0.06$ , and the amplitude is  $\alpha = 0.05$ . The weak harmonic signal is detected by the fast EKF method, the neural network method in Ref. [15], and the SOCP in Ref. [20], respectively.

Fig. 1 shows the results of three methods in the case of SINR = -47.01 dB ( $\alpha = 0.05$ ). For the neural network method in Ref. [15] in Fig. 1 (a), the output power of the weak harmonic signal is only 7 dB higher than the maximum output power of the chaotic interference. The result of the SOCP method [20] and the result of the proposed fast EKF method are

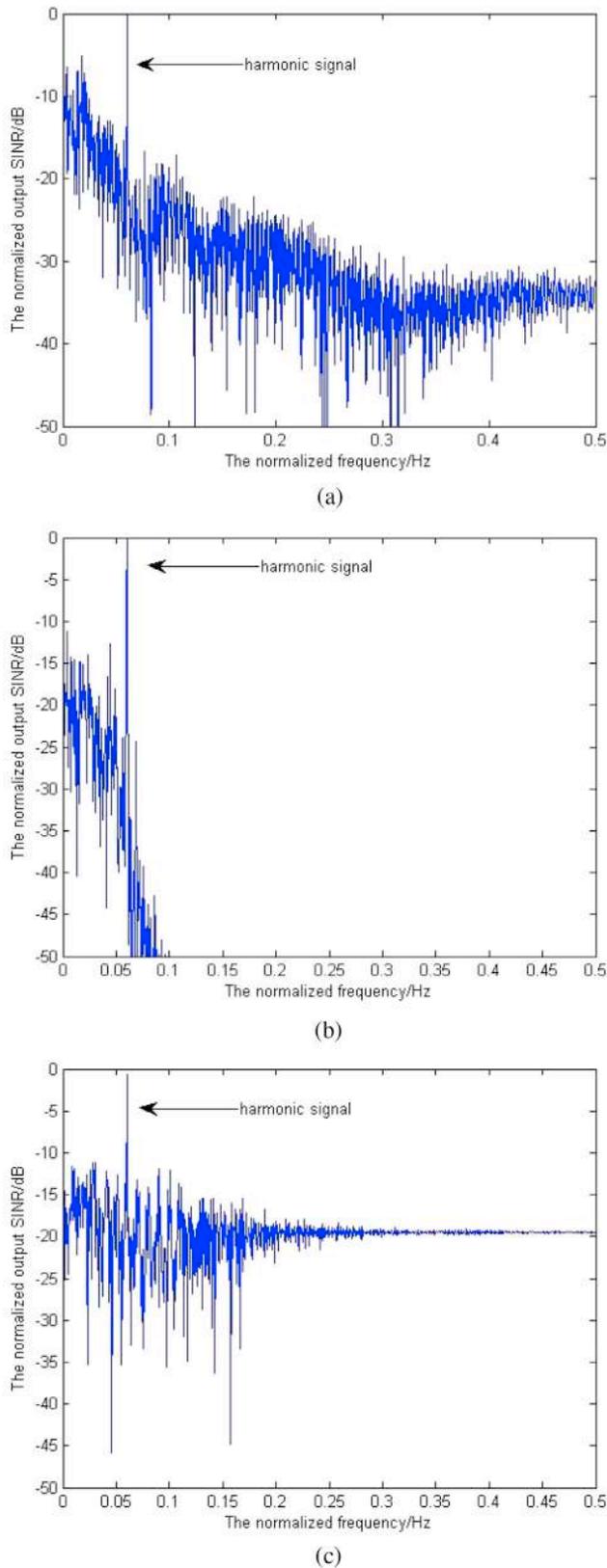


Fig. 1. The detection results of the neural network in Ref. [15], the SOCP in Ref. [20] and the fast EKF.

sho in Fig. 1 (b) and Fig. 1 (c), respectively. They show similar detection performance. The power of the weak harmonic signal with  $f = 0.06$  Hz is about 12 dB higher than the maximum output power of the chaotic interference in both the SOCP method and the proposed fast EKF method.

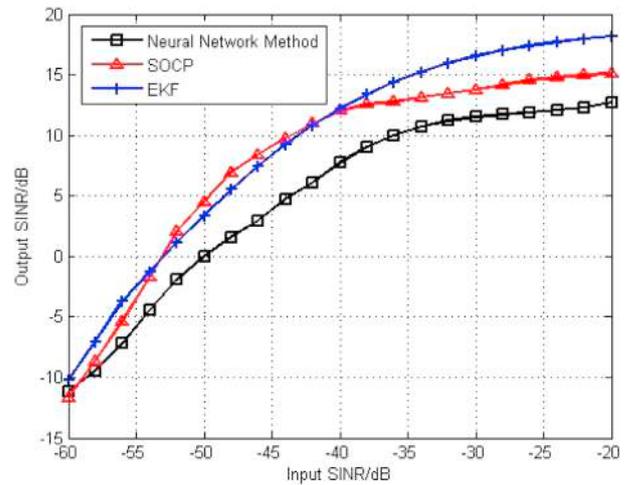


Fig. 2. Input SINR vs. output SINR.

As can be seen from Fig. 1, the proposed fast EKF method has similar detection performance with the SOCP method, but has better detection performance than the neural network method.

Define the ratio of the power of the weak harmonic signal to the maximum output power of chaotic interference as the output SINR. Fig. 2 compares the different SINR of the proposed fast EKF method with the neural network method in Ref. [15] and the SOCP method in Ref. [20], when the weak harmonic frequency is fixed at 0.06 Hz. Only when the output SINR is greater than zero, the weak harmonic signal can be detected accurately. It is shown in Fig. 2 that the proposed fast EKF method has a better detection performance than the neural network method. When the input SINR is below  $-40$  dB, the proposed fast EKF method has the similar detection performance with the SOCP method. When the input SINR is over  $-40$  dB, the proposed fast EKF method has a slightly better detection performance than the SOCP method. Additionally, the proposed fast EKF method also has a lower computational complexity than the SOCP method.

## 6. Conclusion

Good detection performance and low computational complexity are very important in practical applications, but most of the existing methods cannot simultaneously meet both requirements. In order to solve this problem, we propose a weak harmonic detection method based on EKF. The method obtains the optimal weight vector by iterations in the measurement equation and the state equation. The weak harmonic signal is accurately detected from the output SINR, and at the same time, the computation complexity is comparatively low.

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## References

- [1] Y. Haiying, C. Guangju, X. Yongle, Feature evaluation and extraction based on neural network in analog circuit fault diagnosis, *J. Syst. Eng. Electron.* 18 (2) (2007) 434–437.
- [2] B. Li, P. Zhang, Z. Wang, S. Mi, P. Liu, Morphological covering based generalized dimension for gear fault diagnosis, *Nonlinear Dynam.* 67 (4) (2012) 2561–2571.
- [3] G. Matchett, P. Wood, General anesthesia suppresses normal heart rate variability in humans, *Chaos, An Interdisciplinary Journal of Nonlinear Science* 24 (2) (2014) 023129.

- [4] S. Zhiqiang, X. Hongwe, S. Xiujian, L. Fengqiang, Engineering approach for human error probability quantification, *J. Syst. Eng. Electron.* 20 (5) (2009) 1144–1152.
- [5] Z. Gongjian, J. Yonggao, D. Huachun, Q. Taifan, HF radar sea clutter rejection by nonlinear projections, *J. Syst. Eng. Electron.* 16 (4) (2005) 733–737.
- [6] J. Guan, N.-B. Liu, Y. Huang, Y. He, Fractal characteristic in frequency domain for target detection within sea clutter, *IET Radar, Sonar & Navigation* 6 (5) (2012) 293–306.
- [7] S. Wentao, T. Ningning, W. Qiang, Weak targets detection research under sea clutter background, *J. Signal Process.* 30 (1).
- [8] L. Yang, Z. Ning, Y. Qiang, Characteristic-knowledge-aided spectral detection of high frequency first-order sea echo, *J. Syst. Eng. Electron.* 20 (4) (2009) 718–725.
- [9] X. Han, X. Chang, A chaotic digital secure communication based on a modified gravitational search algorithm filter, *Inf. Sci.* 208 (2012) 14–27.
- [10] L. Jian, Z. Jiliu, H. Kun, Q. Qiang, Detection of phase randomly distributed weak transient signal using chaos, *J. Syst. Eng. Electron.* 16 (3) (2005) 527–532.
- [11] H. Ren, D. Liu, Synchronization of chaos using radial basis functions neural networks, *J. Syst. Eng. Electron.* 18 (1) (2007) 83100–83188.
- [12] X. Hongyan, X. Wei, The neural networks method for detecting weak signals under chaotic background, *Acta Phys. Sin.* 7 (2007) 023.
- [13] G.W. Chang, C.-I. Chen, Y.-F. Teng, Radial-basis-function-based neural network for harmonic detection, *IEEE Trans. Ind. Electron.* 57 (6) (2010) 2171–2179.
- [14] L. Zilong, L. Guozhong, L. Jie, Adaptive tracking controller using bp neural networks for a class of nonlinear systems, *J. Syst. Eng. Electron.* 15 (4) (2004) 598–604.
- [15] Z. Hongli, X. Hongyan, X. Wei, Detection of weak signal embedded in chaotic background using echo state network, *J. Signal Process.* 31 (3).
- [16] H. Leung, N. Dubash, N. Xie, Detection of small objects in clutter using a GA-RBF neural network, *IEEE Trans. Aero. Electron. Syst.* 38 (1) (2002) 98–118.
- [17] D. Li, M. Han, J. Wang, Chaotic time series prediction based on a novel robust echo state network, *IEEE Transactions on Neural Networks and Learning Systems* 23 (5) (2012) 787–799.
- [18] I. Hoteit, D.-T. Pham, G. Triantafyllou, G. Korres, A new approximate solution of the optimal nonlinear filter for data assimilation in meteorology and oceanography, *Mon. Weather Rev.* 136 (1) (2008) 317–334.
- [19] H. Leung, T. Lo, S. Wang, Prediction of noisy chaotic time series using an optimal radial basis function neural network, *IEEE Trans. Neural Network.* 12 (5) (2001) 1163–1172.
- [20] Z. Shi, M. Han, Tikhonov-type regularization in local model for noisy chaotic time series prediction, in: 2007 46th IEEE Conference on Decision and Control, IEEE, 2007, pp. 2223–2228.
- [21] H. Jinfeng, Z. Yaxuan, L. Huiyong, Y. Miao, X. Wei, L. Jun, Harmonic signal detection method from strong chaotic background based on optimal filter, *Acta Phys. Sin.* 64 (22).
- [22] S.V. Gonchenko, J. Meiss, I. Ovsyannikov, Chaotic dynamics of three-dimensional Hénon maps that originate from a homoclinic bifurcation, *Regul. Chaotic Dyn.* 11 (2) (2006) 191–212.
- [23] R. Vali, S.M. Berber, S.K. Nguang, Analysis of chaos-based code tracking using chaotic correlation statistics, *IEEE Transactions on Circuits and Systems I: Regular Papers* 59 (4) (2012) 796–805.
- [24] S.A. Vorobyov, A.B. Gershman, Z. Luo, Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem, *IEEE Trans. Signal Process.* 51 (2) (2003) 313–324.